Constraint-Handling in Evolutionary Aerodynamic Design

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Abstract. Constraint-handling techniques for evolutionary multiobjective aerodynamic and multidisciplinary designs are focused. Because number of evaluations is strictly limited in aerodynamic or multidisciplinary design optimization due to expensive computational fluid dynamics (CFD) simulations for aerodynamic evaluations, very efficient and robust constraint-handling technique is required for aerodynamic and multidisciplinary design optimizations. First, in Section 2, features of aerodynamic design optimization problems are discussed. Then, in Section 3 constraint-handling techniques used for aerodynamic and multidisciplinary designs are overviewed. Then, an efficient constraint-handling technique suitable to aerodynamic and multidisciplinary designs is introduced with real-world aerodynamic and multidisciplinary applications. Finally, an efficient geometry-constraint-handling technique commonly used for aerodynamic design optimizations is presented.

Keywords: real-world design optimization, aerodynamic design optimization, Pareto-based constraint handling.

1 Problem Statement

Without losing generality, constrained real-number optimization problems are written as:

Find **x** that minimizes

$$\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), \dots, f_m(\mathbf{x}), \dots, f_{m_{max}}(\mathbf{x}))$$
(1)

subject to

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$$g_1(\mathbf{x}) \le 0, \dots, g_n(\mathbf{x}) \le 0, \dots, g_{n_{max}}(\mathbf{x}) \le 0$$
(2)

where $\mathbf{x} = (x_1, \dots, x_l, \dots, x_{l_{max}})$ is a vector of design parameters of the solution that minimizes the objective function(s) while satisfying the inequalities (2). l_{max} , m_{max} and n_{max} are numbers of design parameter(s), objective function(s) and constraint(s), respectively.

2 Features of Aerodynamic Design Optimization Problems

Most of real world aerodynamic or multidisciplinary design optimization problems are multiobjective and multi-constraint design optimization problems. For example, a typical transonic aircraft wing design involves maximization of drag divergence Mach number, minimization of mission block fuel, maximum take-off weight, and wing box weight while constraints on flutter speed, structural strength, manufacturing capability, fuel tank volume, etc. must be met. Another example is the supersonic transportation design presented in [33], which has four objectives (drag coefficients at transonic and supersonic cruise speeds, wing root bending moment and pitching moment) and constraints on lift coefficients at transonic and supersonic cruise speeds as well as wing thickness. Many other multiobjective and multi-constraint design optimization problems can be easily found, such as low-boom supersonic business jet design [5], expendable launcher design [10], and multistage compressor design [26].

A multiobjective optimization problem (MOP) simultaneously involves several competing objectives. While a single objective optimization problem may have a unique optimal solution, MOPs present a set of compromised solutions, largely known as the tradeoff surface, Pareto-optimal solutions or non-dominated solutions. The goal of MOPs is to find as many Pareto-optimal solutions as possible to provide useful information of the problem to the designers.

Other features of real-world aerodynamic or multidisciplinary design optimization problems are;

- Number of evaluations is severely limited because aerodynamic function evaluation using computational fluid dynamics (CFD) simulations are very expensive.
- Objective and constraint functions are highly multimodal due to nonlinearity of the flow governing equations.
- Design variables, objectives and constraints are typically real numbers.

For example, in the multidisciplinary design optimization of main wing of the regional jet that is under development in Japan (aimed entry to service is in 2013) [4], the objective and constraint function evaluations include; 1) aerodynamic evaluations, 2) aeroelasticity evaluation, 3) wing weight evaluation, and 4) flight envelope analysis (Fig.1), which required more than 100 hours of computational time for each design candidate evaluation even on a vector supercomputer (when one processing element is used). Therefore, in the example in [4], population size and number of generations are limited to 8 and 19, respectively. It should be noted that, in real world, evolutionary optimization with such small population size and number of



Fig. 1 Flowchart in the objective/constraint function evaluation module for the regional jet design [4]



Fig. 2 Comparison of shock wave visualizations colored by entropy under the transonic cruising flight condition between the base design (left) and an optimized design (right) [4]. Weaker shock wave is observed for the optimized design

generations can give useful information to the designers. In fact, authors of [4] significantly improved block fuel (3.6% improvement) from the base design which was designed in conventional design manner (Fig.2).

3 Constraint-Handling Techniques Used for Aerodynamic Design Optimization

Real-world aerodynamic or multidisciplinary design optimization problems involve multiple objectives and multiple constraints. Among many design optimization

approaches, therefore, evolutionary algorithms (EAs) are getting popular in aerodynamic and multidisciplinary design optimizations [3, 4, 10, 14–16, 18, 22, 24, 26–28, 31,33,35]. EAs are particularly suited for MOPs because they can uniformly sample various Pareto-optimal solutions in one optimization by maintaining a population of design candidates and using a fitness assignment based on the Pareto-optimality concept. In addition, EAs have other advantages such as robustness, efficiency, as well as suitability for parallel computing.

EAs, however, do not have any explicit mechanism to handle design constraints. A considerable amount of research on constraint handing techniques that incorporate objective function(s) and constraint(s) into the fitness function of design candidates has been carried out (good summaries are given in Coello [7] and Mezura-Montes [20]).

The simplest way to handle constraints is to remove infeasible design candidates out of optimization by applying fitness function of zero (for maximization problem) [15, 26]. However, this approach is not efficient because it wastes information that infeasible design candidates have, i.e., direction from infeasible region to the feasible region. As described in the previous section, number of design candidate evaluations is strictly limited in real-world aerodynamic or multidisciplinary design optimizations. Therefore this approach is not suitable to such design optimization problems.

Three constraint-handling approaches making use of information infeasible designs have been used for aerodynamic and multidisciplinary design optimizations as far as the author knows; penalty function approach (for example, see [9]), Deb's constraint-handling approach [8], and Oyama's constraint-handling approach [30].

Traditional and the most popular approach for handling design constraints in aerodynamic and multidisciplinary design optimizations is the penalty function method [9], where the fitness of a design candidate is determined based on scale function vector $\mathbf{F}(\mathbf{x}) = (F_1(\mathbf{x}), \dots, F_m(\mathbf{x}), \dots, F_{m_{max}}(\mathbf{x}))$, which is a weighted sum of the objective function value and the amount of design constraint violations. A typical scale function for minimization problem is given by

$$F_m(\mathbf{x}) = f_m(\mathbf{x}) + \sum_{n=nm1}^{nm2} \alpha_n \cdot max(g_n(\mathbf{x}), 0)$$
(3)

where α_n are the positive penalty function coefficients and constraints related to the objective function f_m is $(g_{nm1}, \ldots, g_{nm2})$. Though this approach is widely used in aerodynamic and multidisciplinary designs [14, 16, 18, 22, 31], this method requires careful tuning of the penalty function coefficients to obtain satisfactory designs. For example, if the penalty function coefficients are too small, the optimized designs would not satisfy the constraints. On the other hand, if the penalty function coefficients are too large, the optimized designs would not have satisfactory objective function values. In addition to the balance between the objective functions and the constraints, the balance among the constraints must also be carefully tuned so that the optimized designs satisfy all of the constraints.

Another constraint-handling approach used for aerodynamic evolutionary optimizations is Deb's constraint handling method [8]. This approach ranks design candidates using the following definition of domination between two design candidates,

Definition 1. a solution i is said to constrained-dominate a solution j if any of the following conditions is true,

- 1. Solutions *i* and *j* are feasible and solution *i* dominates solution *j*.
- 2. Solution i is feasible and solution j is not.
- 3. Solutions i and j are both infeasible, but solution i has a smaller constraint violation.

where

Definition 2. a solution i is said to dominate a solution j if both of the following conditions are true,

1. Solution *i* is no worse than solution *j* in all objectives, i.e.,

$$\forall f_m(\mathbf{x}_i) \le f_m(\mathbf{x}_j) \tag{4}$$

2. Solution *i* is strictly better than solution *j* in at least one objective, i.e.,

$$\exists f_m(\mathbf{x}_i) < f_m(\mathbf{x}_j) \tag{5}$$

Flow chart of a procedure using this technique is presented in Fig.3. This approach does not require any penalty function coefficients to be tuned as long as the number of constraint is one. In this sense, this approach is very useful for EA-based design optimizations. In fact, Oyama et al obtained rotor blade designs that significantly outperform the baseline design using an EA coupled with Deb's constraint-handling technique [27, 28]. However, in [8], no approach for a problem with multiple constraints is not presented. Thus, this approach requires careful tuning of the weight



Fig. 3 Flow chart of a ranking procedure using Deb's constraint-handling technique

coefficients if weighted sum of the constraints is used to determine the constraint violation.

The last constraint-handling approach used in aerodynamic and multidisciplinary design optimization problems is Oyama's constraint handling method [30]. This approach is superior to the previous two approaches in the sense that no parameter tuning is required. This approach has been successfully applied to spaceplane conceptual design [30], aerodynamic compressor blade design [29], and aerodynamic airfoil shape optimization [35]. In the next section, this approach is described in detail and then two real-world applications are presented.

4 Oyama's Constraint-Handling Approach

4.1 Approach

Oyama's constraint-handling approach simply apply the idea of non-dominance and niching concepts in the objective function space to the constraint function space. This method bases on the following non-dominance concept.

Definition 3. Solution i is said to constrained-dominate solution j if any of the following conditions is true,

- 1. Solutions *i* and *j* are both feasible and solution *i* dominates solution *j* in the objective function space.
- 2. Solution *i* is feasible and solution *j* is not.
- 3. Solutions *i* and *j* are both infeasible, but solution *i* dominates solution *j* in the constraint space.

where dominance in the objective function space is defined as Definition 2 while dominance in the constraint space is defined as follows.

Definition 4. Solution *i* is said to dominate solution *j* in the constraint space if both of the following conditions are true,

1. Solutions *i* is no worse than solution *j* in all constraints, i.e.,

$$\forall G_n(\mathbf{x}_i) \le G_n(\mathbf{x}_j) \tag{6}$$

2. Solution i is strictly better than solution j in at least one constraint, i.e.,

$$\exists G_n(\mathbf{x}_i) < G_n(\mathbf{x}_j) \tag{7}$$

where

$$G_n(\mathbf{x}) = max(0, g_n(\mathbf{x})) \tag{8}$$

Oyama's constraint-handling approach applies niching based on the amount of constraint violations to infeasible solutions. Here, a standard fitness sharing [13] is applied to the infeasible designs based on their constraint violations as

$$rank'(\mathbf{x}_i) = rank(\mathbf{x}_i) * penalty(\mathbf{x}_i)$$
(9)

$$penalty(\mathbf{x}_i) = 1 + \sum_{j=1, j \neq i}^{npop} sh_{ij}$$
(10)

$$sh_{ij} = \begin{cases} 1 - (d_{ij}/\sigma_{share})^{\alpha} & d_{ij} < \sigma_{share} \\ 0 & d_{ij} \ge \sigma_{share} \end{cases}$$
(11)

$$\sigma_{share} = \sum_{n=1}^{n_{max}} (gmax_n - gmin_n) / npop$$
(12)

$$d_{ij} = \sqrt{\sum_{n=1}^{n_{max}} (g_n(\mathbf{x}_i) - g_n(\mathbf{x}_j))^2}$$
(13)

$$gmax_n = max(g_n(\mathbf{x}_1), \dots, g_n(\mathbf{x}_i), \dots, g_n(\mathbf{x}_{npop}))$$
(14)

$$gmin_n = min(g_n(\mathbf{x}_1), \dots, g_n(\mathbf{x}_i), \dots, g_n(\mathbf{x}_{npop}))$$
(15)

where *npop* is population size and α is set to 0.3. If the present approach is applied to a multiobjective optimization problem, a fitness sharing is used to the feasible designs based on their objective function values. Flow chart of a ranking procedure using this technique is presented in Fig. 4. Because this method simply uses the idea of non-dominance and niching concepts in the constraint function space, this idea can be coupled with most multiobjective EAs. For example, any ranking procedure can be used for ranking among feasible designs as well as infeasible designs. In addition, the use of stochastic ranking [32] may further improve efficiency and robustness.

The proposed method has a number of advantages.

- It does not require any coefficients to be tuned if a parameterless sharing method such as [12] is used. Even if a sharing method that has coefficients to be tuned is used, according to the author's experience, the parameter values used for sharing in the objective space can be used for sharing in the constraint function space.
- The number of objectives is not increased since non-dominance ranking is applied to feasible designs and infeasible designs separately. If the number of objectives is increased, it will be more difficult to obtain non-dominated solutions due to lower selection pressure.
- It is efficient and robust even when all individuals in the initial population are infeasible due to severe constraints because niching strategy is used in the constraint space. When all individuals are infeasible, the population could lose diversity in the next generation, if diversity in constraint space is not considered.
- It is efficient and robust even when the degree of violation of each constraint is very different because the total amount of constraint violation is not used. If total amount of constraint violation is considered and the degree of violation of each constraint is different, it is very difficult to obtain feasible solutions satisfying



Fig. 4 Flow chart of ranking procedure using Oyama's constraint-handling technique

constraints that has smaller violation in average. There are some approaches that use total amount of constraint violation with dynamically tuned weights of constraints. However, such methods may lose diversity in the population when most of the population are infeasible because niching in the constraint space is not applicable.

• Implementation is easy because Pareto-ranking and sharing method based on the objective functions can be usually applicable to Pareto-ranking and sharing based on the constraint function.

Though this method may increase the computational time required for an EA, the increase is usually negligible in real-world aerodynamic design optimization problems where the computational time required for objective and constraint function evaluations is very large. In the next two subsections, real-world design optimizations using an EA coupled with this constraint-handling approach are presented.

4.2 Conceptual Design Optimization of a Two-Stage-To-Orbit Spaceplane

In this subsection, conceptual design optimization of a two-stage-to-orbit (TSTO) spaceplane is presented. The TSTO spaceplane consists of a booster with airbreathing engines and an orbiter with rocket engines. The orbiter is separated from the booster at a certain altitude to reach low-earth-orbit (LEO) for delivering the payload (Fig. 5).

4.2.1 Formulation of the Design Optimization Problem

The present TSTO mission is to put a payload of 10t into an equatorial orbit at the altitude of 400km. For simplicity, the take-off and landing sites are assumed to be on

the equator. The engine of the booster is an air-turbo-ramjet engine with expander cycle (ATREX) [36], which is under development in Japan. The objective is to minimize the gross take-off weight of the spaceplane. The separation time is constrained to be smaller than 550 seconds. The maximum thrust of the booster is also constrained to be smaller than 2.5 MN. The gross take-off weight, separation time and maximum thrust of the booster are iteratively computed from the propulsion, aero-dynamics, trajectory and structure modules [19, 34] as shown in Fig. 6. Propulsion, trajectory and airframe configuration parameters (ten parameters in total) are considered as design variables.

4.2.2 Optimization Approach

The present EA uses floating-point representation [21] to represent design parameters of design candidates where an individual is characterized by a vector of real numbers. Random parental selection and the best-*N* selection [37], where the best *N* individuals are selected for the next generation among *N* parents and *N* children based on Pareto-optimality, are used. The blended crossover (BLX-0.5) [11] is used with crossover rate of 1 for reproduction. Since strong elitism is used, a high mutation rate of 0.2 is applied and a random disturbance is added to the parameter in the amount up to $\pm 20\%$ of the design space. The initial population is generated randomly over the entire design space. The capability of the present EA to find quasi-optimal solutions has been well validated [23, 25].

The rank of each design candidate is defined according to Definition 3. Fonseca and Fleming's Pareto-based ranking [12] is used to rank feasible designs as well as infeasible designs. The population size and number of generations are set to 50.



Fig. 5 Image of the TSTO spaceplane (left) and its mission (right)

4.2.3 Results

Optimization is repeated one hundred times with different initial populations. The present EA coupled with Oyama's constraint-handling approach found feasible designs in each run. Average weight of the optimized designs, weight of the best optimum and standard deviation of the optimum weight were 371.19 kt, 369.00 kt, and 1.5787 kt, respectively.

For comparison of constraint-handling techniques, the result is compared with that obtained with the same EA with different constraint-handling techniques; Deb's approach [8], Coello's approach [6], and dynamic penalty approach [17]. To handle multiple constraints with the Deb's approach, the constraints are combined into one constraint violation function where all weights are 1. For the dynamic penalty approach, all weights in equation (3) are 1. The parameter values used in the dynamic penalty approach are C=0.2, $\alpha=2$, and $\beta=2$. The result is summarized in Table 1. The dynamic penalty approach and Deb's approach failed to find feasible designs in 100 optimizations for this design problem. The reason is probably that both methods adopt simple sum of the amounts of constraint violation of different order of magnitude. On the other hand, Oyama's approach and Coello's approach got good



Fig. 6 Multidisciplinary TSTO evaluation

Approach	Number of suc- cesses	Average weight [kt]	Weight of the best design [kt]	Standard Devi- ation [kt]
Oyama's approach	100	371.19	369.00	1.5787
Coello's approach	99	371.29	369.04	1.6239
Deb's approach	No feasible design is found			
Dynamic penalty	No feasible design is found			

Table 1 Comparison between the constraint-handling methods

scores, while the proposed method was slightly better than Coello's approach in every measure.

4.3 High-Fidelity Aerodynamic Design Optimization of an Axial Compressor Blade

4.3.1 Formulation of the Design Optimization Problem

The next optimization problem is to seek a redesign of NASA rotor67 [38], which is a low-aspect-ratio transonic axial-flow fan rotor and is the first-stage rotor of a two-stage fan. The fan was designed and tested to help provide the technology to develop efficient, lightweight engines for short-haul aircraft in 1970s. The rotor 67 was designed by using a streamline-analysis computational procedure, which provides an axisymmetric, compressible-flow solution to the continuity, energy, and radial equilibrium equations.

The rotor design pressure ratio is 1.63 at a mass flow of 33.25 kg/sec. The design rotational speed is 16043 rpm, which yields a tip speed of 429 m/sec and an inlet tip relative Mach number of 1.38. The rotor has 22 blades and aspect ratio of 1.56 (based on average span/root axial chord). The rotor solidity varies from 3.11 at the hub to 1.29 at the tip. The inlet and exit hub/tip radius ratios are 0.375 and 0.478, respectively. Reynolds number is 1,797,000 based on the blade axial chord at the hub.

The objective of the aerodynamic rotor shape design optimization problem is to minimize the flow loss manifested via entropy generation. Here, mass-averaged entropy production from inlet to exit at the design point of rotor67 is considered as the objective function to be minimized. Because an optimized rotor design should meet the required mass flow rate and pressure ratio, they are maintained by specifying constraints on them:

$$\frac{|massflowrate_{design} - massflowrate_{rotor67}|}{massflowrate_{rotor67}}| \le 0.005$$
(16)

$$\left|\frac{pressureratio_{design} - pressureratio_{rotor67}}{pressureratio_{rotor67}}\right| \le 0.010 \tag{17}$$

In addition, thickness of the optimized design is constrained to be equal to or larger than that of the rotor 67;

$$\sum max(0, thickness_{rotor67} - thickness_{design}) \le 0$$
(18)

where thickness of the designs and rotor 67 is measured at 10%, 20%, ..., 90% chord positions on 57 blade profiles from root to tip.

4.3.2 Blade Shape Parameterization

Here a rotor blade shape is represented by four blade profiles, respectively at 0%, 31%, 62%, and 100% spanwise stations (all spanwise locations discussed here are measured from the hub), the spanwise twist angle distribution, and the stacking line. Each of these sectional profiles can be uniquely defined by using a mean camber line and a thickness distribution. Here, they are parameterized by the third-order B-Spline curves and positions of control points of the B-Spline curves are considered as the design parameters. As illustrated in Fig. 7, five control points are used for the mean camber line. For the thickness distribution, two control points are added at the leading edge and the trailing edge so that these points represent leading edge and trailing edge radii, respectively. Chordwise locations of the control points at leading edge and trailing edge are frozen to zero and one, respectively. The thickness control points at the leading and trailing edges are defined so that the leading and trailing radii of the designs are identical to those of the rotor 67. These profiles are linearly interpolated from hub to tip. Stagger angles are defined at 0%, 33%, 67%, and 100% spanwise stations and linearly interpolated. Spanwise chord length distribution remains identical to that of the rotor 67. Final blade shape is defined by stacking the blade profiles around the center of gravity of each profile. Here, streamwise and circumferential the stacking lines are defined by B-Spline curves as shown in Fig. 7, respectively. As a result, each blade shape is represented with 49 design parameters.



Fig. 7 B-Spline curves for mean camber line (left) and thickness (middle) distributions and stacking line definition (right)

4.3.3 Optimization Approach

The same EA described in 4.2.2 is used. The parameter values used in the EA are also same except for the population size. The population size is increased from 50 to 64 because number of the design parameters is increased. To handle constraints on mass flow rate and pressure ratio, Oyama's constraint-handling approach is used. For blade thickness constraints, an approach for geometry constraints described in section 5 is used.

4.3.4 Aerodynamic Evaluation

The three-dimensional Navier-Stokes (N-S) code used in the present research is TRAF3D [1,2]. Capability of the present code has been validated by comparing the computed results to some experiments such as the Goldman annular vane with and without end wall contouring, the low speed Langston linear cascade [1] as well as the NASA rotor67 [2]. Detail of this code is described in [27].

The three-dimensional grids are obtained by stacking two-dimensional grids generated on the blade-to-blade surface. These two-dimensional grids are of C-type and are elliptically generated, with controlled grid spacing and orientation at the wall. The number of the grid points is 201 chordwise \times 53 tangential \times 57 spanwise. The computational grid for the NASA rotor 67 is shown in Fig. 8. All computations were performed on the NEC SX-6 supercomputer consisting of 128 vector processing elements (PEs) located at JAXA Institute of Space and Astronautics Science in Japan. Aerodynamic evaluations of design candidates at each generation is parallelized using the simple master-slave concept; the grid generations and the flow calculations associated to the design candidates of a generation are distributed into 32 PEs of the NEC SX-6 machine. Aerodynamic function evaluation of each design candidate took about 40 minutes on one PE of the NEX SX-6 machine (For 50 generations, it took more than 66 hours of computational time on 32 vector PEs of the supercomputer).



Fig. 8 Computational grid over NASA rotor67. Every other line is shown

Fig. 9 Comparison of the optimization histories



4.3.5 Result

Figure 9 presents optimization history in terms of the objective function (entropy production) compared with the NASA rotor 67. In the same figure, optimization history of the same EA coupled with Deb's constraint-handling approach is presented for comparison purpose where constraint violation CV is defined as

$$CV = 2 \cdot CV_{mass\,flowrate} + CV_{pressure ratio} \tag{19}$$

For both cases, the optimized designs obtained after the eighth generation satisfied all the constraints. However, the final design obtained by Oyama's approach has a smaller entropy production than the NASA rotor 67 while the optimized design by Deb's approach could not improve this result in 50 generations. It may be because diversity in the population is lost before a feasible solution is found at the ninth



Fig. 10 Comparison of the spanwise entropy distributions

generation when Deb's approach is used. Spanwise entropy distributions of the optimized design and the NASA rotor 67 are compared in Fig. 10. The figure shows that the entropy production can be reduced mainly between 60% to 90% span while it is increased near the tip.

5 Geometry-Constraint-Handling Used for Aerodynamic Design Optimization

In general, aerodynamic design optimization problem involves aerodynamic constraints and other constraints which don't require CFD simulation for function evaluation. A typical example is geometry constraint. For example, aerodynamic drag minimization of a transonic wing without any geometric constraints would result in very thin wing shape. However, such wing shape does not have enough structural strength to withstand the bending moment due to the lift force on the wing. In many cases, structural strength is guaranteed by constraint on minimum wing thickness or minimum wing profile area. In such cases, the constraint function can be evaluated without CFD evaluation of candidate wings. Therefore, in some aerodynamic design optimizations, geometry constraints are evaluated as soon as a new design candidate is generated and if the design candidate does not satisfy the geometry constraints, it is discarded and another design candidate is generated until the new design candidate satisfies all geometry constraints. By doing this procedure, all design candidates satisfy geometry constraints and thus, expensive aerodynamic evaluations can be significantly saved [3,4,24,29].

6 Conclusions

In this chapter, features of aerodynamic design optimization problems were presented and constraint-handling techniques for evolutionary multiobjective aerodynamic and multidisciplinary designs were overviewed. Because number of evaluations is limited in aerodynamic and multidisciplinary design optimizations, a very efficient and robust constraint-handling technique is required for aerodynamic and multidisciplinary design optimizations. Oyama's constraint-handling approach is suitable to aerodynamic and multidisciplinary design optimizations in this sense. Conceptual design of TSTO spaceplane and high-fidelity aerodynamic rotor blade design optimization demonstrated that Oyama's approach is better than traditional constraint-handling methods for real-world aerodynamic and multidisciplinary design optimization problems.

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