# Appendix A
# Numerical Values for Overlap-Free Words

## A.1 Numerical Values of Chapter 7

We introduce the following auxiliary matrices. For the sake of simplicity our notations do not follow exactly those of [26].

$$D_1 = \begin{pmatrix} 0&0&0&0&0&0&0&1&2&1 \\ 0&0&1&1&0&1&1&0&0&0 \\ 0&0&0&0&0&0&0&1&1&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 1&2&0&0&1&0&0&0&0&0 \\ 0&0&1&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \end{pmatrix},$$

$$B_1 = \begin{pmatrix} 0&0&0&0&0&0&0&1&2&1 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&1&0&0&0 \\ 0&0&1&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0 \end{pmatrix},$$

$$C_1 = \begin{pmatrix} 0&0&0&0&0&0&0&2&4&2 \\ 0&0&1&1&0&1&1&0&0&0 \\ 0&0&0&0&0&1&1&1&1&0 \\ 0&0&1&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&1&0&0&0&0&0 \\ 1&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&2&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \end{pmatrix},$$

$$B_2 = \begin{pmatrix} 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&1&1 \\ 0&1&1&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 1&0&0&0&0 \\ 0&0&0&0&0 \end{pmatrix}, C_2 = \begin{pmatrix} 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&2&0 \\ 0&1&0&0&0 \\ 0&0&0&0&0 \end{pmatrix},$$

$$C_4 = \begin{pmatrix} 0&1&1&1&1 \\ 0&0&0&1&1 \\ 0&1&1&0&0 \\ 1&0&0&0&0 \\ 1&0&0&0&0 \end{pmatrix}.$$

Now, defining

$$F_0 = \left(\begin{array}{cc|cc} C_1 & 0_{10\times 10} & C_2 & 0_{10\times 5} \\ D_1 & B_1 & 0_{10\times 5} & B_2 \\ \hline 0_{5\times 10} & 0_{5\times 10} & C_4 & 0_{5\times 5} \\ 0_{5\times 10} & 0_{5\times 10} & 0_{5\times 5} & 0_{5\times 5} \end{array}\right),$$

$$F_1 = \left(\begin{array}{cc|cc} D_1 & B_1 & 0_{10\times 5} & B_2 \\ 0_{10\times 10} & C_1 & 0_{10\times 5} & C_2 \\ \hline 0_{5\times 10} & 0_{5\times 10} & 0_{5\times 5} & 0_{5\times 5} \\ 0_{5\times 10} & 0_{5\times 10} & 0_{5\times 5} & C_4 \end{array}\right), \tag{A.1}$$

$$w = (1,2,2,2,1,2,2,1,2,1,0_{1\times 20})^T,$$

$$y_8 = (4,4,4,2,0,2,2,0,2,0,0,0,0,0,2,6,4,4,2,4,2,0,4,2,2,0,0,0,0,0)^T,$$

$$y_9 = (6,4,4,2,4,2,0,4,2,2,0,0,0,0,0,8,4,4,2,0,4,4,4,0,0,0,0,0,0,0)^T,$$

$$y_{10} = (8,4,4,2,0,4,4,4,0,0,0,0,0,0,0,8,4,6,4,8,2,0,4,2,4,0,0,0,0,0)^T,$$

$$y_{11} = (8,4,6,4,8,2,0,4,2,4,0,0,0,0,0,8,6,6,2,0,2,6,4,2,0,2,0,2,2,0)^T,$$

$$y_{12} = (8,6,6,2,0,2,6,4,2,0,2,0,2,2,0,10,6,4,4,8,2,0,4,2,4,0,0,0,0,0)^T,$$

$$y_{13} = (10,6,4,4,8,2,0,4,2,4,0,0,0,0,0,12,6,4,4,0,6,6,4,2,0,0,0,0,0,0)^T,$$
$$y_{14} = (12,6,4,4,0,6,6,4,2,0,0,0,0,0,0,10,6,8,6,12,4,0,0,4,4,0,0,0,0,0)^T,$$
$$y_{15} = (10,6,8,6,12,4,0,0,4,4,0,0,0,0,0,8,10,6,6,0,4,8,4,4,0,2,2,0,0,0)^T,$$

and introducing the recurrence relation

$$y_{2n} = F_0 y_n, \quad y_{2n+1} = F_1 y_n, \quad n \geq 8,$$

one has the relation [26]:

$$u_{n+1} = w^T y_n. \tag{A.2}$$

We finally introduce two new matrices in $\mathbb{R}^{20\times 20}$ that rule the asymptotics of $u_n$ :

$$A_0 = \begin{pmatrix} C_1 & 0_{10\times 10} \\ D_1 & B_1 \end{pmatrix}, A_1 = \begin{pmatrix} D_1 & B_1 \\ 0_{10\times 10} & C_1 \end{pmatrix}. \tag{A.3}$$

## A.2 The Ellipsoidal Norm

Define

$$P_1 = \begin{pmatrix} 313 & 75 & 23 & 33 & -4 & -3 & 3 & 4 & 37 & 03 \\ 75 & 577 & 100 & 63 & 184 & 350 & 163 & -58 & 138 & 50 \\ 23 & 100 & 599 & 113 & 4 & 292 & 42 & 101 & 82 & 08 \\ 33 & 63 & 113 & 485 & 46 & 135 & 108 & 20 & 69 & 10 \\ -4 & 184 & 4 & 46 & 364 & 235 & 226 & 44 & 89 & -12 \\ -3 & 350 & 292 & 135 & 235 & 1059 & 384 & 95 & 337 & 61 \\ 3 & 163 & 42 & 108 & 226 & 384 & 590 & 27 & 174 & 92 \\ 4 & -58 & 101 & 20 & 44 & 95 & 27 & 386 & 148 & -17 \\ 37 & 138 & 82 & 69 & 89 & 337 & 174 & 148 & 575 & 86 \\ 3 & 50 & 8 & 10 & -12 & 61 & 92 & -17 & 86 & 423 \end{pmatrix},$$

$$P_2 = \begin{pmatrix} -104 & -17 & -181 & -4 & -58 & -51 & -49 & -8 & -27 & -9 \\ -111 & -224 & -82 & -147 & -99 & -303 & -167 & -113 & -169 & -66 \\ -22 & -164 & -158 & -50 & -85 & -72 & -54 & -185 & -35 & -34 \\ -2 & -136 & -52 & -90 & -107 & -146 & -92 & -16 & -113 & -11 \\ -46 & -170 & -130 & -91 & -6 & -112 & -239 & -70 & -121 & 3 \\ -59 & -264 & -274 & -174 & -310 & -376 & -280 & -44 & -273 & -74 \\ -14 & -193 & -116 & -108 & -223 & -179 & -117 & -113 & -120 & -98 \\ -63 & 21 & 17 & -34 & 32 & -76 & 2 & -52 & -31 & -14 \\ -74 & -159 & -47 & -67 & -122 & -173 & -116 & -53 & -68 & -16 \\ 13 & -57 & -36 & -32 & -4 & -61 & -90 & -14 & -69 & 4 \end{pmatrix},$$

$$P_4 = \begin{pmatrix} 291 & 83 & -16 & 48 & -13 & -44 & 6 & 17 & 75 & 11 \\ 83 & 473 & 136 & 28 & 117 & 198 & 174 & 6 & 100 & 37 \\ -16 & 136 & 466 & 104 & 65 & 249 & 118 & 65 & 125 & 14 \\ 48 & 28 & 104 & 476 & 51 & 80 & 76 & 51 & 37 & 18 \\ -13 & 117 & 65 & 51 & 328 & 195 & 194 & 76 & 67 & -2 \\ -44 & 198 & 249 & 80 & 195 & 648 & 162 & 114 & 138 & 68 \\ 6 & 174 & 118 & 76 & 194 & 162 & 567 & 76 & 122 & 65 \\ 17 & 6 & 65 & 51 & 76 & 114 & 76 & 387 & 112 & -10 \\ 75 & 100 & 125 & 37 & 67 & 138 & 122 & 112 & 556 & 42 \\ 11 & 37 & 14 & 18 & -2 & 68 & 65 & -10 & 42 & 438 \end{pmatrix},$$

$$P = \begin{pmatrix} P_1 & P_2 \\ P_2^T & P_4 \end{pmatrix}.$$

Then one has the relations:

$$A^t PA - (2.5186)^{28} P \prec 0, \quad \forall A \in \Sigma^{14}.$$

As explained in Chapter 2 Section 2.3, this suffices to prove that $\rho(\Sigma) \leq 2.5186$.

## A.3 The Vector $x$

Define

$$x = (153, 0, 60, 0, 50, 56, 99, 0, 58, 1, 157, 81, 0, 113, 0, 72, 0, 99, 0, 0)^T.$$

Then, for all $B \in \Sigma^6$ and $A \in \Sigma^{16}$, one has the relation

$$\begin{aligned} B(Ax - rx) &\geq 0, \\ x &\geq 0, \end{aligned} \tag{A.4}$$

with $r = 2.41^{16}$. This proves that $\check{\rho}(\Sigma) \geq 2.41$.