Milk Run Optimization with Delivery Windows and Hedging Against Uncertainty

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1 Introduction

Milk runs are an important transportation concept, e.g. in the automotive industry. Trucks start from a depot, pick up goods at different suppliers, and deliver those goods to a single customer. Therefore, milk runs are technically a pick up and delivery problem with multiple pick ups and a single delivery. They make it possible to deliver small lots efficiently and thus lower average inventory levels. Prerequisite is that there are several frequent orders from suppliers that are closely located, otherwise transshipment centers will be used in spite of handling costs. Pickup&Delivery problems calculate routes for single days, sometimes with the additional restriction of time windows for delivery. Models and algorithms for these problems exist and can help in practice as lead-times are usually very short, in most cases only one day. It has been discussed whether it would be better to allow for delivery windows of a few days so that carriers have more leeway for route optimization (cf. [1]). Now the routing problem is extended with the problem of allocating orders to days. The integrated solution requires vehicles to make multiple trips and is formulated as the VRP with Multiple Trips (VRPM)[1]. The VRPM has found only little attention so far: "Although in practice multiple route assignment is common, there is a shortage of papers covering this feature."[2] It is even more interesting to look at the problem from a dynamic point of view, i.e. to iterate through the days and to assign incoming orders to days without having information on future orders.
Some researchers have tackled similar problems. An early work has

[1] The VRPM is sometimes also referred to as the Vehicle Routing Problem with Multiple Use of Vehicles

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been presented by Brandão and Mercer[3]. Gendreau et al. [4],[5] have repeatedly investigated the problem and presented algorithms for optimal solutions. Zhao et al. [6] also stress the fact that the problem of vehicle routing with multiple use of vehicles within the concerned time horizon is of practical relevance but seldomly studied. They present a tabu search algorithm. A similar approach is given by Taillard et al. [7]. A genetic algorithm for the VRPM is presented by Salhi and Petch [8]. An introduction to online VRP can be found in Allulli et al. [9].

2 Problem Formulation

The four-index vehicle flow formulation is based on the one given by Zhao et al. [6] for the MTCDVRP[2]. It adds three restrictions (4, 7, and 8) which will be discussed. The problem will be called VRPPDMTW[3] and DVRPPDMTW when it adresses the online version.
The objective funtion is:

$$min. \sum_{j}^{N} \sum_{l}^{L} \sum_{t}^{T} x_{0jlt} * f + \sum_{i}^{N} \sum_{j}^{N} \sum_{l}^{L} \sum_{t}^{T} c_{ij} * x_{ijlt} \qquad (1)$$

N is the set of nodes including the depot and the customer. f are the fix costs for each truck. c_{ij} are the costs for traveling from i to j. x_{ijlt} is binary and indicates if truck l travels from i to j in period t. Node 0 will be regarded as the depot, node 1 as the customer. Nodes 2 to n are suppliers.

$$\sum_{i}^{N} \sum_{l}^{L} \sum_{t}^{T} x_{ijlt} = 1 \qquad \forall j \in N \setminus Depot, Customer \qquad (2)$$

$$\sum_{i}^{N} x_{iplt} - \sum_{j}^{N} x_{pjlt} = 0 \qquad \forall p \in N, l \in L, t \in T \qquad (3)$$

Restrictions 2 and 3 let every node except depot and customer be visited exactely once. Also, each node that is visited has to be left as well.

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[2] Multi trip capacity and distance vehicle routing problem

[3] VRP with pickup and delivery and multiple use of vehicles and time windows; note that the solution given in this paper particularly addresses milk runs and is not a generalized pickup and delivery problem

$$\sum_{i}^{N}\sum_{j}^{N} x_{ijlt} \leq x_{1,0,l,t} * M \qquad \forall l \in L, t \in T \tag{4}$$

Restriction 4 assures that every truck in every period travels from the customer to the depot in case it travels to any supplier, thus forming a valid milk run that starts and ends at the depot and visits the customer last.

$$\sum_{i}^{N}\sum_{j}^{N} (d_i * x_{ijlt}) \leq C \qquad \forall l \in L, t \in T \tag{5}$$

$$\sum_{i}^{N}\sum_{j}^{N} (c_{ij} * x_{ijlt}) \leq D \qquad \forall l \in L, t \in T \tag{6}$$

Restrictions 5 and 6 enforce tours to be within capacity limits set by the amount of goods that can be transported by a truck and the distance it can travel.

$$a_j \leq \sum_{i}^{N}\sum_{l}^{L}\sum_{t}^{T} (x_{ijlt} * t) \qquad \forall j \in N \tag{7}$$

$$b_j \geq \sum_{i}^{N}\sum_{l}^{L}\sum_{t}^{T} (x_{ijlt} * t) \qquad \forall j \in N \tag{8}$$

Restrictions 7 and 8 set the time windows for each order. a is the earliest day for pick up, b the latest.

$$\sum_{i \in S}\sum_{j \in S} x_{ijlt} \leq |S| - 1 \qquad S \subseteq N, |S| \geq 2 \qquad \forall l \in L, t \in T \tag{9}$$

Subtour elimination is guaranteed by restriction 9.

3 Test Instance

The data set includes 1,000 orders from 10 suppliers that are within a range of 100 km around the depot. The orders cover 100 days and are served by a fleet of 10 trucks. A truck can transport 300 units over a distance of 500 km. Order sizes are uniformly distributed between 10 and 50 units. 10% of the orders do not have a time window, i.e. the day they have to be served is fixed. If there is a time window, its size is uniformly distributed between 2 and 4 days. 10% of the orders have no leadtime, i.e. they are announced one day in advance. The rest has a leadtime of 2 to 5 days which again is uniformly distributed.

4 Heuristic

The heuristic can run in two different modes. The first mode runs in two stages. In the first stage incoming orders for a certain period are randomly assigned to days. Orders within one day are optimized using the parallel Clark and Wright Savings algorithm. Orders that cannot be served are put on hold. Then, orders are shifted between days using a tabu search algorithm. These stages run in a loop until all orders are assigned to a day or ultimatively have to be rejected. All accepted orders are then fixed which means they cannot be moved in future periods. After that the orders of the next period are looked at. If the period length is the total number of days the solution represents the offline solution. The second mode does not use the tabu search part. It daily receives new orders which have to be fixed on a date. Orders are sorted by descending sizes and then assigned to that day within their time window which currently holds the shortest distance to travel. At this point the method deviates from the tabu search optimization. The latter will only in certain cases shift orders to empty days because this creates tours from and to the depot and customer. This contradicts the myopic goal of shortening routes. The idea is to spread orders over the days to leave buffers in every day so that orders arriving later can still be served. Not being able to serve an order is the worst case.
In contrast to most VRP implementations, the number of vehicles is limited so that the situation may arise that orders have to be rejected. This mimics reality where only a limited number of trucks is available. Further capacity has to sourced from the so-called spot market at a higher price. This resource is currently not included.

5 Results

All results were produced using the heuristic because the size of data sets solvable to optimum by mathematical solvers is limited and less than the size used in this work. The quality of results is expressed by the following formula:

$$\frac{TotalOrderSize - RejectedOrderSize}{TotalDistance} \tag{10}$$

It rewards the amount of orders served and penalizes long tours and rejected orders. The test set described in 3 produced the average results given here:

Table 1. Results

Approach	Quality	Rejected orders
1 day horizon	0.32	105
2 day horizon	0.34	58
5 day horizon	0.49	0
10 day horizon	0.53	0
Offline (= 100 days)	0.57	0
1 day equal distribution	0.56	0

As expected, the offline heuristic performs best and results are getting better the longer the horizon is. The competitive ratio is 1.78 and there are 10.5% missed orders on average when employing a single day planning. The approach of trying to spread the orders equally over all days performs considerably well. Its competitive ratio is 1.02.

6 Conclusion and Outlook

A heuristic was presented which can solve special pickup and delivery problems, so-called milk runs, with time windows and the multiple use of vehicles. It has been shown that methods that deliver good results for the offline version do not necessarily have to deliver good results in the online case, i.e. if decisions have to be made which affect the future which holds considerable uncertainty in terms of additional incoming orders. A simple method, assigning orders to the least booked day, delivered very good results in a test case. However, the impact of certain parameters such as the size of the time window or the leadtime has to be examined in detail. Also, the question of whether the results remain equally good when workload is increased has to be studied closely. Last, it might be interesting to include the spot market to see when orders should be handled externally. The major challenge however is to include other distributions than the uniform distribution regarding the share of orders each supplier holds, the order sizes, and the time windows and leadtimes. This allows for more sophisticated hedging methods and probability calculations.

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