Embodied Minds and Dancing Brains: New Opportunities for Research in Mathematics Education

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Prelude: This chapter reports on an initiative in educational research in mathematics education that is augmenting traditional methods of educational research with methods of cognitive neuroscience and psychophysiology. Background and motivation are provided for this initiative—referred to here as mathematics educational neuroscience. Relations and differences between cognitive neuroscience and educational neuroscience are proposed that may have some bearing as to how this area unfolds. The key role of embodied cognition as a theoretical framework is discussed in some detail, and some methodological considerations are presented and illustrated as well. Overall, mathematics educational neuroscience presents exciting new opportunities for research in mathematics education and for educational research in general.

Introduction

There has been much research in mathematics education that has addressed a wide variety of affective, cognitive, and social issues (e.g., Grouws 1992), and there have been a breathtaking variety of phenomenological, anthropological, ethnographic, behavioral, cognitive, and social interactionist approaches taken toward understanding these issues (Sierpinska and Kilpatrick 1998). Over the years, there have also been a number of efforts to incorporate cognitive science and cognitive technologies into research in mathematics education (e.g., Davis 1984; Schoenfeld 1987; Pea 1987).

Until quite recently, however, there has been very little to be found in the mathematics education research literature exploring or drawing out some of the possible implications of neuroscience or cognitive neuroscience for mathematics education. Indeed, the term "neuroscience" is not to be found at all in the indexes of either of the following seminal publications: Grouws (1992); Sierpinska and Kilpatrick (1998). Perhaps more surprisingly, despite much hoopla over the 1990's being des-

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ignated as “the decade of the brain,” and an enthusiastic though often naïve “brain-based education” movement, there is no mention of that term (viz., “neuroscience”) whatsoever in the most recent mathematics education research compendium (viz., Gutiérrez and Boero 2006).

To date, then, as wide as the frameworks and perspectives are in mathematics education research, it appears many researchers remain largely unaware, uninterested, or uninformed by growing bodies of research into the nature and processes of mathematical cognition and learning, not only in cognitive psychology *per se* (e.g., Campbell 2004a), but especially in the areas of cognitive neuroscience (e.g., Dehaene 1997), and neurogenetics (e.g., Gordon and Hen 2004). Phenomena pertinent to mathematics education are being studied from perspectives that are tightly aligned with the neurosciences, or that are becoming much more so. Mathematics education will be well served as these disparate areas are integrated, focused, and extended to more effectively inform and expand upon research and practice in mathematics education. This chapter presents mathematics educational neuroscience as a new area of inquiry with that end in view, and, accordingly, considers new opportunities for mathematics education research.

I present this chapter in six sections. First, I address the question as to why, as educators and educational researchers, we should bother to concern ourselves with developments in, say, psychophysiology and cognitive neuroscience? In so doing, I summarize three common arguments posed by Byrnes (2001) as to why one might think that we need not bother at all, along with ways in which those arguments can be refuted. In the second section I provide additional rationale and brief overview as to why those of us concerned with new frontiers in mathematics education research *should* bother.

In the third section of this chapter I turn to discussing why cognitive neuroscience has emerged to be of such importance and potential relevance to educational research. In this section I also allude to some potentially fundamental differences between cognitive neuroscience and what could be taken as a bona fide *educational* neuroscience. Central to the view of educational neuroscience that I am interested in pursuing as an educational researcher is a radical view of embodiment and embodied cognition. Various views on embodiment are presented and what I see as a fundamental entailment of embodied cognition for educational neuroscience, constitute the main concerns of section four.

The last two sections of this chapter interweave and expand upon issues raised in the previous sections in an effort to bring *mathematics* educational neuroscience into better focus. There are a number of international initiatives currently underway, and, finally, beyond constructivism, there is a well-established advocacy and wide spread receptivity for matters concerning embodied cognition, especially in Canada, Along with these various considerations, there remain important methodological and technical issues and challenges to confront and address in bringing mathematics educational neuroscience out of the realms of conceptualization and into the mainstream of educational research, and subsequently still, to eventually better inform educational practice.

There are many aspects and numerous concerns regarding the relevance of the neurosciences to education. One might ask: what does the brain have to do with learning? And one might glibly answer: try learning without one! This question, however, is not as naïve as one might think. It can be asked quite rightly in the spirit of wondering how, or in what ways, knowledge of neurons, synapses, and calcium channels could possibly inform the teaching of traditional topics found in the K-12 curriculum. There is a grand gulf to bridge between education and the neurosciences. The author sympathizes with such concerns and understands that healthy scepticism is warranted, but also requests the reader bear an open mind. In this chapter I restrict my considerations to helping make a case for educational neuroscience as an important new area of educational research replete with new opportunities. The case presented herein is intended to be suggestive, not definitive; illustrative, not comprehensive; and generative, not final.

First: Why Bother?

Prima facie, educational neuroscience requires that educational researchers and practitioners engage the neurosciences in some manner or another. Some practitioners, recognizing the importance and value of doing so, and with little or no well-established alternatives, have in various degrees bought into various claims of what has come to be known as the “brain-based education” movement. There is a certain appeal and there are typically grains of truth to these claims, insofar as staying hydrated and eating well contributes to healthy brains. More critically minded educators, however, recognize that there is a grand gulf between neurons, studied in terms of physiological mechanisms, and children, approached in more edifying psychological terms, with the inherent rights and dignity typically bestowed upon human beings. Given the span of this grand gulf, the question inevitably arises: Why bother working toward bridging this gap? Is it, truly, still a “bridge too far” (Bruer 1997). Byrnes (2001) posits, and then endeavours to refute, three common arguments *against* the relevance of brain research to the psychology of cognition and learning.

Byrnes’s first argument pertains to the computer analogy, whereby brain is identified with hardware, and mind is identified with software. Accordingly, as educational researchers interested in matters of mind, we can restrict our consideration to the software/mind, independently of the hardware/brain. Byrnes then counter-argues that this computational view is “anti-biological.” Embodied views of cognition and learning, however, are becoming more widely accepted, and these views entail *biological* foundations (discussed in more detail below). Byrnes notes further that interdependencies between computer software and hardware are much greater than commonly supposed.

Byrnes’s second argument against the relevance of neuroscience to psychology, and education more generally, is that these two areas address different levels of analysis, and as such, they provide very different answers to the same questions. He illustrates this argument through the different kinds of answers that a physicist, physiologist, and psychologist attending a baseball game might provide to the question “Why did [the pitcher] throw a curve ball?” Educational researchers are typically loath to reduce psychological questions to matters of physiology, let alone biology, chemistry, or physics. Byrnes, in refuting this argument, suggests there are important insights to be gained from studies seeking understandings of *interfaces* between different levels of analysis, and *especially* between psychology and physiology in particular. One need not be a positivist or a reductionist to maintain that such interfaces must interrelate and interact in coherent ways.

The third common argument Byrnes posits for ignoring relationships between psychology and physiology, and neurophysiology in particular, is that too little is known about the brain at this point, and as brain science is in such flux, psychologists should just "forge ahead alone." Byrnes, once again in refutation, quite rightly emphasises that psychology and the neurosciences have much to offer each other. This is a key point: psychologists, and educational psychologists in particular, have cognitive and psychometric models that can help guide physiological investigations, and reciprocally, the results of those investigations can help substantiate and refine models of cognition and learning developed by educational psychologists.

Indeed, to paraphrase Byrne in further regard to his third counterargument, collaborations between cognitive psychologists and neuroscientists have been "forging ahead together," resulting in the vibrant and rapidly expanding new field of cognitive neuroscience. Cognitive neuroscientists are deeply engaged in connecting cognitive function and performance with brain and brain behaviour. Education will likely benefit greatly from these developments, and it seems untoward for educational researchers to simply stand on the sidelines.

Much has been gained from qualitative research in mathematics education, and one must not neglect the value of such research in improving the depth, if not the scope, of our understanding of mathematical cognition and learning. Having said that, however, when protocols and data obtained from qualitative research into mathematical cognition and learning consist of "think-aloud" reports, and our cognitive models of learners' thinking remain essentially behavioral, analytical, or speculative in nature (e.g., Campbell and Zazkis 2002; Zazkis and Campbell 2006), one must ask: how accurate, general, and robust can qualitative research into subjective mentalities ultimately prove to be? Methods and tools of cognitive neuroscience, some of which are illustrated below, are becoming more accessible to educational researchers to address these kinds of concerns.

Second: Some Preliminary Rationale

It is well known that many learners, and perhaps most especially, those who aspire to teach elementary school children, are deeply afflicted by or are at least prone to mathematics-related anxieties. For those who have experienced such anxieties, their deeply embodied nature has been self-evident. On the other hand, the delight of mathematical comprehension exemplified by 'aha' moments serves as a clear emotional counterpoint to math anxiety. These are also deeply embodied experiences. Some embodied behaviours are more overt than others. That is to say, some are readily observable as vocalizations, facial expressions, and other changes in musculature, especially those associated with movements of head and limbs. Other embodied aspects of subjective experience are more subtle, hence, more difficult to observe. These aspects of embodied behaviour are typified by physiological changes that are constantly occurring within the human body.

Grounded in the limbic system of the brain, embodied manifestations of emotions such as anxiety are most readily evident in physiological changes in organs of the body connected to the brain through the peripheral nervous system, especially the skin, heart, and lungs. Embodied manifestations of anxiety include changes in skin conductance, cardiovascular activity, and respiratory difficulties ranging from breathlessness and shallow breathing to hyperventilation. Closely connected with embodied emotional responses are changes in brain behaviour associated with various cognitive functions, such as perception, memory, learning, creativity, reasoning, and so on. The embodied manifestations of human cognition within the human brain, most notably, the neocortex, are evident though the collective activities of neural assemblages. Brain behaviour has become more transparent through recent advances in brain imaging technologies, which can reveal brain state fluctuations that can be reliably correlated with various aspects of affective and cognitive function.

Methods for studying brain and body behaviour, such as EEG and EKG have traditionally fallen under the purview of cognitive neuroscience and psychophysiology. These disciplines focus on identifying brain and body mechanisms underlying cognition and affect. Recently, however, methods such as EEG and EKG have also been brought to bear in educational research in an initiative that is coming to be known as educational neuroscience (see below).

The fundamental presupposition of educational neuroscience as considered herein is that human cognition is embodied cognition. That is to say, every subjective sensation, memory, thought, and emotion—anything at all that any human being can ever experience—is *in principle* enacted in some objective, observable, way as embodied behaviour. Although all embodied behaviours are ‘part and parcel’ of the on-going subjective flux of lived experience, beyond the empirical study of overt behaviour, deeper insight into cognition and learning warrant measurements, analyses, and interpretations of physiological changes with methods like EEG and EKG.

General research questions for educational neuroscience that could be addressed include assessing and critiquing the effectiveness and implications of neuropharmalogical drugs, or nootropics, in both abnormal and normal populations. One way of conceiving *mathematics* educational neuroscience would be to identify and assess interrelations between mathematics-related anxieties and mathematical understanding in teachers and learners of mathematics. Research questions here would include: what ways and to what extent do mathematics-related anxieties impede mathematical understanding; and, conversely, in what ways and to what extent can better understandings mitigate mathematics-related anxieties. More specific questions include: what kinds and to what extent do positive and negative emotions promote or impede various aspects of engagement, reasoning, and performance in mathematical problem-solving activities.

Various manners in which these questions can be unpacked and put even more specifically constitute a program of research that will likely involve many years of study—perhaps decades. There are many other questions that will need to be addressed along the way. Consider, for example: is it possible to discern the manner and extent to which learners are attending to a visual stimuli or reflecting on (i.e., thinking about) that stimuli at any given moment; is it possible to discern the manner and extent to which participants are attending and/or thinking spatially or symbolically. Applying tools that are becoming more readily available to educational researchers for observing and recording various aspects of brain and body behaviour, most notably, perhaps, electroencephalography (EEG), electrocardiography (EKG), and eye-tracking (ET), coupled with audiovisual recordings (AV), can be very generative of such questions.

Third: Cognitive and Educational Neuroscience

Cognitive psychologists, computer scientists and neuroscientists, psychophysiologists, geneticists, and others, have been making remarkable advances in understanding mental function, brain structure, and physiological behaviour. Furthermore, substantial progress is being made in understanding of the relations between these traditionally diverse and separate realms of disciplined inquiry (Gazzaniga 2004). These interdisciplinary efforts in cognitive neuroscience are being fuelled by an increasing knowledge base from lesion studies and technological advances in brain imaging.

Brain lesions, i.e., neural damage, can result in various ways from developmental abnormalities, impact injuries, surgery, strokes, or disease. Lesions, be they local or widespread within the brain, typically result in altered or compromised mental functioning of those who suffer them. Lesions can have rather bizarre implications for cognitive function, some of which have been widely popularised by authors such as Oliver Sacks (e.g., 1990, 1995). Yet, in many cases, the mental life of those with brain lesions can be remarkably robust and quite adaptable as well (e.g., Sacks 1989). The bottom line here is that there is a broad and multidimensional range of correlations between local and widespread damage to neural assemblies with specific and general aspects of mental functioning. Although technological innovations in brain imaging are providing new insights, neuroscientists are working hard to identify various mechanisms behind such correlations, and some psychologists remain at odds with neuroscientists (e.g., Uttal 2001), and some neuroscientists at odds amongst themselves (e.g., Cohen and Tong 2001), regarding various assumptions about the nature of those correlations.

Nevertheless, brain imaging techniques have opened new windows on brain structure and brain behaviour. From hemodynamic (blood mediated) techniques such as functional magnetic resonance imaging (fMRI) and positron emission tomography (PET), to electromagnetic techniques such as magnetoencephalography (MEG) and electroencephalography (EEG), significant strides are being made in our understandings of correlates between brain anatomy, brain behaviour, and mental function (Gazzaniga 2004). Of particular interest here, as shall become more evident below, are brain oscillations in human cortex, which are closely, if not causally, associated with mental phenomena characteristic of mathematical thinking ranging from profound insight to deep aversion. Such oscillations are readily detectable using EEG, within certain constrained experimental conditions and thresholds of signal and noise.

Concerned as it is with psychological, computational, neuroscientific, and genetic bases of cognition, cognitive neuroscience is now recognized as a well-established interdisciplinary field of study with its own society and annual meetings. Indeed, the Cognitive Neuroscience Society (CNS) presents itself on the welcome page of its website as "a network of scientists and scholars working at the interface of mind, brain, and behavior research" (CNS 2007). As such, it would seem, then, that cognitive neuroscientists share many areas of common interest with educational researchers, especially with regard to educational psychology and psychometrics.

Yet, on the same web page, the CNS also sees its members as "engaged in research focused on elucidating the biological underpinnings of mental processes" (ibid.), thereby suggesting that their approach may be more foundationally reductionist than interactionist in nature. Educational researchers such as myself want to be informed by biological mechanisms and processes underlying learning, and we also want to have access to the methods of cognitive neuroscience. As an educational researcher, however, my *primary* focus is not on the biological mechanisms and processes underlying or associated with cognition and learning. Rather, it is on the lived experiences of teaching and learning, along with the situational contexts and outcomes of those experiences.

Cognitive neuroscience, approached from a "hard" scientific orientation, has the luxury of focusing on various aspects of brain behavior in terms of neural structure, mechanisms, processes, and functions. On the other hand, neuroscience approached from a more humanistic orientation would have the luxury of not having to be concerned with trying to explain, or explain away, the lived experience of learners solely in terms of biological mechanisms or computational processes underlying brain behavior.

The above considerations suggest the possibility of a more humanist-oriented *educational* neuroscience, as a new area of *educational* research that is both informed by the results of cognitive neuroscience, and has access to the methods of cognitive neuroscience, specifically conscripted for the purposes of educational research into the lived experiences of embodied cognition and learning. As such, educational neuroscience could be portrayed as more akin to a full-fledged neurophenomenology (cf., Varela 1996; Varela and Shear 1999; Lutz and Thompson 2003). On the other hand, educational neuroscience can also be viewed as an applied cognitive neuroscience, insofar as the tools, methods, and predominantly mechanistic and functionalist frameworks of cognitive neuroscience are applied to educational problems.

Either way, educational neuroscience will likely prove to be a foundational new area of educational research. Indeed, a general consensus is emerging on two basic points. First, educational neuroscience should be characterized by soundly reasoned and evidence-based research into ways in which the neurosciences can inform educational practice, and, importantly, also vice versa. Secondly, educational research in cognitive psychology informed by, and informing, cognitive neuroscience should constitute the core of educational neuroscience (cf., e.g., Berninger and Corina 1998; Bruer 1997; Geake and Cooper 2003). New centres and labs toward this end have recently opened in England www.educ.cam.ac.uk/neuroscience/index.htm, Germany www.znl-ulm.de, the U.S. www.dartmouth.edu/~numcog,[1] Canada www.engrammetron.net, and elsewhere. This appears to be a very timely development, as there has been increasing interest and emphasis on informing educational practice and policy making through advances in the neurosciences (NRC 2000; OECD 2002), along with increasing concern that much educational research, especially of the qualitative ilk, is lacking in a scientific "evidence-based" foundation (NRC 2002).

Fourth: Embodied Cognition

As part of a general shift in emphasis from concerns with curriculum to concerns with learners and learning, constructivism, despite its various versions (Phillips 1995), has held sway in education and educational research for most of the past three decades. At least this is the case in mathematics education, especially with the pioneering efforts of Ernst von Glasersfeld, Les Steffe, and Paul Cobb (e.g., von Glasersfeld 1991; Steffe et al. 1983).

With learners and learning comprising major foci of educational research, the initial cognitivist emphases of Piagetian-inspired constructivists like von Glasersfeld (1982) has come to be augmented by research concerned with the social and economic environments within which learning takes place, with concomitant emphases placed on the roles of language and communication (e.g., Kirshner and Whitson 1997; Sfard 2008; Wertsch 1991).

Over the past decade, enactivist notions of embodied cognition have also entered into the mainstream in mathematics education research in Canada (e.g., Brown and Reid 2006; Campbell 2002a, 2002b; Campbell and Dawson 1995; Campbell and Handscomb 2007, April; Davis 1995a, 1995b; Davis and Sumara 2007; Gerofsky and Gobel 2007; Hackenberg and Sinclair 2007; Kieren 2004; Kieren and Simmt 2001; Reid 1996; Reid and Drodge 2000; Simmt and Kieren 2000). Enactivist views need not supplant constructivist views, whether they be cognitivist or situativist in orientation. Rather, admitting the embodiment of lived experience affirms the biological and ecological ground of cognition, recognising body as a situational locus, and ecology as a broader context.

Educational neuroscience, conceived less as an applied cognitive neuroscience, and more as a transdisciplinary enterprise, may provide an opportunity to set aside foundational dualisms that have traditionally served to undermine unified studies of subjective human experience and objectively observable behavior. In order to

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[1] Daniel Ansari has relocated his lab http://psychology.uwo.ca/faculty/ansari_res.htm.

do so educational neuroscience could adhere to maxims that: (1) embed mind in body (with a special emphasis on brain); (2) situate embodied minds within human cultures; and (3) recognize the biological emergence of humanity from within and our dependence on the natural world (e.g., Merleau-Ponty 1962, 1968; Varela et al. 1991; Campbell and Dawson 1995; Núñez et al. 1999).

To help illustrate the unifying power of this embodied view with regard to *mathematics* educational neuroscience, consider Eugene Wigner's renown reflections on the "unreasonable effectiveness of mathematics in the natural sciences" (1960). If mind (*res cogitans*) is fundamentally (i.e., ontologically) distinct from the material world (*res extensa*), it remains a great grand mystery as to why mathematics can be *applied* to the world so effectively. If mind is embedded within the material world, as the embodied view entails, mystery dissolves into expectation (Campbell 2001). Moreover, in considering the *embodied mind* as a unified ontological primitive, there is no need to treat consciousness as an erstwhile inexplicable and apparently useless epiphenomenon, supervening upon mechanical neural processes (e.g., Jackendoff 1987). A radical implication of embodied cognition is that first person lived experiences of learners partake and manifest in third person observable structures and processes (Campbell 2001, 2002a, 2003b; Campbell and Handscomb 2007, April).

Embodied cognition has largely come to the fore in mathematics education research since the seminal publication of Francisco Varela et al.'s (1991) *The Embodied Mind: Cognitive Science and Human Experience*. I have written on this work in much detail elsewhere (e.g., Campbell 1993; Campbell and Dawson 1995), and most of my scholarship and research since has been oriented toward delving more deeply into the origins, assumptions, and implications of this view (e.g., Campbell 1998, 2001, 2003b; Campbell and the ENL Group 2007).

Not surprisingly, the notion of embodiment can be viewed in a variety of different ways. Embodiment can be considered in terms of concrete particulars. For instance, a chalk stroke on a blackboard can be considered as a concrete embodiment of the concept of a line, or a marble can be considered the embodiment of a sphere. This view of embodiment is very much akin to the Platonic view, whereby concrete particulars are mere shadows of ideas, which have transcendent existence of their own. Embodiment can also be viewed as being akin to the Aristotelian view, where ideas are somehow embodied, *qua* immanent, *within* concrete particulars. In this view, mathematical manipulatives, popular in mathematics education, embody mathematical ideas.

Another view of embodiment widely propounded by Lakoff and colleagues Lakoff and Johnson 1999; Lakoff and Núñez 2000, turns the Platonic and Aristotelian views upside down. For these thinkers, bodies are prior to ideas, rather than ideas being prior to embodiments, and ideas are primarily grounded upon metaphors of embodiment. Turning ideas upside down to make a point like this, for instance; or the embodied activity of blazing a trail in the forest to serve as a metaphor for a number line; similarly, the concept of a limit considered as the embodied experience of approaching an obstacle. Another view along these lines is Egan's notion of *somatic understanding* coupled with the notion of *binary opposites* (Egan 1997). In Egan's view, for instance, notions like big and small, tall and short, hot and cold, and so on, have their grounding in somatic understandings that things are variously ‘bigger than my body,’ ‘smaller than my body,’ ‘taller than my body,’ and so on.

The notion of embodiment underlying educational neuroscience can be viewed in fundamental ontological terms of both being *of* and being *in* the world (Campbell 2002a, 2002b; Campbell and Handscomb 2007, April). That is to say, in this non-dualist view, the world is within us, in an idealist sense, and we are within the world, in a realist sense. Epistemologically, our experience of and within the world is empirically grounded. Our rational reflections upon the world, however, are not arbitrary constructs. As we are of the world and to the extent that the world is within us, our reflections are nothing less than *that part of the world that we are* reflecting upon itself. These abilities result from an on-going history of *structural coupling* and *co-emergence* with and within the world (Varela et al. 1991). This scientific-phenomenological, or perhaps more rightly, humanistic, view of embodiment carries with it a fundamental implication. That is, changes in subjective experience, be they sensory, emotive, or intellectual, must objectively manifest in some way through embodied action, i.e., overt behaviour, including brain/body behaviours that have been difficult or impossible to observe with methods traditionally available to educational researchers.

It may be helpful, at this point, to briefly compare this humanistic notion of embodiment with orthodox neuroscientific and religious views of embodiment. With regard to neuroscientific views of embodiment, cognitive functions are mediated through sensorimotor activity and neuronal mechanisms. The radical view of embodiment suggested herein is consistent with this view, but differs in rejecting strict behaviouralist and material reductionist views that forego broader considerations of mind, such as the subjectivity of lived experience, agency, and the exercise of volition. On the other hand, to connect mind and body as one thing might be viewed as heretical in some religious circles. Embodiment, for those so concerned, does not seem patently inconsistent with perennial theological beliefs such as resurrection and reincarnation.

In accord with this naturalistic embodied, situated, and emergent view, when meaning is constructed, transformations are postulated to take place in minds that are manifest through bodies (especially through changes in brain and brain behaviour). It remains possible, of course, that such embodied—viz., objectively observable and measurable—manifestations of mind, remain but shadows of subjectivity, analogous in some sense to the way in which the exterior of an extensible object is but a external manifestation of that object’s interior. Scratch away at the surface of an extensible object as much as one might, some aspects of the interior always remains “hidden.” The bottom line here is that brain and brain behaviour are made progressively more manifest to investigation through close observation and study of embodied action and social interaction, be they in clinical, classroom, or ecological contexts. In accord with this view, with advances in brain imaging, the shadows of mind are becoming much sharper.

Embodied cognition provides mathematics educational neuroscience with a common perspective from which the lived subjective *experience* of mind is postulated to be manifest in objectively observable aspects of embodied *actions* and *behaviour*. Adopting such a framework could enable educational neuroscience to become a bona fide transdisciplinary inquiry (Gibbons et al. 1994), in that it has the potential to integrate and to extend well beyond traditional ontologically disjoint frameworks, be they solely of mind (i.e., phenomenology), brain (i.e., neuroscience), function (i.e., functionalism), or behavior (i.e., behaviorism). Moreover, there is no need to attempt to reduce mind to brain (physicalism), or brain to mind (idealism). An embodied perspective keeps learners in mind, and in body.

In sum, people objectively manifest subjective experiences of thinking and learning in many ways. Beyond the more overt behaviours, demographics, and self-reports that have traditionally comprised the spectra of data in educational research, there are neurological and physiological activities connected with thinking and learning for educational researchers to observe and account for. According to the theoretical framework of embodiment underlying educational neuroscience, observations, measurements, and analyses of physiological activities associated with brain and body behaviour can provide insights into lived subjective experiences pertaining to cognition and learning in general, and mathematical thinking in particular. The challenge is to seek out and identify such embodiments.

Fifth: Toward Defining Mathematics Educational Neuroscience

Brain research is relevant to the field of psychology and education to the extent that it fosters better understandings of mind, development and learning (Byrnes 2001). The validity, reliability, and relevance of theories of teaching and learning in education research may variously be corroborated, refined, or refuted through neuroscientific studies or the use of neuroscientific tools and methods to test hypotheses of any particular theoretical account.

With recent advances in brain-imaging and eye-tracking technologies, there has been a strong emergence of interest regarding the role of neuroscience in informing education and vice versa (e.g., Blakemore and Frith 2005; Byrnes 2001; Geake 2005; Goswami 2004; Lee 2003; McCandliss et al. 2003), and the same holds true regarding mathematics education (e.g., Campbell 2005a; Iannece et al. 2006). Initiatives seeking to forge links between these two very broad fields of research have been falling under the general rubric of educational neuroscience (e.g., Campbell 2005a, 2005b; Varma et al. 2008).

Recent initiatives in *mathematics* educational neuroscience have been cultivated in part by research in cognitive psychology (e.g., Campbell 2004a, 2004b), and cognitive neuroscience research in mathematical cognition and learning (Dehaene 1997; Butterworth 1999). There have also been some cognitive psychologists and cognitive neuroscientists (e.g., Ansari and Dhital 2006; Szücs and Csépe 2004), and some educational researchers who are applying methods of cognitive neuroscience to mathematics education research (e.g., Campbell 2006b, 2006c; Campbell and the ENL Group 2007; Lee et al. 2007; Liu et al. 2006; van Nes and de Lange 2007; van Nes and Gebuis 2006).

Mathematics educational neuroscience has the potential to become an important, if not revolutionary, new area of research in mathematics education (Campbell

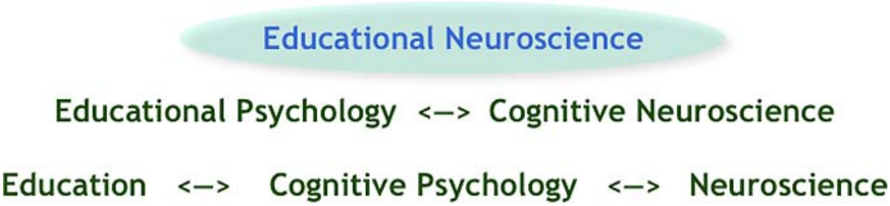

Fig. 1 Interdisciplinary progressions (after Campbell and the ENL Group 2007)

2005a, 2005b, 2006b, 2006c, 2008a, 2008b; Campbell and the ENL Group 2007; Campbell et al. 2009a, 2009b; Shipulina et al. 2009). As we have seen above, a fundamental implication of embodied cognition, radically conceived, is that changes in lived experience will manifest through changes in bodily state in various ways, some quite obvious and others more subtle, and many in between. A major task of mathematics educational neuroscience is to help investigate and establish such connections, thereby providing more evidence-based ground to the research in mathematics education. It follows that augmenting mathematics education research with physiological data sets like eye-tracking, pupillary response, electroencephalography, electrocardiography, skin response, respiration rates, and so on, can provide deeper and better understandings of the psychological aspects of teaching and learning mathematics. At a very basic level, it would be a significant advance in mathematics education research to have evidence-based measures that could reliably and practically distinguish amongst, say, various aspects of perception, reasoning, and understanding.

More generally, educational neuroscience is viewed here primarily as a new area of *educational research*, perhaps not so much in terms of building a bridge between neuroscience and education, but rather, as helping fill a gap between these vast areas. As discussed above, given that cognitive psychology provides the most natural connection between education and neuroscience, and given that educational neuroscience should be viewed as and strive to be something more than applied cognitive neuroscience, the following progression of interdisciplinary fields suggests itself (Fig. 1).

Educational neuroscience should, so it seems to me, prioritise learners' lived experience in relation to cognitive function over the neural mechanisms underlying them. That is, it should be informed by, but not geared toward identifying neural mechanisms underlying and accounting for cognitive function and behaviour—which is quite rightfully the task of *cognitive* neuroscience.

Despite many similarities and overlaps between educational and cognitive neuroscience, some fundamental differences can be exemplified by the latter's quandaries regarding the function of consciousness and how it arises from, and even how it can possibly arise from the activity of neural mechanisms. Educational neuroscience, in contrast, can take the lived reality and unity of consciousness as given (cf., Kant 1933/1787), as a place to start from and work with, and, as noted above, not something to explain, or to explain away. Furthermore, with the exception of research in special education, with their foci on various learning disabilities, educational neuroscience may quite reasonably assume that learners' lived experiences are unified experiences. That is to say, the so-called "binding" problem remains a foundational problem for cognitive neuroscience, not for educational neuroscience.

What, then, would foundational problems of a transdisciplinary educational neuroscience look like? Given the entailments of embodiment, such that changes in lived experience are manifest in brain, body, and behaviour in some way, one such problem concerns just what these "manifests" are, and to what extent are they shared amongst learners. This problem is akin to the problem of "correlates" between cognitive function, brain, and brain behaviour in cognitive neuroscience. In fact, these two problems can be seen as one and the same, viewed from different philosophical frameworks.

Naturally, to the extent that educational neuroscience makes use of the tools and methods of cognitive neuroscience, there will be many shared methodological concerns, and many of those of a technical nature that require expertise from fields such as physics, electrical engineering, mathematics, and philosophy to help resolve. Perhaps foremost in this regard concerns the mathematical modelling underlying all brain imaging methods. What makes this problem particularly salient is that mathematical modelling harbours perhaps the most well established and entrenched of dualist views, based as it is on the notion that mathematical idealisations (*res cogitans*) model real world applications (*res extensa*). From an embodied perspective, the relationships between mathematical thinking and the world in which we live, through which that very mode of thinking has emerged, may be much more profoundly intimate (Campbell 2001, 2002a, 2003b).

The aforementioned comments regarding embodied manifestations of lived experience and allusions to non-dualist reconceptualizations of mathematical modelling are offered in a provocative spirit of challenging, though *not* rejecting, some commonly accepted assumptions about science and mathematics. Educational neuroscience can draw upon accomplishments of cognitive neuroscience while simultaneously investigating the radical empirical ground of lived experience. After all, it seems more appropriate for educators to ask "What are learners/teachers experiencing and doing when learning/teaching?" than it is to ask, "What brain mechanisms are giving rise to learning/teaching behaviours?" Educational researchers should not relinquish this humanistic orientation, even with educational neuroscience conceived as an applied cognitive neuroscience. With an embodied view of (mathematical) thinking, I have been suggesting, (mathematics) educational neuroscience can have the best of both worlds.

It is reasonable for a sceptic to ask: Why bother with the notion of lived experience if it makes no difference whatsoever in the pursuit of science to leave it out? For those who recall the emergence of cognitive psychology from radical behaviourism, this objection should carry a familiar ring. Reducing lived experience to cognitive function has served to eliminate such troublesome subjectivities, and has ensured that mechanistic scientific assumptions remain intact. Even such patently humanistic activities as goal formation and the exercise of choice can be viewed in purely functionalist terms. Machines can be built on these cognitive models, and they can work just fine, absent any semblance of lived experience. The shadows of mind are becoming sharper in more ways than one. What of mind itself? Do we dispense with the very experiential ground through which *our* thinking is rendered? And even if we can, should we?

Beyond philosophical considerations, there are methodological challenges in isolating brain activity with electroencephalography (EEG), and in integrating EEG with psychophysiological data sets such as electrooculography (EOG), for measuring eye movements, and eye-tracking (ET) with data sets more familiar to educational researchers, like audiovisual (AV) recordings. What is involved in integrating these methods in ways that can bring educational research into the 21st century?

Sixth: New Questions and New Tools

Incorporating tools and methods of psychophysiology and cognitive neuroscience can provide researchers in mathematics education with new questions pertaining to investigations into teaching and learning mathematics. Consider, for example, what kinds of detectable, measurable, and recordable psychophysiological manifestations may be evident in learners' minds and bodies during mathematical concept formation—that is, when various mental happenings coalesce into pseudo or bona fide understandings of some aspect of mathematics. For instance, what observable and measurable changes in brain activity associated with and indicative of concept formation, hypothesis generation, or moments of insight might be detectible using electroencephalography (EEG)? How might eye-tracking technology, electrocardiography (EKG), and galvanic skin response (GSR) help to observe and measure responses to task engagement, cognitive load, or anxiety reactions. Capturing embodied manifestations of learners' cognitive and affective processes and states can provide rich and important insights into learners' experiences and behavior and afford exciting new venues for research in mathematics education. Figure 2, for instance, illustrates the rich data sets that are now possible to acquire using these kinds of methods to augment traditional educational research methods. Here, a participant in my lab is being observed in a mathematical study while his eye movements are being tracked (overlaying the stimulus in blue), and his brain waves recorded.

These data are integrated in a time synchronous manner enabling simultaneous playback and playforward in a step-by-step, frame-by-frame, manner, or at a variety of speeds, incorporating coding and a variety of qualitative and quantitative analysis methods. It is not the purpose of this chapter to present one particular study or another. It should suffice to point out that seeking new behavioural patterns in data such as these is to the traditional educational research methods based on audiovisual recordings as audiovisual recordings were to the traditional educational research methods based on field notes. In examining these data, it should be evident that what is of primary interest is to search for or identify various signatures or correlates of cognition and affect that are embodied and made manifest in teaching and learning. Understanding and grounding these manifestations in various brain mechanisms,

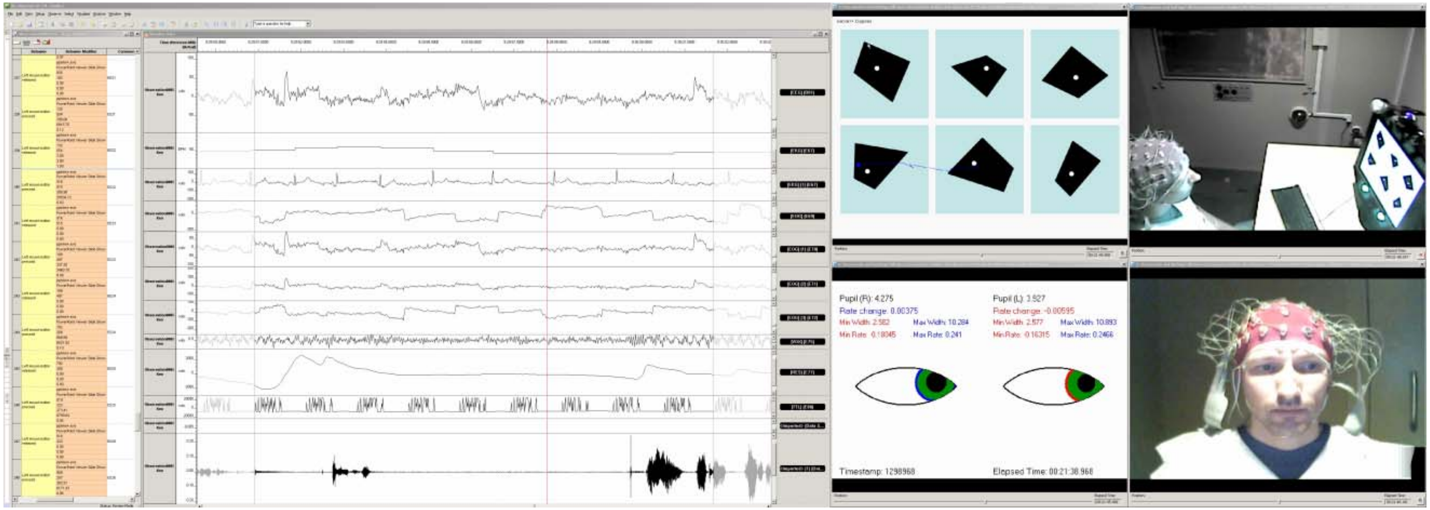

Fig. 2 Integrated and time synchronized data set from a mathematics educational neuroscience study capturing an “aha” moment

however, while certainly of interest and serving to inform educational neuroscience, would be the job of cognitive neuroscience.

What is gained from using methods of psychophysiology and cognitive neuroscience, such as EEG, EKG, ET, and GSR in educational research, are new means for operationalising the psychological and sociological models educational researchers have traditionally developed for interpreting the mental states and social interactions of teachers and learners in the course of teaching and learning mathematics. This statement holds for qualitative educational researchers and quantitative educational psychometricians alike. It bears emphasis that educational neuroscience can augment traditional qualitative and quantitative studies in cognitive modelling in general, and more specifically, in mathematics education research. McVee et al. (2005) have argued that schema theory, the mainstay of cognitive modelling, remains of fundamental relevance to contemporary orientations towards social and cultural theories of learning. Holding fast to a *humanistic* orientation, educational neuroscience concerns the psychological, sociological, and naturalistic dimensions of learning, only now, using methods and tools, and informed by results, of cognitive neuroscience, all the while guided by, and yet also serving to test and refine, more traditional educational models, questions, problems, and studies.

Cultivating mathematical ability is the main task and mandate of mathematics education. Most of this culturally acquired understanding in mathematics goes beyond what is currently known about the biological and psychophysiological groundings of mathematical cognition that is typically studied by cognitive neuroscientists (e.g., Dehaene 1997; Butterworth 1999). It seems important that these culturally acquired understandings of mathematics should be consistent in connecting with and building upon the biological and psychophysiological underpinnings of mathematical thinking that cognitive neuroscientists have been working so hard to uncover (e.g., Dehaene et al. 2004). What is likely the case, and this may constitute a central and guiding hypothesis for mathematics educational neuroscience, is that there may be a variety of "disconnects" between our inherited biological predispositions for mathematics and the culturally derived mathematics comprising the K-12 mathematics curriculum (cf., Campbell 2006c).

As a case in point, there is an emerging consensus in cognitive neuroscience that the human brain naturally supports two key distinct mathematical processes (interrelated and mediated in various ways by linguistic processes): a discrete incrementing process, which generates countable quantities, and a continuous accumulation process, which generates continuous quantities (Dehaene 1997). Gallistel and Gelman (2000) have noted an emerging synthesis between these two processes, and the tensions between them, have been "central to the historical development of mathematical thought" and "rooted in the non-verbal foundations of numerical thinking" in both non-verbal animals and humans. These processes also appear to be implicated in Lakoff and Núñez's (2000) four fundamental "grounding metaphors" of object construction and collection (viz. discrete) and measuring and motion (viz. continuous). In research in mathematics education it is well documented that many children and adults have notorious difficulties in moving from whole number arithmetic (qua, working with quantities) to rational number arithmetic (qua, working with magnitudes) (e.g., Campbell 2002b). It is common practice in mathematics education, in accord with a relatively quite recent development in our cultural history of mathematics, to view whole numbers as a "subset" of rational numbers. This subsuming of whole numbers to rational numbers, which, insofar as the latter are conceived in terms of the number line, may constitute a classic disconnect between our natural biological predispositions and K-12 mathematics curriculum and instruction. If so, this could potentially account, at least in part, as to why this progression from whole number arithmetic and rational number arithmetic is so problematic for learners from early childhood into adolescence and beyond (Campbell 2006c). Identifying and reconciling disconnects such as these could be taken as central issues and concerns in defining mathematics educational neuroscience (Campbell and the ENL Group 2007). But how best for educational researchers to go about it?

Tools of particular interest for educational researchers are EEG and ET systems, and for a variety of reasons. First, relative to most other methods, EEG and ET instrumentation fall within the realm of affordability. Secondly, they are relatively easy and safe to use, involving minimal risk to participants. Thirdly, with sampling rates in the millisecond range, both EEG and ET are well suited for capturing the psychophysiological dynamics of attention and thought in real time. Both methods basically offer temporal resolution at the speed of thought and place fewer spatial constraints on participants than other methods. Furthermore, as evidence of increasing confidence in both the reliability and robustness of these methods, many "turnkey" acquisition and analysis systems are now readily available, placing fewer technical burdens on educational researchers venturing to use such systems.

Eye-tracking (ET) studies have commonly used methods that severely limit head movement (e.g., Hutchinson 1989). More recently, less constraining, non-intrusive, methods have been developed for remotely measuring eye movements in human-computer interactions (e.g., Sugioka et al. 1996). These remote-based methods have become quite reliable, robust, and easy to set up (e.g., Ebisawa 1998). Most instructional software today can be variously offered through computer-based environments. Remote-based ET, therefore, is bound to become an important and well established means for evaluating the instructional design and use of computer-based mathematics learning environments (cf., Campbell 2003a).

With EEG, cognitive neuroscientists have developed a viable approach to studying complex cognitive phenomena through electromagnetic oscillation of neural assemblies (e.g., Fingelkurts and Fingelkurts 2001; Klimesch 1999; Niebur 2002; Ward 2003). One key to this approach is the notion of event related desynchronization/synchronization (ERD/S) (Pfurtscheller and Aranibar 1977). In the course of thinking, the brain produces a fluctuating electromagnetic field that is not random, but rather appears to correlate well within distinct frequency ranges with cognitive function in repeatable and predictable ways.

As previously noted, brain oscillations in human cortex may be correlated with mental phenomena characteristic of mathematical thinking ranging from insight (Jung-Beeman et al. 2004) to aversion (Hinrichs and Machleidt 1992). There have been increasing efforts to tease out a "neural code" for such correlates of affect and mentation (such as emotional response, working memory, attention, anxiety, intelligence, cognitive load, problem solving, and so on) of synchronic brain behaviour in distinct frequency bands, typically identified as Delta ($<$1–4 Hz), Theta ($\sim$4–8 Hz), Alpha ($\sim$8–13 Hz), Beta ($\sim$13–30 Hz), and Gamma ($\sim$30–60 $>$ Hz).

A prerequisite to understanding and using this method is a basic mathematical understanding of signal processing, such as sampling, aliasing, Nyquist frequencies, and spectral analysis. There are basically two fundamental pitfalls in signal processing. The first is mistaking noise for signal, and the second is mistakenly eliminating meaningful signals. The first pitfall is typically a matter of faulty interpretation, whereas the second is typically a matter of faulty data acquisition and/or analysis (Campbell 2004a, 2004b). Gaining an elementary level of expertise in such matters should be relatively straightforward for researchers in mathematics education with mathematics, physics, or engineering degrees. For those researchers in mathematics education with insufficient prerequisite expertise, there is always the option of seeking out cognitive neuroscientists with expertise in EEG, and in other, more sophisticated methods as well, such as time-frequency analyses, independent component analyses, and beamforming. As mathematically sophisticated as some of these aspects of signal processing are, they should not be considered *a priori* as beyond the purview of researchers in mathematics education. Indeed, it is likely that those who undertake to familiarise themselves with the basic ideas and methods of signal processing will find them more intuitive than the basic ideas and methods of statistics.

As powerful as the tools and methods of cognitive neuroscience are, however, and as promising as the prospects for filling and bridging gaps in our understanding between education and neuroscience may be, some philosophical problems and preconceptions appear as intransigent and recalcitrant as ever. What are we to make of an embodied "mindbrain"? What does such a thing actually look like? Well, it looks like a brain. And how does it think? Well, it thinks like a mind. Like quicksand, questions like these can easily and quite readily draw the unwary back into classical dualist conundrums.

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