Discovering Affinities between Perceptual Granules

L_2 Norm-Based Tolerance Near Preclass Approach

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The effectiveness of the pattern recognition process depends mainly on proper representation of these patterns, using the set of characteristic features.

– K.A. Cyran and A. Mrozek, 2001.

Abstract. This paper proposes an approach to detecting affinities between perceptual objects contained in perceptual granules such as images with tolerance near preclasses. A *perceptual object* is something perceptible to the senses or knowable by the mind. Perceptual objects that have similar appearance are considered perceptually near each other, i.e., perceived objects that have perceived affinities or, at least, similar descriptions. A *perceptual granule* is a finite, non-empty set containing sample perceptual objects with common descriptions. Perceptual granules originate from observations of the objects in the physical world. Similarities between perceptual granules are measured within the context of what is known as a tolerance near space. This form of tolerance space is inspired by C.E. Zeeman's work on visual perception and Henri Poincaré's work on the contrast between mathematical continua and the physical continua in a pragmatic philosophy of science that laid the foundations for tolerance spaces. The perception of nearness or closeness that underlies tolerance near relations is rooted in Maurice Merleau-Ponty's work on the phenomenology of perception during the mid-1940s, and, especially, philosophical reflections on description of perceived objects and the perception of nearness. Pairs of perceptual granules such as images are considered near each other to the extent that tolerance near preclasses of sufficient magnitude can be found. The contribution of this paper is the introduction of L_2 norm-based tolerance near preclasses in detecting affinities between images.

Keywords: affinities, image, L_2 norm, perceptual granule, tolerance near preclass.

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1 Introduction

This paper introduces a tolerance near preclass approach to solving the image correspondence problem, i.e., where one uses image tolerance preclasses to detect affinities between pairs of images. Recently, it has been shown that near sets can be used in a perception-based approach to discovering correspondences between images (see, e.g., [6, 7, 8, 16, 17]). Sets of perceptual objects where two or more of the objects have matching descriptions are called near sets. Detecting image resemblance and image description are part of the more general pattern recognition process enunciated by Krzysztof Cyran and Adam Mrózek in 2001 [2]. Work on a basis for near sets began in 2002, motivated by image analysis and inspired by a study of the perception of the nearness of perceptual objects carried out in cooperation with Z. Pawlak in [12]. This initial work led to the introduction of near sets [14], elaborated in [13, 16, 18]. A perception-based approach to discovering resemblances between images leads to a tolerance class form of near sets [16] that models human perception in a physical continuum viewed in the context of image tolerance spaces. A tolerance space-based approach to perceiving image resemblances hearkens back to the observation about perception made by Ewa Orłowska in 1982 [10] (see, also, [11]), i.e., classes defined in an approximation space serve as a formal counterpart of perception.

The term *tolerance space* was coined by E.C. Zeeman in 1961 in modeling visual perception with tolerances [25]. A tolerance space is a set X supplied with a binary relation $\simeq$ (i.e., a subset $\simeq \subset X \times X$) that is reflexive (for all $x \in X$, $x \simeq x$) and symmetric (i.e., for all $x, y \in X$, $x \simeq y$ implies $y \simeq x$) but transitivity of $\simeq$ is not required [23]. For example, it is possible to define a tolerance space relative to subimages of an image. This is made possible by assuming that each image is a set of fixed points. Let O denote a set of perceptual objects (e.g., gray level subimages) and let $\overline{gr}(x)$ = average gray level of subimage x. Then define the tolerance relation

$$\simeq_{\overline{gr},\varepsilon} = \{(x,y) \in O \times O \mid |\overline{gr}(x) - \overline{gr}(y)| \leq \varepsilon\},$$

for some tolerance $\varepsilon \in \Re$ (reals). Then $(O, \simeq_{\overline{gr},\varepsilon})$ is a sample tolerance space. Formulation of a tolerance relation is at the heart of the discovery process in searching for affinities between perceptual granules. The basic idea is to find objects such as images that resemble each other with a tolerable level of error. Sossinsky [23] observes that main idea underlying tolerance theory comes from Henri Poincaré [19]. Physical continua (e.g., measurable magnitudes in the physical world of medical imaging [4]) are contrasted with the mathematical continua (real numbers) where almost solutions are common and a given equation has no exact solution. An *almost solution* of an equation (or a system of equations) is an object which, when substituted into the equation, transforms it into a numerical ‘almost identity, i.e., a relation between numbers which is true only approximately (within a prescribed tolerance) [23]. Equality in the physical world is meaningless, since it can never be verified either in practice or in theory. The study of image tolerance near spaces is directly related to recent work on tolerance spaces (see, e.g., [1, 3, 4, 15, 16, 17, 20, 21, 22, 26]). The contribution of this paper is the introduction of L_2 norm-based tolerance near preclasses useful in detecting affinities between images.

This paper is organized as follows. Section 2 presents the basic framework used to define L_2 norm-based tolerance near relations (Sect. 2.2), tolerance near sets (Sect. 2.3), tolerance near preclasses (Sect. 2.4), and image resemblance measurement (Sect. 2.5).

2 L_2 Norm-Based Tolerance Near Relations

This section gives a brief review of tolerance near sets [16, 17] and introduces tolerance near preclasses. The notation used in this paper is summarized in Table 1.

Table 1 Relation Symbols

Symbol	Interpretation
$\mathbb{R}$	Set of real numbers
O	Set of perceptual objects
X	$X \subseteq O$, set of sample objects
x	$x \in O$, sample object
$\mathbb{F}$	A set of functions representing object features
$\mathscr{B}$	$\mathscr{B} \subseteq \mathbb{F}$
ϕ_i	$\phi_i \in \mathscr{B}$, where $\phi_i : O \longrightarrow \mathbb{R}$, probe function
$\phi(x)$	$\phi(x) = (\phi_1(x), \ldots, \phi_i(x), \ldots, \phi_L(x))$, description
$\langle X, \mathbb{F} \rangle$	$\phi(x_1), \ldots, \phi(x_{\lvert X \rvert})$, i.e., perceptual information system
ε	$\varepsilon \in [0,1]$
$\simeq_{\mathscr{B},\varepsilon}$	$= \{(x,y) \in O \times O \mid \parallel \phi(x) - \phi(y) \parallel \leq \varepsilon\}$, perceptual tolerance relation
$\cong_{\mathscr{B},\varepsilon}$	L_2 norm-based weak tolerance nearness relation
$\underline{\underline{\bowtie}}_{\mathscr{B},\varepsilon}$	generic nearness relation
C^x	$\{(x,y) \in X \times Y \mid x \underline{\underline{\bowtie}} y\}$, tolerance near preclass
$\mathbb{C}^x$	$\{C^x_1, \ldots, C^x_i, \ldots, C^x_k\}$, collection of tolerance near preclasses
$x_{/\cong_{\mathscr{B},\varepsilon}}$	$= \underset{i}{\operatorname{argmax}}\{\lvert C^x_i \rvert \mid C^x_i \in \mathbb{C}^x\}$, maximal tolerance near preclass
$C^{x,th}_i$	$\underset{i}{\operatorname{argmax}}\{\lvert C^x_i \rvert \geq th \mid C^x_i \in \mathbb{C}^x\}$, threshold tolerance near preclass

2.1 L_2 Norm for Images

This section introduces an L_2 norm for images based on the measurment of the length of each vector of feature-value differences extracted from pairs of images. The difference d_i between image feature values is obtained using

$$d_i(\phi_i(x), \phi_i(y)) = \lvert \phi_i(x) - \phi_i(y) \rvert, \phi_i \in B, (x,y) \in X \times Y, i \leq \lvert B \rvert \,,$$

where d_i denotes the ith difference between image feature values. Let $\mathbf{d}^T, \mathbf{d}$ denote row and column vectors of features value differences, respectively, i.e.,

$$\mathbf{d}^T = (d_1, \ldots, d_k), \mathbf{d} = \begin{bmatrix} d_1 \\ \ldots \\ d_k \end{bmatrix}.$$

Finally, the overall distance is the L_2 norm $\parallel \mathbf{d} \parallel_2$ for a vector $\mathbf{d}$ of features value difference measurements, i.e.,

$$\parallel \mathbf{d} \parallel_2 = (\mathbf{d}^T \mathbf{d})^{\frac{1}{2}} = \sqrt{\sum_{i=1}^{k} d_i^2}. \tag{1}$$

In general, $\parallel \cdot \parallel_2$ denotes the length of a vector in L_2 space [9]. The particular $\parallel \mathbf{d} \parallel_2$ norm in (1) provides what can be viewed as a concrescence (gathering together of features value difference measurements in vector $\mathbf{d}$) used to measure resemblances between images. This overall distance metric provides a formal basis for a variety of similarity measures that have exhibited good performance in the image retrieval experiments reported in this paper. An important, direct benefit of (1) is that it makes it possible to test which combinations of features provide good or poor indices of image similarity or degree of dissimilarity. Obviously, some combinations of features work better than others.

2.2 Perceptual Tolerance Relation

In this section, tolerance near sets are defined within the context of a perceptual information system.

Definition 1 (Perceptual Information System). A *perceptual information system* $\langle O, \mathbb{F} \rangle$ or, more concisely, *perceptual system*, is a real valued total deterministic information system where O is a non-empty set of *perceptual objects*, while $\mathbb{F}$ a countable set of *probe functions*.

A *perceptual tolerance relation* is defined in the context of perceptual systems in (2).

Definition 2 (L_2 Norm-Based Perceptual Tolerance Relation [16, 17]). Let $\langle O, \mathbb{F} \rangle$ be a perceptual system and let $\varepsilon \in \Re$ (set of all real numbers). For every $\mathscr{B} \subseteq \mathbb{F}$ the perceptual tolerance relation $\simeq_{\mathscr{B},\varepsilon}$ is defined as:

$$\simeq_{\mathscr{B},\varepsilon} = \{(x, y) \in O \times O : \parallel \mathbf{d} \parallel_2 \leq \varepsilon\}, \tag{2}$$

where $\parallel \cdot \parallel_2$ is the L_2 norm in (1).

Example 1 (Image Tolerance Classes). Figure 1 shows a pair of images, their tolerance class coverings (Fig. 1b, Fig. 1e) and one selected tolerance class relative to a particular image region (Fig. 1c, Fig. 1f, i.e., left eye). Let $\langle O, \mathbb{F} \rangle$ be a perceptual

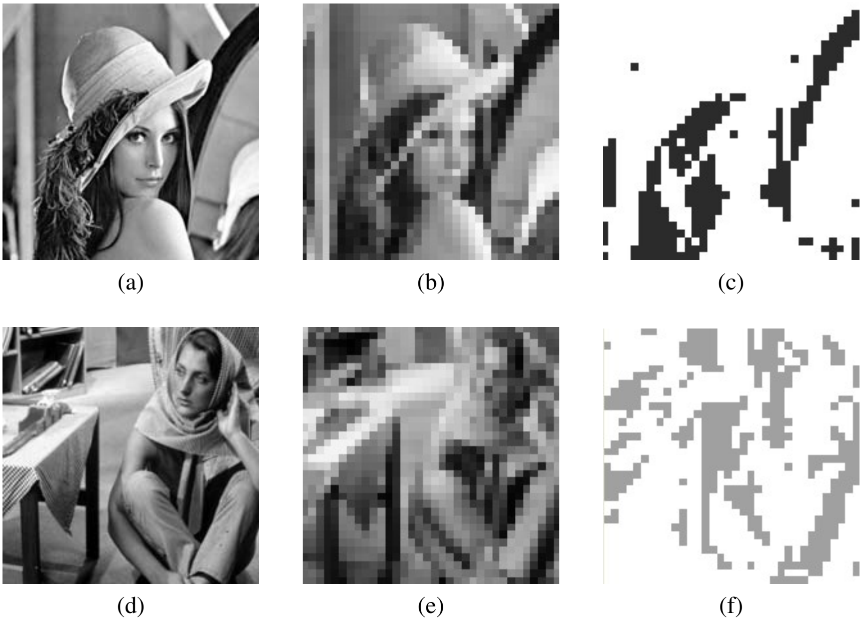

Fig. 1 Sample images and their tolerance classes. **a** Lena (L) **b** Lena Classes **c** L Eye Class **d** Barbara (B) **e** Barb Classes **f** B Eye Class

system where O denotes the set of 25×25 subimages. The image is divided into 100 subimages of size 25×25 and can be shown as a set $X = O$ of all the 100 subimages. Let $\mathscr{B} = \{\phi_1(x)\} \subseteq \mathbb{F}$, where $\phi_1(x) = \overline{gr}(x)$ is the average gray scale value of subimage x between 0 and 255. Let $\varepsilon = 25.5(10\%)$. Observe, for example, the sample tolerance class and containing subimages in Fig. 1c corresponding to Lena's left eye. Again, for example, observe the sample tolerance class and containing subimages in Fig. 1f corresponding to Barbara's left eye. Relative to the subimage containing Lena's eye and Barbara's eye, each tolerance class contains subimages where the difference between average gray scale values of the subimages and the selected subimage are within the prescribed tolerance level ε. Separate image tolerance class coverings for each image provide a basis for measuring the degree that pairs of image resemble each other.

It is now possible to define a weak tolerance nearness relation (see Def. 5), first introduced in [15].

Definition 3 (L_2 Norm-Based Weak Tolerance Nearness Relation [15]). Let $\langle O, \mathbb{F} \rangle$ be a perceptual system and let $\varepsilon \in \Re$ (reals). For every $\mathscr{B} \subseteq \mathbb{F}$ the L_2 norm-based weak tolerance nearness relation $\cong_{\mathscr{B},\varepsilon}$ is defined as:

$$\cong_{\mathscr{B},\varepsilon} = \{(x,y) \in O \times O \mid \ \exists \phi_i \in \mathscr{B}. \ \| \mathbf{d} \|_2 \leq \varepsilon\}, \tag{3}$$

where $\| \cdot \|_2$ is the L_2 norm in (1).

2.3 Tolerance Near Sets

Perceptual systems and tolerance near sets provide a feature-based solution of the image correspondence problem. The basic idea is to discover tolerance classes containing images with descriptions that differ from each other within a preset tolerance. Pairs of images X, Y with partitions defined by a tolerance relation resemble each other in the case where $X \underline{\underline{\bowtie}}_{\mathbb{F}} Y$ for some tolerance ε.

Definition 4 (Tolerance Near Sets [15, 16]). Let $\langle O, \mathbb{F} \rangle$ be a perceptual system and let $X \subseteq O$. A set X is a tolerance near set iff there is $Y \subseteq O$ such that $X \underline{\underline{\bowtie}}_{\mathbb{F}} Y$.

In effect, tolerance perceptual near sets are those sets that are defined by the nearness relation $\underline{\underline{\bowtie}}_{\mathbb{F}}$.

Definition 5 (Weak Tolerance Nearness [15]). Let $\left\langle O, \mathbb{F} \right\rangle$ be a perceptual system and let $X, Y \subseteq O, \varepsilon \in \mathbb{R}$. The set X is perceptually near to the set Y within the perceptual system $\left\langle O, \mathbb{F} \right\rangle$ ($X \underline{\underline{\bowtie}}_{\mathbb{F}} Y$) iff there exists $x \in X$, $y \in Y$ and there is a $\phi \in \mathbb{F}, \varepsilon \in \mathbb{R}$ such that $x \cong_{\mathscr{B}} y$. If a perceptual system is understood, then we say shortly that a set X is perceptually near to a set Y in a weak tolerance sense of nearness.

2.4 Tolerance Near Preclasses

Tolerance preclasses were introduced in [20] and elaborated in [1]. Let $\simeq$ denote a tolerance relation. A subset $C \subseteq \simeq$ defined on a set X is a preclass of the tolerance relation $\simeq$ if, and only if, $\forall \{x, y\} \subseteq C \Rightarrow\ x \simeq\ y$. A maximal preclass of a tolerance relation $\simeq$ is called a class of $\simeq$ (clique induced by $\simeq$ [24]). The main difference between classes in an equivalence relation $\sim$ and tolerance classes in $\simeq$ is the equivalence classes form a pairwise disjoint covering of X. A tolerance near preclass C^x is defined as

$$C^x = \{(x, y) \in X \times X \mid\ x \cong_{\mathscr{B},\varepsilon} y\}.$$

Let $\mathbb{C}^x$ denote a collection of preclasses, i.e.,

$$\mathbb{C}^x = \{C_1^x, \ldots, C_i^x, \ldots, C_k^x\}.$$

Proposition 1. *Let $\cong_{\mathscr{B},\varepsilon}$ be a tolerance near relation defined a set X. A subset $C^x \subseteq X$ if, and only if C^x is contained in some tolerance near class in the covering defined on X by $\cong_{\mathscr{B},\varepsilon}$.*

We can now define a tolerance near class $x_{/\cong_{\mathscr{B},\varepsilon}}$ in terms of a maximal tolerance near preclass, i.e.,

$$x_{/\cong_{\mathscr{B},\varepsilon}} = \underset{i}{\operatorname{argmax}}\{|C_i^x|\ :\ C_i^x \in \mathbb{C}^x\}.$$

In terms of computational efficiency, it makes sense to introduce a threshold-based tolerance near preclass in solving image retrieval and video segmentation problems.

Definition 6 (Threshold-Based Tolerance Near Preclass). Let $\langle O, \mathbb{F} \rangle$ be a perceptual system and let $X \subseteq O$. Let $\mathbb{C}^x$ denote a collection of preclasses in a covering defined on O by a tolerance near relation $\cong_{\mathscr{B},\varepsilon}$. Then $C_i^{x,th} \in \mathbb{C}^x$ denotes a threshold-based tolerance near preclass that is defined as:

$$C_i^{x,th} = \underset{i,th}{\operatorname{argmax}} \{ |C_i^x| \geq th| \ : \ C_i^x \in \mathbb{C}^x \}. \tag{4}$$

The assertion that images X, Y resemble each other is limited to cases where one finds a tolerance near preclass $C_i^{x,th}$ with cardinality above threshold th and stopping short of searching for maximal tolerance near class. This convention assures a more conservative approach to concluding that images are near each other at the $C_i^{x,th}$ preclass level rather concluding that images resemble each other in the minimal preclass cardinality case when $|\mathbb{C}^x| = 2$.

Example 2 (Tolerance Near Preclasses). Assume that we start with a query image X viewed as a set of subimages (denoted by $\circ$ in Fig. 2). The basic approach in detecting pairs of images that sufficient affinity to be classified as near each other (i.e., the degree of resemblance is sufficient), is to define a covering for the pair of images using the tolerance relation $\cong_{\mathscr{B},\varepsilon}$ defined in (3) and then compare a template image Y with the query image X. In other words, we are looking for tolerance near classes containing subimages from X, Y that are within the prescribed tolerance. However, instead of searching for maximal preclasses, the search for images that resemble each other ends whenever we find one or more tolerance near preclasses $C_i^{x,th}$ defined in (4) that have sufficiently high cardinality relative to threshold th. In effect, it is asserted here that the discovery of a tolerance near preclass $C_i^{x,th}$ makes it possible to classify a pair of images as belonging to the same species. The sample scenarios in the bipartite graph in Fig. 2 illustrates this approach, where

$$C_i^{x,1} = \underset{i,1}{\operatorname{argmax}} \{ |C_i^x| \geq 1 \ : \ C_i^x \in \mathbb{C}^x \},$$

$$C_i^{x,2} = \underset{i,2}{\operatorname{argmax}} \{ |C_i^x| \geq 2 \ : \ C_i^x \in \mathbb{C}^x \},$$

$$C_i^{x,3} = \underset{i,3}{\operatorname{argmax}} \{ |C_i^x| \geq 3 \ : \ C_i^x \in \mathbb{C}^x \},$$

$$C_i^{x,4} = \underset{i,4}{\operatorname{argmax}} \{ |C_i^x| \geq 4 \ : \ C_i^x \in \mathbb{C}^x \}.$$

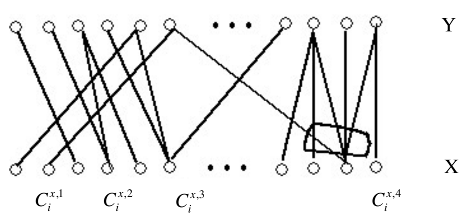

Fig. 2 Near Preclasses

Table 2 Sample Image Nearness Measurements

Features	subimage $n \times n$	tolerance ε	$NM_{\cong_{\mathscr{B},\varepsilon}}(X,Y)$
$\overline{gr}$	25×25	0.01	0.6509 (medium nearness of subimages)
$\overline{gr}, H_S$	25×25	0.01	0.2738
$\overline{gr}, H_S, H_P$	25×25	0.01	0.2299
$\overline{gr}$	25×25	0.10	0.7942 (medium nearness of subimages)
$\overline{gr}, H_S$	25×25	0.10	0.5835
$\overline{gr}, H_S, H_P$	25×25	0.10	0.5830

2.5 *Image Resemblance Measurement*

A complete image nearness measurement system is available at [5]. For example, a nearness measure $NM_{\cong_{\mathscr{B}}}(X,Y)$ with tolerance relation $\cong_{\mathscr{B},\varepsilon}$ [4, 7] (for simplicity, we write $\cong_{\mathscr{B}}$ instead of $\cong_{\mathscr{B},\varepsilon}$) and with weights $|z_{/\cong_{\mathscr{B}}}|$ is:

$$NM_{\cong_{\mathscr{B}}}(X,Y) = \left(\sum_{z_{/\cong_{\mathscr{B}}} \in Z_{/\cong_{\mathscr{B}}}} |z_{/\cong_{\mathscr{B}}}| \right)^{-1} \times \\ \times \sum_{z_{/\cong_{\mathscr{B}}} \in Z_{/\cong_{\mathscr{B}}}} |z_{/\cong_{\mathscr{B}}}| \frac{\min(|[z_{/\cong_{\mathscr{B}}}]_X|, |[z_{/\cong_{\mathscr{B}}}]_Y|)}{\max(|[z_{/\cong_{\mathscr{B}}}]_X|, |[z_{/\cong_{\mathscr{B}}}]_Y|)} . \quad (5)$$

Let $\overline{gr}, H_S, H_P$ denote $n \times n$ subimage average grey level, Shannon entropy, and Pal entropy, respectively. Table 2 gives a summary of measurements carried out on the pair of sample images in Fig. 2.2 and Fig. 2.2. These sample measurements do match our intuition after visual inspection of the images. Not surprisingly, the degree of image nearness decreases as the number of measured features increases. Also, changing ε from very small to larger values tends to inflate the nearness measurements as shown in Table 2.

3 Conclusion

This article introduces an approach to measuring the resemblance between pairs of images using an L_2 norm-based tolerance nearness relation. For the first time, tolerance near preclasses are introduced. It is conjectured that measuring image resemblance can be simplified and computation time reduced by stopping short of comparing maximal tolerance near preclasses made possible with the use of a threshold on preclass size. One obvious benefit of the proposed approach is a more conservative estimation of the resemblance between images. That, if all threshold-based tolerance near preclasses have cardinality below a preset threshold for a sample pair of images, then one concludes *absence of resemblance* for the sample images. This approach has promising implications for segmenting videos, especially in applications where grouping images in a video depends on very refined measurements over many separate images contained in a video.

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