Hybrid Modelling, Power Management and Stabilization of Cognitive Radio Networks*

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Abstract. In this paper, we deal with hybrid modelling, optimal control and stability in cognitive radio networks. Networks that are based on cognitive radio communications are intelligent wireless communication systems. They are conscious about changes in the environment and are able to react in order to achieve an optimal utilization of the radio resources. We provide a general hybrid model of a network of nodes operating under the cognitive radio paradigm. The model abstracts from the physical transmission parameters of the network and focuses on the operation of the control module. The control problem consists in minimizing the consumption of the network, in terms of average transmitted power or total energy spent by the whole network. A hybrid optimal control problem is solved and the power-optimal control law is computed. We introduce the notion of network configuration stability and derive a control law achieving the best compromise between stability and optimal power consumption. Finally, we apply our results to the case of a cognitive network based on UWB technology.

1 Introduction

In recent years, much interest has arisen in cognitive networks and their applications. The cognitive terminology was coined by Joseph Mitola III [1] and refers to radio devices that are able to sense the external environment, learn from history and make intelligent decisions in order to adjust their transmission parameters according to the current state of the environment [2]. The main features of cognitive networks have been mostly studied from the radio perspective (see, for example, [3], [4] and [5]). Some of the topics that have been investigated are spectrum management, cognitive architecture, power control, security issues. A successful approach is given by the game theory [6]. The power control problem, in wireless (not necessarily cognitive) contexts, has been addressed in

* This work has been partially supported by the Center of Excellence for Research DEWS, University of L'Aquila, 67040 Monteluco di Roio, L'Aquila (AQ), Italy.

R. Majumdar and P. Tabuada (Eds.): HSCC 2009, LNCS 5469, pp. 76–89, 2009.

many works, mainly as a noncooperative game [7], [8], in partially hybrid contexts [9] or in UWB networks with cognitive features [10].

In [13] and [14], we introduced hybrid modelling of self-organizing communication networks, and more specifically of overlay UWB networks. In this paper, we expand the above model by applying the concepts to any network of nodes operating under the cognitive radio paradigm. In particular, we provide a model that abstracts from the physical transmission parameters of the network and focuses on the operation of the control module. The control problem consists in minimizing the consumption of the network, in terms of average transmitted power or total energy spent by the whole network. For simplicity, the focus is on the uplink communications and the network topology is a star, that is, the control action is centralized in a Central Node (usually referred to as CNode). The CNode selects at each time t one out of several sets of transmission parameters that must be used by the other nodes for their transmissions. In particular, the selection is made on the basis of power minimization. We provide an optimal solution to the power minimization problem for a generally defined cognitive network. Based on the observation that the optimal solution lacks providing stability guarantees, we further refine the model by introducing an energetic cost that weighs the energy loss provoked by switching from one set of transmission parameters to another. We then derive the solution to the power minimization problem under stability constraints, and compare it to the original optimal solution.

The paper is organized as follows. In Section 2, we review some definitions of hybrid systems and provide a complete description of the cognitive network model. In Section 3, we introduce the energy minimization problem for the hybrid system and compute the hybrid power-optimal control strategy. Then, we introduce the notion of configuration stability, and use this concept to find a sub-optimal configuration-stabilizing solution of the optimum problem. Section 4 addresses the case study of a cognitive network based on UWB technology. Section 5 offers some concluding remarks.

2 Hybrid Modelling

2.1 Basic Definitions

We define the class of hybrid systems we consider in this paper, following the framework introduced by [11]. Our definition includes continuous control input, continuous disturbance and continuous output. Moreover, both discrete control inputs and discrete disturbances act on the system.

Definition 1 (Hybrid System). *A hybrid system $\mathcal{H}$ is a collection*

$$\mathcal{H} = (Q \times X, Q_0 \times X_0, U, D, Y, Inv, S, \Sigma, E, R) \quad (1)$$

where

- $Q \times X$ is the hybrid state space, where $Q \subset \mathbb{N}$ is a finite set of discrete states and X is the continuous state space. $Q_0 \times X_0 \subseteq Q \times X$ is the set of initial discrete and continuous conditions.
- U, D, Y are subsets of finite dimensional vector spaces and are respectively the continuous input, disturbance and output space. We denote by $\mathcal{U}_c$ the set of piecewise continuous control functions $u : \mathbb{R} \longrightarrow U$ and by $\mathcal{U}_d$ the set of disturbance functions $d : \mathbb{R} \longrightarrow D$.
- $Inv : Q \longrightarrow 2^X$ is a map associating to each discrete state $q \in Q$ a domain of acceptable continuous states.
- $S = \{S_q\}_{q \in Q}$ associates to each discrete state $q \in Q$ the nonlinear time-variant continuous system

$$S_q : \begin{cases} \dot{x}(t) = f_q(t, x(t), u(t)) \\ y(t) = h_q(t, x(t), u(t), d(t)) \end{cases}$$

  where $t \in \mathbb{R}$, $x(t) \in X$, $u(t) \in U$, $d(t) \in D$. Given $q \in Q$, $f_q(\cdot)$ is a function such that, $\forall u(\cdot) \in \mathcal{U}_c$, the solution $x(t)$ exists and is unique for all $t \in \mathbb{R}$. Given $q \in Q$, $t \in \mathbb{R}$, $x(t) \in X$, $u(t) \in U$, $d(t) \in D$, $y(t) = h_q(t, x(t), u(t), d(t)) \in Y$, where $h_q : T \times X \times U \times D \longrightarrow Y$.
- Σ is the finite set of discrete inputs, collecting discrete control inputs and discrete disturbances. Each input is associated to one or more edges $e \in E$.
- $E = E_c \cup E_d \subset Q \times \Sigma \times Q$ is a collection of edges, including the set of the controlled transitions E_c, determined by discrete control inputs, and the set of the switching transitions E_d, determined by discrete disturbances. We assign higher priority to switching transitions with respect to controlled ones: if a switching transition and a controlled transition occur at the same time, only the switching one is considered, while the controlled one is ignored.
- $R : E \times X \rightarrow X$ is a deterministic map called *reset*.

Definition 2 (Execution). *An execution of the hybrid system $\mathcal{H}$ is a collection $\chi = (\tau, q, \sigma, x, y, u, d)$, consisting of a set of switching times $\tau = \{t_i\}_{i=0}^{L}$ and the functions $q(\cdot) : [t_0, t_L) \longrightarrow Q$, $\sigma(\cdot) : [t_0, t_L) \longrightarrow \Sigma$, $x(\cdot) : [t_0, t_L) \longrightarrow X$, $y(\cdot) : [t_0, t_L) \longrightarrow Y$, $u(\cdot) : [t_0, t_L) \longrightarrow U$, $d(\cdot) : [t_0, t_L) \longrightarrow D$, satisfying the following conditions:*

- *Initial condition:* $(q(t_0), x(t_0)) \in Q_0 \times X_0$.
- *Discrete evolution: for all* $i = 1, ..., L-1$
  1. $q(\cdot)$ *and* $\sigma(\cdot)$ *are constant over the intervals* $[t_i, t_{i+1})$;
  2. $(q^-(t_i), \sigma(t_i), q(t_i)) \in E$;
  3. $x(t_i) = R(q^-(t_i), \sigma(t_i), q(t_i), x^-(t_i))$ *where* $q^-(t_i) = \lim_{t \longrightarrow t_i^-} q(t)$ *and* $x^-(t_i) = \lim_{t \longrightarrow t_i^-} x(t)$.
- *Continuous evolution: for all* $i = 1, ..., L-1$, *at time* $t \in [t_i, t_{i+1})$
  1. $x(t)$ *is the (unique) state trajectory of the dynamical system* $S_{q(t_i)}$ *with initial time* t_i, *initial state* $x(t_i)$ *and control law* u;
  2. $x(t) \in Inv(q(t_i))$;
  3. $y(t) = h_{q(t_i)}(t, x(t), u(t), d(t))$.

2.2 Hybrid Modelling of Cognitive Networks

We model the set of wireless nodes as a social network, forming one single entity [13]. We consider a self-organizing network of nodes that adopt a multiple access scheme in which coexistence is foreseen, that is signals originating from different users share in principle a same resource in terms of time and frequency. Users separation is obtained by appropriate coding.

An important hypothesis, that is fundamental in our model, is the possibility of selecting among different sets of transmission parameters, which can range from coding, to modulation formats, to pulse shaping. We therefore assume W different configurations, that is W different sets of transmission parameters w_q, with $q = 1, ..., W$. We associate an energetic cost $c \geq 0$ to the operation of switching from one set to another.

We assume that system performance is described by a specification on the system behavior, for example the level of signal to noise ratio or the transmission delay.

The topology of the network is a star, that is, nodes communicate through the CNode, and implement the cognitive radio paradigm. If a new node asks for admission, the CNode evaluates the possibility of admitting it, by checking whether constraints for admission are compatible with network specifications.

At each time t, the CNode communicates to the other nodes the set of transmission parameters $q \in \{1, ..., W\}$, the number of active nodes N and the average power level P_{RX} that it wants to receive. We suppose the signal containing the above information is sent by the controller at a fixed power level that is predetermined and known by all nodes. Each active node j receives this signal and, on the basis of received power level, can estimate the attenuation $a_j(t)$ characterizing its path to the coordinator and can determine the power to be used in its transmissions, namely $p_{TX,j}(t) = a_j(t)\, p_{RX}(t)$. In this work, we disregard w.l.o.g. any assumption about the maximum transmission power of each node [13], in order to decouple completely the power-minimization problem (object of the present paper) and the problem of nodes leaving the network for lack of available power.

Since a node can enter/leave the network several times, one has $j \in \{1, ..., N_{\max}\}$, where $N_{\max}$ is the maximum number of nodes which can be admitted to the network. We define a time-varying attenuation vector $A(t) \in \mathbb{R}^{N_{\max}}$, that includes the attenuations $a_j(t)$ for each node j, and an activity vector $S(t) \in \mathbb{R}^{N_{\max}}$, whose generic element $s_j(t)$ equals 1 if node j is transmitting at time t, and 0 otherwise.

The instantaneous transmission power consumption of the network can expressed as:

$$P_{TX}(t) = \sum_{j=1}^{N_{\max}} s_j(t)\, p_{TX,j}(t) = \left(\sum_{j=1}^{N_{\max}} s_j(t)\, a_j(t) \right) p_{RX}(t) = A^{'}(t) S(t)\, u(t) \,.$$

Following the assumptions, the network can be modeled as a hybrid system as follows:

- The set of discrete states is $Q = \{1, 2, ..., W\}$. Each discrete state $q \in Q$ is associated to a configuration w_q, that is a set of transmission parameters that are used for communication. The continuous state $x \in X = \mathbb{R}^2$ represents the number N of active nodes and the energy spent by the network from the beginning of its life, which includes the energy spent for transmission and for switching among different configurations but does not include the energy spent by nodes to stay in state of idle/receiving.
- The set of initial states is $Q_0 \times X_0 = Q \times \{n \in \mathbb{N},\ n \geq 2\} \times \{0\}$: the network begins its life when there are at least 2 nodes, the minimum for a communication. At the beginning, the energy consumption is zero.
- The domains are

$$Inv\,(q) = \mathbb{N} \times \mathbb{R}^{+} \quad \forall q \in Q.$$

- The continuous dynamics associated to a discrete state $q \in Q$ is

$$S_q : \begin{cases} \begin{bmatrix} \dot{x}_1\,(t) \\ \dot{x}_2\,(t) \end{bmatrix} = f\,(t, u\,(t)) = \begin{bmatrix} 0 \\ A'\,(t) S\,(t)\, u\,(t) \end{bmatrix} \\ \qquad y\,(t) = \begin{bmatrix} h_q\,(x\,(t)\,, u\,(t)) \\ d\,(t) \end{bmatrix} \end{cases}$$

  $f\,(t, u)$ includes a trivial dynamics (the number of nodes may change only as a consequence of a discrete transition) and the instantaneous transmission power consumption of the network $A'\,(t)\,S\,(t)\,u$. The continuous control input $u\,(t) \in U = \mathbb{R}^{+}$ represents the power level $p_{RX}\,(t)$ that the CNode wants to receive from each transmitting node. The output vector $y\,(t) \in Y = Q \times \mathbb{N} \times U \times D$ includes the set of variables sent to each node by the CNode

$$h_q\,(x, u) = \left[q\ x_1\ u\right]^T$$

  and the measurable continuous disturbance vector $d\,(t) \in D \subseteq \mathbb{R}^W$ ($d\,(t)$ is not sent to all active nodes).
- The discrete inputs are $\Sigma = \Sigma_c \cup \{\sigma_d\}$, where σ_d is the discrete disturbance representing the uncontrollable event that a node leaves the network, while $\Sigma_c = \{\sigma_q,\ q \in Q\} \cup \{\sigma_a\}$ is the set of discrete controls:
  - σ_q is the control action occurring when the coordinator decides to commute from the current set of parameters to the set w_q, $q \in Q$;
  - σ_a models the decision to accept a new candidate node in the network; we assume here that the decision procedure, after an admission request, requires a negligible time to be performed.
- The edges are $E = E_c \cup E_d$, where E_c is the set of the *controlled transitions*:

$$E_c = E_{c,W} \cup E_{c,a}$$

$$E_{c,W} = \{(p, \sigma_q, q)\,,\ p,\ q \in Q,\ p \neq q,\ \sigma_q \in \Sigma_c\}$$
$$E_{c,a} = \{(q, \sigma_a, q)\ \ ,\ q \in Q,\ \sigma_a \in \Sigma_c\}$$

  and $E_d = \{(q, \sigma_d, q)\ \ ,\ q \in Q\}$ is the set of the *switching transitions.*

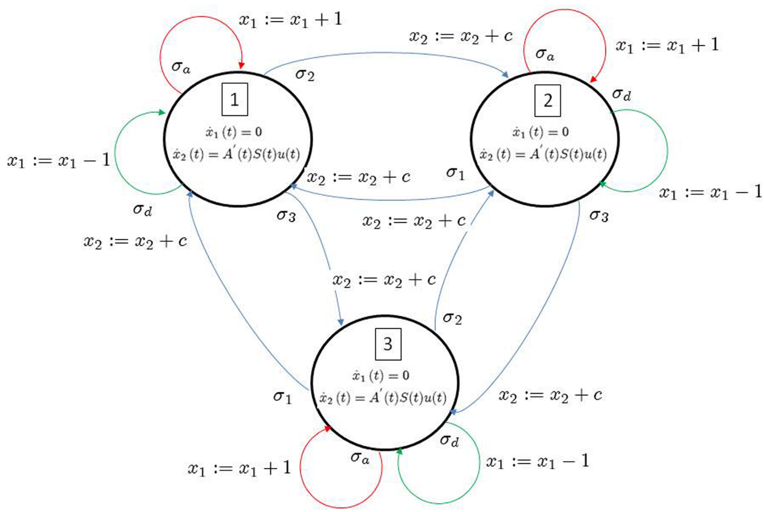

Fig. 1. Hybrid Model

– Reset map: $\forall x \in X$

$$R(e,x) = \begin{cases} \begin{bmatrix} x_1 \\ x_2 + c \end{bmatrix} & e \in E_{c,W} \\ \begin{bmatrix} x_1 + 1 \\ x_2 \end{bmatrix} & e \in E_{c,a} \\ \begin{bmatrix} x_1 - 1 \\ x_2 \end{bmatrix} & e \in E_d \end{cases}$$

Note that the x_2 dynamics is reset only when a change in the transmission parameters occurs, modelling the energetic switching cost c for the network.

The network model is a non-deterministic hybrid system because of the presence of the discrete disturbance σ_d and of the continuous disturbance $d(t)$.

3 Energy Minimization as a Hybrid Optimal Control Problem

Since the admission of a candidate node is allowed only if constraints for admission are compatible with network specifications and none of the current active nodes is forced to leave the network as a result of its admission [13], we focus here on the choice of a discrete control σ_q (a set of transmission parameters) and of a continuous control $u(t)$ that minimize the energy consumption of the network. In this section, we define the energy minimization problem on the hybrid system representing the cognitive network and solve it by defining the discrete and continuous control action.

3.1 Energy Minimization

In the following definitions, we refer to the hybrid system (1). Let $\chi_{T,(q_0,x_0)}$ denote the set of all executions $\chi = (\tau, q, \sigma, x, y, u, d)$ defined on the same time horizon $T = [t_0, t_L) \subset \mathbb{R}$ with initial condition $(q_0, x_0) \in Q_0 \times X_0$, i.e. $(q(t_0), x(t_0)) = (q_0, x_0)$. Given an execution $\chi \in \chi_{T,(q_0,x_0)}$, we define its *value* at time t, and we abuse notation by writing $\chi(t)$, as

$$\chi(t) := (q(t), \sigma(t), x(t), y(t), u(t), d(t))$$

We partition the set of the switching times $\tau = \tau_c \cup \tau_{nc}$, where $\tau_c := \{t_{c,i}\}_{i=1}^{L_c}$ includes all the switching times due to configuration transitions (elements of $E_{c,W}$) and τ_{nc} is its complement $\tau \setminus \tau_c$. Hence τ_{nc} includes switching times due to switching transitions and to admission transitions.

*Problem 1 (**Energy minimization**).* Given the hybrid system $\mathcal{H}$, a set $\chi_{T,(q_0,x_0)}$ including all the executions $\chi = (\tau, q, \sigma, x, y, u, d)$ defined on a time horizon $T = [t_0, t_{fin}) \subset \mathbb{R}$ with $(q_0, x_0) \in Q_0 \times X_0$, where $d \in \mathcal{U}_d$ is a given disturbance function and $\tau \supseteq \tau_{nc}$, where τ_{nc} is given. Let $\varXi$ be the space of all discrete strategies $\sigma : T \longrightarrow \varSigma$ compatible with τ_{nc}, i.e. such that $\sigma(t_{nc,i}) \in \{\sigma_a, \sigma_d\}$ $\forall t_{nc,i} \in \tau_{nc}$, and $\mathcal{U}_{bound}$ the space of all continuous functions u that satisfy a constraint $u(t) \geq u_{LB}(q,t)$ for all $t \in T$. The *energy minimization control problem* consists in minimizing the functional

$$J_e(u, \sigma, \tau_c, c) = \int_T A'(t)\, S(t)\, u(t) dt + c * card(\tau_c) \quad = x_2(t_{fin})$$

over all $\sigma \in \varXi$ and $u \in \mathcal{U}_{bound}$.

Solution 1 (case $c = 0$). We solve the energy minimization problem for the case $c = 0$ and refer to the corresponding optimal strategy as (σ_0^*, u_0^*). With this assumption, we can also write

$$\min_{(\sigma,u)} x_2(t_{fin}) = \min_{(\sigma,u)} J_e(u, \sigma, \tau_c, 0) = \min_{(\sigma,u)} \int_T A'(t)\, S(t)\, u(t) dt$$

so the discrete optimal trajectory and the continuous optimal control action are clearly

$$q_0^*(t) := \arg\min_{q \in Q} u_{LB}(q,t)$$

$$u_0^*(t) = u_{LB}(q_0^*(t), t) \quad \forall t \in T$$

The function $q_0^* : T \longrightarrow Q$ "induces" the set of switching times $\tau_0^* = \tau_c^* \cup \tau_{nc}$, where τ_c^* is the set of controlled switching times with cardinality L_c^*, namely such that for all $i = 1, ..., L_c^*$, $q_0^*(t_{c,i}^*) \neq \lim_{t \longrightarrow (t_{c,i}^*)^-} (q_0^*(t))$. Finally the discrete control function $\sigma_0^* : T \longrightarrow \varSigma$ is a piecewise constant function, in which the control-dependent part is defined by the relation

$$\sigma_0^*(t_{c,i}^*) = \sigma_{q_0^*(t_{c,i}^*)} \in \varSigma_c \qquad i = 1, ..., L_c^*$$

Remark 1. For $c = 0$, the solution to the energy minimization problem (σ_0^*, u_0^*) is obtained by minimizing the power at each time t. Hence, it is computable in real-time and the control (σ_0^*, u_0^*) is indeed achievable. We refer to it as *hybrid power-optimal strategy.*

For $c > 0$, the solution (σ^*, u^*) to the energy minimization problem depends on the values of the time-varying constraint $u_{LB}(q, t)$ over the whole interval $[t_0, t_{fin})$, which may not be known a priori. For example, if the constraint corresponds to a minimum signal-to-noise ratio requirement, $u_{LB}(q, t)$ would depend on disturbances such as external noise and interference. Those disturbances are supposed to be measurable in real-time but cannot be known a priori. Hence, in general, the control strategy (σ^*, u^*) is not computable in real-time and is therefore not achievable. The power-optimal strategy (σ_0^*, u_0^*) is not optimal for $c > 0$, but is a sub-optimal solution of the energy minimization problem as it will be precisely described in the next subsection, where we look for a strategy achieving the best compromise between computability, power consumption and stability of the configuration of the network.

Remark 2. Note that since the dynamic equation in each discrete state and the reset maps are deterministic, an optimal strategy (σ^*, u^*) leads to a unique maximal hybrid execution $(\tau^*, \sigma^*, q^*, x^*, y^*, u^*, d) \in \chi_{T,(q_0,x_0)}$. The discrete optimal trajectory q^* is well-defined if there are no multiple optimal configurations at the same time. If this situation occurs, the optimal configuration can be either chosen arbitrarily among the optimal ones, or chosen according to additional constraints or specifications.

3.2 Configuration Stability

In the previous subsection, we showed that the hybrid strategy (σ_0^*, u_0^*) minimizes both power and energy consumption if the cost of configuration switchings is negligible. However, the discrete control strategy σ_0^* does not assure stability of the network, in the sense that too many switchings may occur in a finite amount of time. Switchings may also be due to switching transitions or admission requests, but since those are uncontrollable transitions, we focus here on ensuring stability at least from the controlled switchings point of view. In this subsection, we propose a sub-optimal hybrid strategy $(\sigma_\delta^*, u_\delta^*)$ that guarentees good performance and stable behavior.

The sub-optimal continuous control u corresponding to any sub-optimal discrete state $\tilde{q}(t) \neq q^*(t)$ is $\tilde{u}(t) := u_{LB}(\tilde{q}(t), t)$, that is the lowest value of continuous control satisfying the constraint. This simple consideration allows us, in the following, to focus only on the discrete control of the system.

Definition 3 (Configuration stability). *Let $\mathcal{H}_c$ be the hybrid system $\mathcal{H}$ controlled by a hybrid strategy (σ, u). If there exists $\delta > 0$ such that, for any execution $(\tau, q, \sigma, x, y, u, d)$, the inequality $0 < \delta \leq t_{c,i+1} - t_{c,i}$ holds $\forall i \in \{1, ..., L_c - 1\}$, then $\mathcal{H}_c$ is said to be δ-configuration stable and σ is said to be a δ-*configuration stabilizing strategy.

Configuration stability is related to the existence of a dwell-time, but with reference only to controlled switching times. The system $\mathcal{H}_c$ controlled by the power-optimal control strategy (σ_0^*, u_0^*) is not necessarily δ-configuration stable, for some $\delta > 0$, since consecutive configuration switchings can be arbitrarily close in time. We propose here to modify the optimal strategy in order to achieve configuration stability.

In the space Ξ of all discrete strategies $\sigma : T \longrightarrow \Sigma$ compatible with τ_{nc}, we consider a pseudometric $d : \Xi \times \Xi \longrightarrow \mathbb{R}^+$:

$$d(x,y) := \lambda(\{t \in T : x(t) - y(t) \neq 0\}) \quad \forall x, y \in \Xi$$

where $\lambda(\cdot)$ is the Lebesgue measure. The chosen pseudometric compares how different two discrete-valued functions are in terms of the duration of the time intervals where they assume different values. In the following, we propose a strategy σ_δ^* that is achieved by deferring any controlled switching so that it occurs not before a time δ from the previous one. If one or more than a switching occur within a time δ, only the last one is taken into account.

Algorithm. Given the collection of optimal switching times $\tau_c^* = \left\{t_{c,i}^*\right\}_{i=1}^{L_c^*}$, we build a sequence $\tilde{\tau}_c = \left\{\tilde{t}_{c,j}\right\}_{j=1}^{\tilde{L}_c}$, depending on τ_c^*, as follows:

Initialization: $\tilde{t}_{c,1} = t_{c,1}^*$; flag=FALSE; $i = 2$; $k = 2$.
Iteration: while $(i \leq L_c^*)$
{ if $(t_{c,i}^* < \tilde{t}_{c,k-1} + \delta)$ then {flag=TRUE; $i++$;}
else {if (flag==TRUE) then $\{\tilde{t}_{c,k} = \tilde{t}_{c,k-1} + \delta$; flag=FALSE;}
else $\{\tilde{t}_{c,k} = t_{c,i}^*$; $i++$;}
$k++$;}
}
Conclusion: if ((flag==TRUE) and $(\tilde{t}_{c,k-1} + \delta < t_{fin})$) then $\tilde{t}_{c,k} = \tilde{t}_{c,k-1} + \delta$;
else $k--$;
$\tilde{L}_c = k$;
define the collection $\tilde{\tau}_c = \left\{\tilde{t}_{c,j}\right\}_{j=1}^{\tilde{L}_c}$;
set $\sigma_\delta^*\left(\tilde{t}_{c,j}\right) = \sigma_{q^*\left(\tilde{t}_{c,j}\right)}$ for all $j = 1, ..., \tilde{L}_c$.

Theorem 1. *Given a hybrid system $\mathcal{H}$, a time horizon $T = [t_0, t_f) \subseteq \mathbb{R}$, an initial condition $(q_0, x_0) \in Q_0 \times X_0$ and the pseudometric space Ξ of all discrete strategies $\sigma : T \longrightarrow \Sigma$ compatible with τ_{nc}. If $\sigma_0^* \in \Xi$ is the power-optimal discrete control strategy, the sub-optimal strategy σ_δ^*, with controlled switching times $\tilde{\tau}_c$, has the following properties:*

1. *σ_δ^* is a δ-configuration stabilizing strategy;*
2. *σ_δ^* does not anticipate σ_0^* (causality principle), i.e. "jumps" in σ_δ^* cannot occur earlier than corresponding "jumps" in σ_0^*;*
3. *if $\tilde{\sigma}$ is any other δ-configuration stabilizing strategy, then $d(\sigma_\delta^*, \sigma_0^*) < d(\tilde{\sigma}, \sigma_0^*)$, i.e. σ_δ^* is at minimum distance from the power-optimal strategy σ_0^*.*

Proof. The algorithm builds the sequence $\tilde{\tau}_c$ such that $0 < \delta \leq \tilde{t}_{c,j+1} - \tilde{t}_j$ $\forall j \in \left\{0, 1, ..., \tilde{L}_c - 1\right\}$. Hence, property 1 is fulfilled by construction. Moreover, the final step of the algorithm shows that σ_δ^* is built causally starting from σ_0^*, so that property 2 holds. Property 3 is also satisfied because it is not possible to find another discrete strategy $\tilde{\sigma}$, not anticipating σ_0^*, such that $d(\tilde{\sigma}, \sigma_0^*) < d(\sigma_\delta^*, \sigma_0^*)$; in fact σ_δ^* equals σ_0^* except for time intervals in which the dwell-time constraint is not fulfilled. In such cases, it guarantees the controlled system to have exactly a dwell time equal to δ. Any other function $\tilde{\sigma}$ such that $d(\tilde{\sigma}, \sigma_0^*) < d(\sigma_\delta^*, \sigma_0^*)$ equals σ_0^* in at least one of the time intervals in which the dwell-time constraint is not fulfilled, so it itself cannot fulfill the constraint. So we can finally deduce that, given the optimal strategy σ_0^*, the strategy σ_δ^* is the "nearest" function satisfying the dwell-time constraint.

Call $T_\delta \subset T$ the subset of the time horizon in which σ_δ^* and σ_0^* are different. Notice that $\lambda(T_\delta) \leq L_c^* \; \delta$. Then consider the transmission energy consumption $J_e(u, \sigma, \tau_c, 0)$ already defined. The δ-configuration stabilizing strategy σ_δ^* uniquely defines the discrete evolution q_δ^* and the continuous sub-optimal control u_δ^*

$$\sigma_\delta^*\left(\tilde{t}_{c,j}\right) = \sigma_{q_\delta^*\left(\tilde{t}_{c,j}\right)} \in \Sigma_c \qquad i = 1, ..., \tilde{L}_c$$

$$u_\delta^*(t) = u_{LB}\left(q_\delta^*(t), t\right).$$

The energy loss between power-optimal and sub-optimal strategy is

$$\begin{aligned}\Delta J_\delta &:= J_e\left(u_\delta^*, \sigma_\delta^*, \tilde{\tau}_c, 0\right) - J_e\left(u_0^*, \sigma_0^*, \tau_c^*, 0\right) = \\ &= \int\limits_{T_\delta} A'(t)\, S(t) \left[u_{LB}\left(q_\delta^*(t), t\right) - u_{LB}\left(q_0^*(t), t\right)\right] dt.\end{aligned}$$

Notice that the integrand is not null in T_δ. Moreover the integration domain is continuous with respect to the variable δ, and $\lim_{\delta \longrightarrow 0} \lambda(T_\delta) = 0$. Hence we can conclude that

Theorem 2. *For any $\varepsilon > 0$, there exists $\delta := \delta(\varepsilon)$ and a δ-configuration stabilizing strategy σ_δ^* such that $\Delta J_\delta := J_e\left(u_\delta^*, \sigma_\delta^*, \tilde{\tau}_c, 0\right) - J_e\left(u_0^*, \sigma_0^*, \tau_c^*, 0\right) < \varepsilon$.*

The previous theorem is a continuity result stating that any energy requirement can be approached with as much precision as desired, by tuning the duration of the dwell-time δ.

Remark 3. Since the functional $J_e(u, \sigma, \tau_c, c)$ is unbounded if the switching cost c grows, then $\forall \delta > 0$, $\exists \bar{c} > 0$ such that for all costs $c \geq \bar{c}$, the sub-optimal control strategy $(\sigma_\delta^*, u_\delta^*)$, with a lower number of switchings than the power-optimal one (σ_0^*, u_0^*), results to be even better than (σ_0^*, u_0^*) in terms of energy consumption:

$$J_e\left(u_\delta^*, \sigma_\delta^*, \tilde{\tau}_c, c\right) \leq J_e\left(u_0^*, \sigma_0^*, \tau_c^*, c\right) \quad \forall c \geq \bar{c}.$$

4 Case Study: Cognitive Networks Based on UWB Technology

Refer, for example, to [10], [13], [14] for the assumptions about UWB Communication.

We consider as set of transmission parameters a set of W waveforms that are used for the pulse shaping. The system specification is expressed in terms of signal-to-noise ratio on a pulse at the correlator output, that has the following expression

$$SNR_p\left(u,q,d,N\right)=\frac{T_S\ u}{d_q+\sigma_m^2\left(q\right)\left(N-1\right)u}$$

where T_S is the chip duration, N is the number of nodes, $\sigma_m^2\left(q\right)$ is the MUI weight for the waveform q and d_q is the external noise power for the waveform q, i.e. the $q-th$ component of the disturbance vector d.

The specification is $SNR_p\left(u,q,d,N\right)\geq SNR_0$ where $SNR_0>0$ is given. It leads to the lower-bound constraint on the minimum received power

$$u\left(t\right)\geq u_{LB}\left(q\left(t\right),t\right)=\frac{SNR_0 d_{q(t)}\left(t\right)}{T_S-SNR_0\sigma_m^2\left(q\left(t\right)\right)\left(N\left(t\right)-1\right)}\qquad\forall t\in T$$

Notice that such a lower bound, corresponding to the minimum signal-to-noise ratio requirement, increases with external noise power and MUI weight. The power-optimal solution is given by the following expressions

$$\begin{cases} q_0^*\left(t\right)=\arg\min\limits_{q\in Q}\left(\frac{SNR_0 d_q(t)}{T_S-SNR_0\sigma_m^2(q)(N(t)-1)}\right) \\ u_0^*\left(t\right)=\frac{SNR_0 d_{q_0^*(t)}(t)}{T_S-SNR_0\sigma_m^2\left(q_0^*(t)\right)(N(t)-1)} \\ \sigma_0^*\left(t_{c,i}^*\right)=\sigma_{q_0^*\left(t_{c,i}^*\right)} \qquad i=1,...,L_c^* \end{cases}\qquad\forall t\in T$$

where τ_c^* is the set of the "induced" controlled switching times, with cardinality L_c^* (see subsection 3.1). Note that the power-optimal hybrid strategy (σ_0^*,u_0^*) can be regarded as an output-feedback hybrid optimal control law.

4.1 Simulations

In order to evaluate the impact of cognition, the cognitive UWB network coexists with several narrowband interferers. The CNode is located at the centre of a circular area with radius $R=10\ m$. The area contains $N=10$ active nodes, not changing during the simulation time. The active users are continuously transmitting data towards the CNode during the whole duration of the simulation. At time t, the N active nodes adopt a generic waveform $w_q(t)$, that can be selected within a set of 6 different waveforms $w_1(t)$, . . . , $w_6(t)$, represented by the first six odd derivatives of the Gaussian pulse. Specifically, we assume that the CNode may order a change in the adopted waveform only at multiples of a given interval.

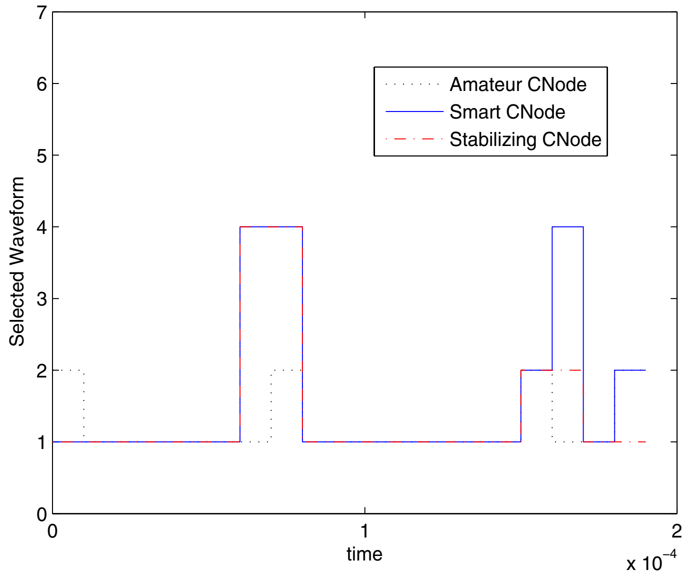

Fig. 2. Waveform Selection

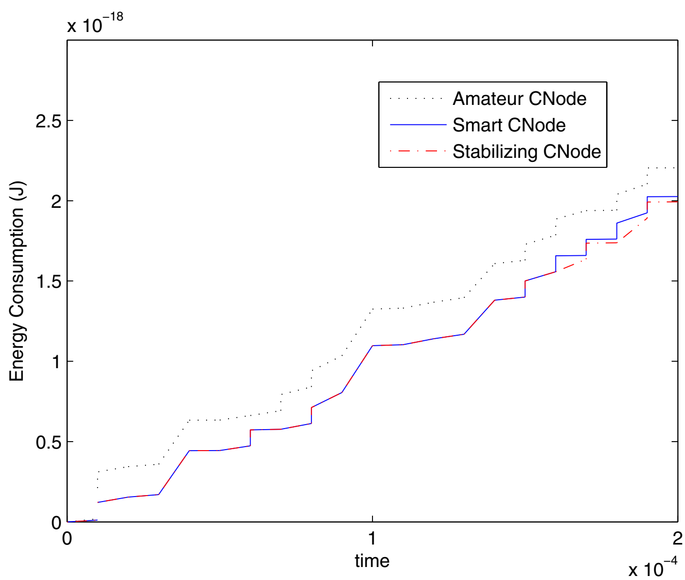

Fig. 3. Power Consumption

The synchronization threshold SNR0 is set to 3 dB, the emitted power of the interfering devices is equal to 10^{-3} W, the simulation is over 20 cognitive intervals, each of 10^{-5} seconds. The minimum dwell-time is set to 2 cognitive intervals and the switching cost is 10^{-19} $Joule$.

We assume that several narrowband interferers are present in the area, not transmitting continuously. In order to highlight the effect of cognition on network coexistence, we consider, for each simulation, four different CNodes that will be compared in terms of performance:

1. The Adaptive CNode (no cognition) initially selects a waveform and does not perform any further selection of the pulse shape during network lifetime.
2. The Amateur CNode (limited cognition) is capable to select the waveform in correspondence of a sub-set of the 6 available waveforms, consisting of the last used waveform and the two adjacent ones. Within this subset, it is capable of selecting the pulse shape minimizing the transmission power.
3. The Smart CNode (optimal cognition) is always capable of selecting the pulse shape that minimizes the transmission power for the active nodes of the network, i.e. it performs the power-optimal hybrid strategy (σ_0^*, u_0^*).
4. The Stabilizing CNode (stabilizing or sub-optimal cognition) guarantees the best trade-off between optimality and stability, performing the sub-optimal control strategy $(\sigma_\delta^*, u_\delta^*)$.

Figures 2 and 3 show the simulation results. The Adaptive CNode (not reported on the plot) chooses the waveform $w_2(t)$ and its network keeps transmitting using this pulse-shaper over the whole simulation. The result is totally unefficient because this choice leads to a power consumption that is about 7 times higher than in the optimal cognitive case. Limited and stabilizing cognitions are much better, requiring between 15% and 20% more power than the optimal CNode. The plots concerning power are not reported due to limited space.

Figure 2 shows that the stabilizing CNode makes only 4 waveform transitions, while the Amateur and the Smart one perform 6 jumps. This leads to a large saving of energy for the stabilizing CNode, such that its final energy consumption $x_2(t_{fin})$ is 2.7 % lower than the one performed by the power-optimal strategy (see Figure 3). This is an example in which Remark (3) holds, and the stabilizing strategy $(\sigma_\delta^*, u_\delta^*)$ is better than the power-optimal one (σ_0^*, u_0^*) in terms of total energy consumption.

5 Conclusions and Open Issues

In this paper, we focused on the hybrid modelling and optimal control of cognitive radio networks. At first, we provided a model of a general network of nodes operating under the cognitive radio paradigm, which abstracts from the physical transmission parameters of the network and focuses on the operation of the control module. Then, we proposed an optimal solution to the power minimization problem. We also introduced the notion of configuration stability and showed that this property is not ensured when the power-optimal control is applied. A control strategy achieving the best compromise between stability and optimality was then derived. Finally, we applied our results to the case of a cognitive network based on UWB technology.

The architecture we analyzed in this paper was centered on the Cognitive Node. An extension of this architecture would be one of multiple CNodes, with each CNode that is responsible for a cluster of nodes. The hierarchical distributed case will be the object of future investigations.

Acknowledgements

The authors would like to thank Daniele Domenicali for interesting discussions on cognitive radio networks and for the use of his UWB Simulator.

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