Dual P Systems

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Abstract. This paper aims to answer the following question: given a P system configuration M, how do we find each configuration N such that N evolves to M in one step? While easy to state, the problem has not a simple answer. To provide a solution to this problem for a general class of P systems with simple communication rules and without dissolution, we introduce the dual P systems. Essentially these systems reverse the rules of the initial P system and find N by applying reversely valid multisets of rules. We prove that in this way we find exactly those configurations N which evolve to M in one step.

1 Introduction

Often when solving a (mathematical) problem, one starts from the end and tries to reach the hypothesis. P systems [4] are often used to solve problems, so finding a method which allows us to go backwards is of interest. When looking at a cell-like P system with rules which only involve object rewriting (of type $u \to v$, where u, v are multisets of objects) in order to reverse a computation it is natural to reverse the rules ($u \to v$ becomes $v \to u$) and find a condition equivalent to maximal parallelism. The dual P system $\widetilde{\Pi}$ is the one with the same membranes as Π and the rules of Π reversed. However, when rules of type $u \to (v, out)$ or $u \to (v, in_{child})$ are used, two ways of reversing computation appear. The one we focus on is to employ a special type of rule reversal and to move the rules between membranes: for example, $u \to (v, out)$ associated to the membrane with label i in Π is replaced with $v \to (u, in_i)$ associated to the membrane with label $parent(i)$ in $\widetilde{\Pi}$. This is described in detail in Section 4. Another way of defining the dual P system is by reversing all the rules without moving them between membranes (and thus allow rules of form $(v, out) \to u$). To capture the backwards computation we have to move objects according to the existence of communicating rules in the P system. The object movement corresponds to reversing the message sending stage of the evolution of a membrane. After that the maximally parallel rewriting stage is reversed. This is only sketched in Section 5 as a starting point for further research.

The structure μ of a P system is represented by a tree structure (with the *skin* as its root), or equivalently, by a string of correctly matching parentheses, placed

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in a unique pair of matching parentheses; each pair of matching parentheses corresponds to a membrane. Graphically, a membrane structure is represented by a Venn diagram in which two sets can be either disjoint, or one a subset of the other. The membranes are labeled in a one-to-one manner. A membrane without any other membrane inside is said to be elementary.

A *membrane system* of degree m is a tuple $\Pi = (O, \mu, w_1, \ldots w_m, R_1, \ldots, R_m, i_o)$ where:

- O is an alphabet of objects;
- μ is a membrane structure, with the membranes labeled by natural numbers $1, \ldots, m$, in a one-to-one manner;
- w_i are multisets over O associated with the regions $1, \ldots, m$ defined by μ;
- $R_1, \ldots, R_m$ are finite sets of rules associated with the membranes with labels $1, \ldots, m$; the rules have the form $u \to v$, where u is a non-empty multiset of objects and v a multiset over messages of the form $(a, here), (a, out), (a, in_j)$;

The membrane structure μ and the multisets of objects and messages from its compartments define a *intermediate configuration* of a P system. If the multisets from its compartments contain only objects, they define a *configuration*. For a intermediate configuration M we denote by $w_i(M)$ the multiset contained in the inner membrane with label i. We denote by $\mathcal{C}^{\#}(\Pi)$ the set of intermediate configurations and by $\mathcal{C}(\Pi)$ the set of configurations of the P system Π.

Since we work with two P systems at once (namely Π and $\widetilde{\Pi}$), we use the notation $R_1^{\Pi}, \ldots, R_m^{\Pi}$ for the sets of rules $R_1, \ldots, R_m$ of the P system Π.

We consider a multiset w over a set S to be a function $w : S \to \mathbf{N}$. When describing a multiset characterized by, for example, $w(s) = 1, w(t) = 2$, $w(s') = 0, s' \in S \backslash \{s, t\}$, we use its string representation $s + 2t$, to simplify its description. To each multiset w we associate its support, denoted by $supp(w)$, which contains those elements of S which have a non-zero image. A multiset is called non-empty if it has non-empty support. We denote the empty multiset by 0_S. The sum of two multisets w, w' over S is the multiset $w + w' : S \to \mathbf{N}, (w + w')(s) = w(s) + w'(s)$. For two multisets w, w' over S we say that w is contained in w' if $w(s) \leq w'(s), \forall s \in S$. We denote this by $w \leq w'$. If $w \leq w'$ we can define $w' - w$ by $(w' - w)(s) = w'(s) - w(s)$. To work in a uniform manner, we consider all multisets of objects and messages to be over

$$\Omega = O \cup O \times \{out\} \cup O \times \{in_j \mid j \in \{1, \ldots, m\}\}$$

Definition 1. *The set $\mathcal{M}(\Pi)$ of membranes in a P system Π together with the membrane structure are inductively defined as follows:*

- *if i is a label and w is a multiset over $O \cup O \times \{out\}$ then $\langle i|w \rangle \in \mathcal{M}(\Pi)$; $\langle i|w \rangle$ is called an* elementary membrane*, and its structure is $\langle \rangle$;*
- *if i is a label, $M_1, \ldots, M_n \in \mathcal{M}(\Pi), n \geq 1$ have distinct labels $i_1, \ldots, i_n$, each M_k has structure μ_k and w is a multiset over $O \cup O \times \{out\} \cup O \times \{in_{i_1}, \ldots, in_{i_n}\}$ then $\langle i|w; M_1, \ldots, M_n \rangle \in \mathcal{M}(\Pi)$; $\langle i|w; M_1, \ldots, M_n \rangle$ is called a* composite membrane*, and its structure is $\langle \mu_1 \ldots \mu_n \rangle$.*

Note that if i is the label of the skin membrane then $\langle i|w; M_1, \ldots, M_n\rangle$ defines an intermediate configuration.

We use the notations $parent(i)$ for the label indicating the parent of the membrane labeled by i (if it exists) and $children(i)$ for the set of labels indicating the children of the membrane labeled by i, which can be empty.

By *simple* communication rules we understand that all rules inside membranes are of the form $u \to v$ where u is a multiset of objects ($supp(u) \subseteq O$) and v is either a multiset of objects, or a multiset of objects with the message in_j ($supp(v) \subseteq O \times \{in_j\}$ for a $j \in \{1, \ldots, m\}$) or a multiset of objects with the message out ($supp(v) \subseteq O \times \{out\}$). Moreover we suppose that the *skin* membrane does not have any rules involving objects with the message *out*.

We use multisets of rules $\mathcal{R} : R_i^{\Pi} \to \mathbf{N}$ to describe maximally parallel application of rules. For a rule $r : u \to v$ we use the notations $lhs(r) = u, rhs(r) = v$. Similarly, for a multiset $\mathcal{R}$ of rules from R_i^{Π}, we define the following multisets over Ω:

$$lhs(\mathcal{R})(o) = \sum_{r \in R_i^{\Pi}} \mathcal{R}(r) \cdot lhs(r)(o) \text{ and } rhs(\mathcal{R})(o) = \sum_{r \in R_i^{\Pi}} \mathcal{R}(r) \cdot rhs(r)(o)$$

for each object or message $o \in \Omega$. The following definition captures the meaning of "maximally parallel application of rules":

Definition 2. *We say that a multiset of rules $\mathcal{R} : R_i^{\Pi} \to \mathbf{N}$ is* valid *in the multiset w if $lhs(\mathcal{R}) \leq w$. The multiset $\mathcal{R}$ is called* maximally valid *in w if it is valid in w and there is no rule $r \in R_i^{\Pi}$ such that $lhs(r) \leq w - lhs(\mathcal{R})$.*

2 P Systems with One Membrane

Suppose that the P system Π consists only of the *skin* membrane, labeled by 1. Since the membrane has no children and we have assumed it has no rules concerning *out* messages, all its rules are of form $u \to v$, with $supp(u), supp(v) \subseteq O$. Given the configuration M in the system $\Pi = (O, \mu, w_1, R_1^{\Pi})$ we want to find all configurations N such that N rewrites to M in a single maximally parallel rewriting step. To do this we define the dual P system $\widetilde{\Pi} = (O, \mu, w_1, R_1^{\widetilde{\Pi}})$, with evolution rules given by:

$$(u \to v) \in R_1^{\widetilde{\Pi}} \text{ if and only if } (v \to u) \in R_1^{\Pi}$$

For each $M = \langle 1|w\rangle \in \mathcal{C}^{\#}(\Pi)$, we consider the dual intermediate configuration $\widetilde{M} = \langle 1|w\rangle \in \mathcal{C}^{\#}(\widetilde{\Pi})$ which has the same content ($w = w_1(\widetilde{M}) = w_1(M)$) and membrane structure as M. Note that the dual of a configuration is a configuration. The notation $\widetilde{M}$ is used to emphasize that it is an intermediate configuration of the system $\widetilde{\Pi}$.

The name *dual* is used for the P system $\widetilde{\Pi}$ under the influence of category theory, where the dual category is the one obtained by reversing all arrows.

Remark 1. Note that using the term of *dual* for $\widetilde{\Pi}$ is appropriate because $\widetilde{\widetilde{\Pi}} = \Pi$.

When we reverse the rules of a P system, dualising the maximally parallel application of rules requires a different concept than the *maximal validity* of a multiset of rules.

Definition 3. *The multiset* $\mathcal{R} : R_i^{\Pi} \to \mathbf{N}$ *is called* reversely valid *in the multiset* w *if it is valid in* w *and there is no rule* $r \in R_i^{\Pi}$ *such that* $rhs(r) \leq w - lhs(\mathcal{R})$.

Note that the difference from *maximally valid* is that here we use the right-hand side of a rule r in $rhs(r) \leq w - lhs(\mathcal{R})$, instead of the left-hand side.

Example 1. Consider the configuration $M = \langle 1|b + c\rangle$, in the P system $\widetilde{\Pi}$ with $O = \{a, b, c\}$, $\mu = \langle\rangle$ and with evolution rules $R_1^{\Pi} = \{r_1, r_2\}$, where $r_1 : a \to b$, $r_2 : b \to c$. Then $\widetilde{M} = \langle 1|b + c\rangle \in \mathcal{C}(\widetilde{\Pi})$, with evolution rules $R_1^{\widetilde{\Pi}} = \{\widetilde{r_1}, \widetilde{r_2}\}$, where $\widetilde{r_1} : b \to a$, $\widetilde{r_2} : c \to b$. The valid multisets of rules in $w_1(\widetilde{M}) = b + c$ are $0_{R_1^{\widetilde{\Pi}}}, \widetilde{r_1}, \widetilde{r_2}$ and $\widetilde{r_1} + \widetilde{r_2}$. The reversely valid multiset of rules $\widetilde{\mathcal{R}}$ in $w(\widetilde{M_1})$ can be either $\widetilde{r_1}$ or $\widetilde{r_1} + \widetilde{r_2}$. If $\widetilde{\mathcal{R}} : \widetilde{r_1}$ then $\widetilde{M}$ rewrites to $\langle 1|a + c\rangle$; if $\widetilde{\mathcal{R}} : \widetilde{r_1} + \widetilde{r_2}$ then $\widetilde{M}$ rewrites to $\langle 1|a + b\rangle$. These yield the only two configurations that can evolve to M in one maximally parallel rewriting step (in Π). This example clarifies why reversely valid multisets of rules must be applied: validity ensures that some objects are consumed by rules $\widetilde{r}$ (dually, they were produced by some rules r) and reverse validity ensures that objects like b (appearing in both the left and right-hand sides of rules) are always consumed by rules $\widetilde{r}$ (dually, they were surely produced by some rules r, otherwise it would contradict maximal parallelism for the multiset $\mathcal{R}$).

Note that if $M' = \langle 1 \mid 2a\rangle$ in the P system Π, then there is no multiset of rules $\widetilde{\mathcal{R}}$ valid in $w_1(\widetilde{M'}) = 2a$ for the dual $\widetilde{M'}$. This happens exactly because there is no configuration N' such that N' rewrites to M' by applying at least one of the rules r_1, r_2.

We present the operational semantics for both maximally parallel application of rules (mpr) and inverse maximally parallel application of rules ($\widetilde{mpr}$) on configurations in a P system with one membrane.

Definition 4

- $\langle 1|w\rangle \xrightarrow{\mathcal{R}}_{mpr} \langle 1|w - lhs(\mathcal{R}) + rhs(\mathcal{R})\rangle$ *if and only if* $\mathcal{R}$ *is maximally valid in* w*;*
- $\langle 1|w\rangle \xrightarrow{\mathcal{R}}_{\widetilde{mpr}} \langle 1|w - lhs(\mathcal{R}) + rhs(\mathcal{R})\rangle$ *if and only if* $\mathcal{R}$ *is reversely valid in* w*.*

The difference between the two semantics is coming from the difference between the conditions imposed on the multiset $\mathcal{R}$ (maximally valid and reversely valid, respectively).

For a multiset $\mathcal{R}$ of rules over R_1^{Π} we denote by $\widetilde{\mathcal{R}}$ the multiset of rules over $R_1^{\widetilde{\Pi}}$ for which $\widetilde{\mathcal{R}}(u \to v) = \mathcal{R}(v \to u)$. Then $lhs(\mathcal{R}) = rhs(\widetilde{\mathcal{R}})$ and $rhs(\mathcal{R}) = lhs(\widetilde{\mathcal{R}})$.

Proposition 1. $N \xrightarrow{\mathcal{R}}_{mpr} M$ *if and only if* $\widetilde{M} \xrightarrow{\widetilde{\mathcal{R}}}_{\widetilde{mpr}} \widetilde{N}$*.*

Proof. If $N \xrightarrow{\mathcal{R}}_{mpr} M$ then $\mathcal{R}$ is maximally valid in $w_1(N)$ and $w_1(M) = w_1(N) - lhs(\mathcal{R}) + rhs(\mathcal{R})$; then $w_1(M) - rhs(\mathcal{R}) = w_1(N) - lhs(\mathcal{R})$. By duality, we have

$w_1(M) = w_1(\widetilde{M})$ and $rhs(\mathcal{R}) = lhs(\widetilde{\mathcal{R}})$; it follows that $w_1(\widetilde{M}) - lhs(\widetilde{\mathcal{R}}) = w_1(N) - lhs(\mathcal{R}) \geq 0$, therefore $lhs(\widetilde{\mathcal{R}}) \leq w_1(\widetilde{M})$, and so $\widetilde{\mathcal{R}}$ is valid in $\widetilde{M}$. Suppose $\widetilde{\mathcal{R}}$ is not reversely valid in $w_1(\widetilde{M})$, i.e., there exists $\widetilde{r} \in R_1^{\widetilde{\Pi}}$ such that $rhs(\widetilde{r}) \leq w_1(\widetilde{M}) - lhs(\widetilde{\mathcal{R}})$, which is equivalent to $lhs(r) \leq w_1(M) - rhs(\mathcal{R})$. Since $w_1(M) - rhs(\mathcal{R}) = w_1(N) - lhs(\mathcal{R})$ it follows that $\mathcal{R}$ is not maximally valid in $w_1(N)$, which yields a contradiction.

If $\widetilde{M} \xrightarrow{\widetilde{\mathcal{R}}}_{\widetilde{mpr}} \widetilde{N}$ then $\widetilde{\mathcal{R}}$ is reversely valid in $w_1(\widetilde{M})$; since $w_1(N) - lhs(\mathcal{R}) = w_1(\widetilde{M}) - lhs(\widetilde{\mathcal{R}}) \geq 0$ it follows that $\mathcal{R}$ is valid in $w_1(N)$. If we suppose that $\mathcal{R}$ is not maximally valid in $w_1(N)$ then, reasoning as above, we obtain that $\widetilde{\mathcal{R}}$ is not reversely valid in $w_1(\widetilde{M})$(contradiction). $\square$

3 P Systems without Communication Rules

If the P system has more than one membrane but it has no communication rules (i.e., no rules of form $u \to v$, with $supp(v) \subseteq O \times \{out\}$ or $supp(v) \subseteq O \times \{in_j\}$) the method of reversing the computation is similar to that described in the previous section. We describe it again but in a different way, since here we introduce the notion of a (valid) system of multisets of rules for a P system Π. This notion is useful for P systems without communication rules, and is fundamental in reversing the computation of a P system with communication rules. This section provides a technical step from Section 2 to Section 4.

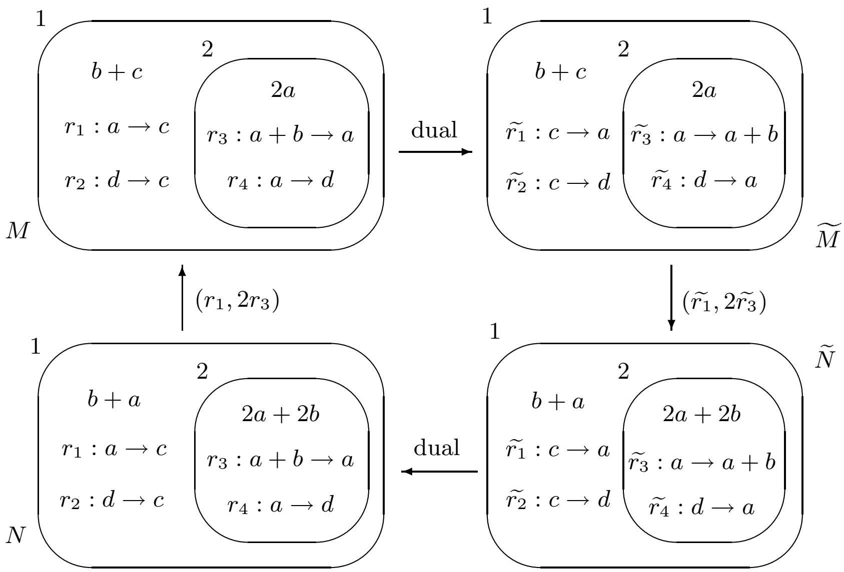
Definition 5. *A* system *of multisets of rules for a P system Π of degree m is a tuple $\mathcal{R} = (\mathcal{R}_1, \mathcal{R}_2, \ldots, \mathcal{R}_m)$, where each $\mathcal{R}_i$ is a multiset over R_i^{Π}, $i \in \{1, \ldots, m\}$.*

A system of multisets of rules $\mathcal{R}$ is called *valid, maximally valid* or *reversely valid* in the configuration M if each $\mathcal{R}_i$ is valid, maximally valid or reversely valid in the multiset $w_i(M)$, which, we recall, is the multiset contained in the inner membrane of configuration M which has label i.

The P system $\widetilde{\Pi}$ dual to the P system Π is defined analogously to the one in Section 2: $\widetilde{\Pi} = (O, \mu, w_1, \ldots w_m, R_1^{\widetilde{\Pi}}, \ldots, R_m^{\widetilde{\Pi}})$ where $(u \to v) \in R_1^{\widetilde{\Pi}}$ if and only if $(v \to u) \in R_1^{\Pi}$. Note that $\widetilde{\widetilde{\Pi}} = \Pi$.

If $\mathcal{R} = (\mathcal{R}_1, \ldots, \mathcal{R}_m)$ is a system of multisets of rules for a P system Π, we denote by $\widetilde{\mathcal{R}}$ the system of multisets of rules for the dual P system $\widetilde{\Pi}$ given by $\widetilde{\mathcal{R}} = (\widetilde{\mathcal{R}_1}, \ldots, \widetilde{\mathcal{R}_2})$.

Example 2. Consider the configuration $M = \langle 1|b + c; N\rangle$, $N = \langle 2|2a\rangle$ of the P system Π with evolution rules $R_1^{\Pi} = \{r_1, r_2\}, R_2^{\Pi} = \{r_3, r_4\}$, where $r_1 : a \to c$, $r_2 : d \to c$, $r_3 : a + b \to a$, $r_4 : a \to d$. Then $\widetilde{M} = \langle 1|b + c; \langle 2|2a\rangle\rangle$, with evolution rules $R_1^{\widetilde{\Pi}} = \{\widetilde{r_1}, \widetilde{r_2}\}$, $R_2^{\widetilde{\Pi}} = \{\widetilde{r_3}, \widetilde{r_4}\}$, where $\widetilde{r_1} : c \to a$, $\widetilde{r_2} : c \to d$, $\widetilde{r_3} : a \to a + b$, $\widetilde{r_4} : d \to a$. In order to find all membranes which evolve to M in one step, we look for a system $\widetilde{\mathcal{R}} = (\widetilde{\mathcal{R}_1}, \widetilde{\mathcal{R}_2})$ of multisets of rules, which is reversely valid in the configuration $\widetilde{M}$. Then $\widetilde{\mathcal{R}_1}$ can be either $0_{R_1^{\widetilde{\Pi}}}, \widetilde{r_1}$ or $\widetilde{r_2}$ and the only possibility for $\widetilde{\mathcal{R}_2}$ is $2\widetilde{r_3}$. We apply $\widetilde{\mathcal{R}}$ to the *skin* membrane $\widetilde{M}$ and we obtain three possible configurations $\widetilde{P}$ such that $P \Rightarrow M$; namely, P can be either $\langle 1|b+c; \langle 2|2a+2b\rangle\rangle$ or $\langle 1|b+a; \langle 2|2a+2b\rangle\rangle$ or $\langle 1|b+d; \langle 2|2a+2b\rangle\rangle$.



We give a definition of the operational semantics for both maximally parallel application of rules (mpr) and inverse maximally parallel application of rules ($\widetilde{mpr}$) in a P system without communication rules. We use $\mathcal{R}$ as label to suggest that rule application is done simultaneously in all membranes, and thus to prepare the way toward the general case of P systems with communication rules.

Definition 6. *For $M, N \in \mathcal{C}(\Pi)$ we define:*

- $M \xrightarrow{\mathcal{R}}_{mpr} N$ *if and only if* $\mathcal{R} = (\mathcal{R}_1, \ldots, \mathcal{R}_m)$ *is maximally valid in* M *and* $w_i(N) = w_i(M) - lhs(\mathcal{R}_i) + rhs(\mathcal{R}_i)$*;*
- $M \xrightarrow{\mathcal{R}}_{\widetilde{mpr}} N$ *if and only if* $\mathcal{R} = (\mathcal{R}_1, \ldots, \mathcal{R}_m)$ *is reversely valid in* M *and* $w_i(N) = w_i(M) - lhs(\mathcal{R}_i) + rhs(\mathcal{R}_i)$*.*

The two operational semantics are similar in their effect on the membranes, but differ in the conditions required for the multisets of rules $\mathcal{R}$.

Proposition 2. *If $N \in \mathcal{C}(\Pi)$, then*

$$N \xrightarrow{\mathcal{R}}_{mpr} M \text{ if and only if } \widetilde{M} \xrightarrow{\widetilde{\mathcal{R}}}_{\widetilde{mpr}} \widetilde{N}$$

Proof. If $N \xrightarrow{\mathcal{R}}_{mpr} M$ then $\mathcal{R}$ is maximally valid in the configuration N, which means that $\mathcal{R}_i$ is maximally valid in $w_i(N)$, and $w_i(M) = w_i(N) - lhs(\mathcal{R}_i) + rhs(\mathcal{R}_i)$. By using the same reasoning as in the proof of Proposition 1 it follows that $\widetilde{\mathcal{R}_i}$ is reversely valid in $w_i(\widetilde{M})$, for all $i \in \{1, \dots, m\}$. Therefore $\widetilde{\mathcal{R}}$ is reversely valid in the configuration $\widetilde{M}$ of the dual P system $\widetilde{\Pi}$. Moreover, we have $w_i(\widetilde{N}) = w_i(\widetilde{M}) - lhs(\widetilde{\mathcal{R}_i}) + rhs(\widetilde{\mathcal{R}_i})$, so $\widetilde{M} \xrightarrow{\widetilde{\mathcal{R}}}_{\widetilde{mpr}} \widetilde{N}$.

If $\widetilde{M} \xrightarrow{\widetilde{\mathcal{R}}}_{\widetilde{mpr}} \widetilde{N}$ the proof follows in the same manner. □

4 P Systems with Communication Rules

When the P system has communication rules we no longer can simply reverse the rules and obtain a reverse computation; we also have to move the rules between membranes. When saying that we move the rules we understand that the dual system can have rules $\widetilde{r}$ associated to a membrane with label i while r is associated to a membrane with label j (j is either the parent or the child of i, depending on the form of r). We need a few notations before we start explaining in detail the movement of rules.

If u is a multiset of objects ($supp(u) \subseteq O$) we denote by (u, out) the multiset with $supp(u, out) \subseteq O \times \{out\}$ given by $(u, out)(a, out) = u(a)$, for all $a \in O$. More explicitly, (u, out) has only messages of form (a, out), and their number is that of the objects a in u. Given a label j, we define (u, in_j) similarly: $supp(u, in_j) \subseteq O \times \{in_j\}$ and $(u, in_j)(a, in_j) = u(a)$, for all $a \in O$.

The P system $\widetilde{\Pi}$ dual to the P system Π is defined differently from the case of P systems without communication rules: $\widetilde{\Pi} = (O, \mu, w_1, \dots w_m, R_1^{\widetilde{\Pi}}, \dots, R_m^{\widetilde{\Pi}})$ such that:

1. $\widetilde{r} = u \to v \in R_i^{\widetilde{\Pi}}$ if and only if $r : v \to u \in R_i^{\Pi}$;
2. $\widetilde{r} : u \to (v, out) \in R_i^{\widetilde{\Pi}}$ if and only if $r : v \to (u, in_i) \in R_{parent(i)}^{\Pi}$;
3. $\widetilde{r} : u \to (v, in_j) \in R_i^{\widetilde{\Pi}}$ if and only if $r : v \to (u, out) \in R_j^{\Pi}$, $i = parent(j)$;

where u, v are multisets of objects. Note the difference between rule duality when there are no communication rules and the current class of P systems with communication rules.

Proposition 3. *The dual of the dual of a P system is the initial P system:*

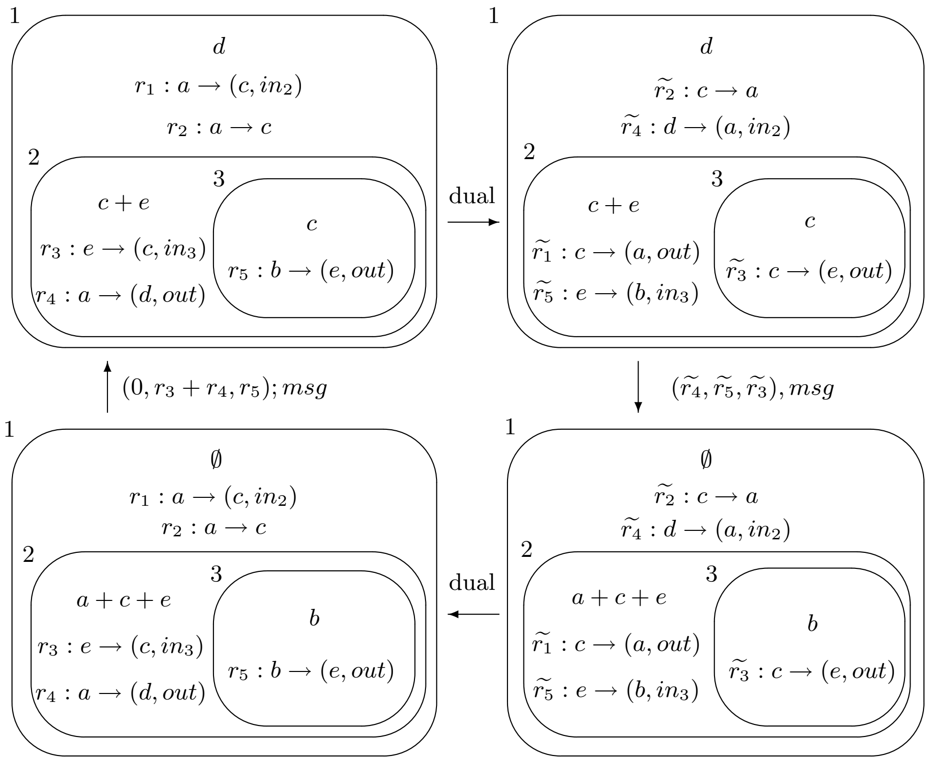
$$\widetilde{\widetilde{\Pi}} = \Pi$$

Proof. Clearly, $u \to v \in R_i^{\widetilde{\widetilde{\Pi}}}$ iff $u \to v \in R_i^{\Pi}$. Moreover, $\widetilde{\widetilde{r}} : u \to (v, out) \in R_i^{\widetilde{\widetilde{\Pi}}}$ iff $\widetilde{r} : v \to (u, in_i) \in R_{parent(i)}^{\widetilde{\Pi}}$ which happens iff $r : u \to (v, out) \in R_i^{\Pi}$ (the condition related to the parent amounts to $parent(i) = parent(i)$). Then, $\widetilde{\widetilde{r}} : u \to (v, in_j) \in R_i^{\widetilde{\widetilde{\Pi}}}$ iff $\widetilde{r} : v \to (u, out) \in R_j^{\widetilde{\Pi}}$ and $i = parent(j)$, which happens iff $r : u \to (v, in_j) \in R_{parent(j)=i}^{\Pi}$. □

If $\mathcal{R} = (\mathcal{R}_1, \dots, \mathcal{R}_m)$ is a system of multisets of rules for a P system Π we also need a different dualisation for it. Namely, we denote by $\widetilde{\mathcal{R}}$ the system of multisets of rules for the dual P system $\widetilde{\Pi}$ given by $\widetilde{\mathcal{R}} = (\widetilde{\mathcal{R}_1}, \dots, \widetilde{\mathcal{R}_2})$, such that:

– if $\widetilde{r} : u \to v \in R_i^{\widetilde{\Pi}}$ then $\widetilde{\mathcal{R}_i}(\widetilde{r}) = \mathcal{R}_i(r)$;
– if $\widetilde{r} : u \to (v, out) \in R_i^{\widetilde{\Pi}}$ then $\widetilde{\mathcal{R}_i}(\widetilde{r}) = \mathcal{R}_{parent(i)}(r)$;
– if $\widetilde{r} : u \to (v, in_j) \in R_i^{\widetilde{\Pi}}$ then $\widetilde{\mathcal{R}_i}(\widetilde{r}) = \mathcal{R}_j(r)$.

Example 3. Consider $M = \langle 1|d; N\rangle$, $N = \langle 2|c + e; P\rangle$, $P = \langle 3|c\rangle$ in the P system Π with $R_1^{\Pi} = \{r_1, r_2\}$, $R_2^{\Pi} = \{r_3, r_4\}$ and $R_3^{\Pi} = \{r_5\}$, where $r_1 : a \to (c, in_2)$, $r_2 : a \to c$, $r_3 : e \to (c, in_3)$, $r_4 : a \to (d, out)$ and $r_5 : b \to (e, out)$. Then $\widetilde{M} = \langle 1|d; \langle 2|c + e; \langle 3|c\rangle\rangle\rangle$ in the dual P system $\widetilde{\Pi}$, with $R_1^{\widetilde{\Pi}} = \{\widetilde{r_2}, \widetilde{r_4}\}$, $R_1^{\widetilde{\Pi}} = \{\widetilde{r_1}, \widetilde{r_5}\}$, $R_3^{\widetilde{\Pi}} = \{\widetilde{r_3}\}$, where $\widetilde{r_1} : c \to (a, out)$, $\widetilde{r_2} : c \to a$, $\widetilde{r_3} : c \to (e, out)$, $\widetilde{r_4} : d \to (a, in_2)$ and $\widetilde{r_5} : e \to (b, in_3)$. For a system of multisets of rules $\mathcal{R} = (r_1 + r_2, 2r_4, 3r_5)$ in Π the dual is $\widetilde{\mathcal{R}} = (2\widetilde{r_4} + \widetilde{r_2}, \widetilde{r_1} + 3\widetilde{r_5}, 0_{R_3^{\widetilde{\Pi}}})$.



The definitions for validity and maximal validity of a system of multisets of rules are the same as in Section 3. However, we need to extend the definition of reverse validity to describe situations arising from a rule being moved.

Definition 7. *A system of multisets of rules* $\mathcal{R} = (\mathcal{R}_1, \ldots, \mathcal{R}_n)$ *for a P system* Π *is called* reversely valid *in the configuration* M *if:*

– $\mathcal{R}$ *is valid in the configuration* M *(i.e.,* $lhs(\mathcal{R}_i) \leq w_i(M)$*);*
– $\forall i \in \{1, \ldots, m\}$*, there is no rule* $r : u \to v \in R_i^{\Pi}$ *such that* $rhs(r) = v \leq w_i(M) - lhs(\mathcal{R}_i)$*;*

– $\forall i \in \{1,\ldots,m\}$ *such that there exists* $parent(i)$, *there is no rule* $r : u \rightarrow (v, in_i) \in R^{\Pi}_{parent(i)}$ *such that* $v \leq w_i(M) - lhs(\mathcal{R}_i)$;
– $\forall i,j \in \{1,\ldots,m\}$ *such that* $parent(j) = i$, *there is no rule* $r : u \rightarrow (v, out) \in R^{\Pi}_j$ *such that* $v \leq w_i(M) - lhs(\mathcal{R}_i)$.

While this definition is more complicated than the one in Section 3, it can be seen in the proof of Proposition 4 that it is exactly what is required to reverse a computation in which a maximally parallel rewriting takes place.

Example 3 continued. We look for $\widetilde{\mathcal{R}}$ reversely valid in $\widetilde{M}$. Since $\widetilde{\mathcal{R}}$ must be valid, $\widetilde{\mathcal{R}_1}$ can be equal to $0_{R^{\widetilde{\Pi}}_1}$ or $\widetilde{r_4}$; $\widetilde{\mathcal{R}_2}$ equal to $0_{R^{\widetilde{\Pi}}_2}$, $\widetilde{r_1}$, $\widetilde{r_5}$ or $\widetilde{r_1} + \widetilde{r_5}$; $\widetilde{\mathcal{R}_3}$ equal to $0_{R^{\widetilde{\Pi}}_3}$ or $\widetilde{r_3}$. According to Definition 7, we can look at any of those possibilities for $\mathcal{R}_i$ to see if it can be a component of a reversely valid system $\mathcal{R}$. In this example the only problem (with respect to reverse validity) appears when $\widetilde{\mathcal{R}_2} = 0_{R^{\widetilde{\Pi}}_2}$ or when $\widetilde{\mathcal{R}_2} = \widetilde{r_1}$, since in both cases we have $e \leq w_2(\widetilde{M}) - lhs(\widetilde{\mathcal{R}_2})$ and rule $c \rightarrow (e, out) \in R^{\widetilde{\Pi}}_3$. Let us see why we exclude exactly these two cases. Suppose $\widetilde{\mathcal{R}_2} = \widetilde{r_1}$ and, for example, $\widetilde{\mathcal{R}_1} = \widetilde{r_4}$, $\widetilde{\mathcal{R}_3} = \widetilde{r_3}$. If $\widetilde{\mathcal{R}}$ is applied, $\widetilde{M}$ rewrites to $\langle 1|(a, in_2); \langle 2|(a, out) + e; \langle 3|(e, out)\rangle\rangle\rangle$; after message sending, we obtain $\langle 1|a; \langle 2|a + 2e; \langle 3|0_O\rangle\rangle\rangle$ which cannot rewrite to M while respecting maximal parallelism (otherwise there would appear two c's in the membrane P with label 3). The same thing would happen when $\widetilde{\mathcal{R}_2} = 0_{R^{\widetilde{\Pi}}_2}$.

In P systems with communication rules we work with both rewriting and message sending. We have presented two semantics for rewriting in Section 3: $\rightarrow_{mpr}$ (maximally parallel rewriting) and $\rightarrow_{\widetilde{mpr}}$ (inverse maximally parallel rewriting). They are also used here, with the remark that the notion of *reversely valid system* has been extended (see Definition 7).

Before giving the operational semantics for message sending we present a few more notations. Given a multiset $w : \Omega \rightarrow \mathbf{N}$ we define the multisets $obj(w)$, $out(w)$, $in_j(w)$ which consist only of objects (i.e., $supp(obj(w))$, $supp(out(w))$, $supp(in_j(w)) \subseteq O$), as follows:

– $obj(w)$ contains all the objects from w: $obj(w)(a) = w(a), \forall a \in O$;
– $out(w)$ contains all the objects a which are part of a message (a, out) in w: $out(w)(a) = w(a, out), \forall a \in O$;
– $in_j(w)$ contains all the objects a which are part of a message (a, in_j) in w: $in_j(w)(a) = w(a, in_j), \forall a \in O, \forall j \in \{1,\ldots,m\}$.

Definition 8. *For a intermediate configuration* M, $M \rightarrow_{msg} N$ *if and only if*

$$w_i(N) = obj(w_i(M)) + in_i(w_{parent(i)}(M)) + \sum_{j \in children(i)} out(w_j(M))$$

To elaborate, the message sending stage consists of erasing messages from the multiset in each inner membrane with label i, adding to each such multiset the objects a corresponding to messages (a, in_i) in the parent membrane (inner membrane with label $parent(i)$) and furthermore, adding the objects a corresponding to messages (a, out) in the children membranes (all inner membranes with label j, $j \in children(i)$).

Proposition 4. *If M is a configuration of Π then*

$$M \xrightarrow{\mathcal{R}}_{mpr} \rightarrow_{msg} N \text{ implies } \widetilde{N} \xrightarrow{\widetilde{\mathcal{R}}}_{\widetilde{mpr}} \rightarrow_{msg} \widetilde{M}.$$

If $\widetilde{N}$ is a configuration of $\widetilde{\Pi}$ then

$$\widetilde{N} \xrightarrow{\widetilde{\mathcal{R}}}_{\widetilde{mpr}} \rightarrow_{msg} \widetilde{M} \text{ implies } M \xrightarrow{\mathcal{R}}_{mpr} \rightarrow_{msg} N.$$

Proof. We begin by describing some new notations. Consider a system of multisets of rules $\mathcal{R} = (\mathcal{R}_1, \ldots, \mathcal{R}_m)$ for a P system Π with evolution rules $R_1^\Pi, \ldots, R_m^\Pi$. We define the following multisets of objects:

$$lhs^{obj}(\mathcal{R}_i), rhs^{obj}(\mathcal{R}_i), lhs^{out}(\mathcal{R}_i), rhs^{out}(\mathcal{R}_i), lhs^{in_j}(\mathcal{R}_i), rhs^{in_j}(\mathcal{R}_i)$$

such that, for u, v multisets of objects:

$$lhs^{obj}(\mathcal{R}_i)(a) = \sum_{r:u\rightarrow v \in R_i^\Pi} R_i(r) \cdot u(a);$$
$$rhs^{obj}(\mathcal{R}_i)(a) = \sum_{r:u\rightarrow v \in R_i^\Pi} R_i(r) \cdot v(a),$$
$$lhs^{out}(\mathcal{R}_i)(a) = \sum_{r:u\rightarrow (v,out) \in R_i^\Pi} R_i(r) \cdot u(a);$$
$$rhs^{out}(\mathcal{R}_i)(a) = \sum_{r:u\rightarrow (v,out) \in R_i^\Pi} R_i(r) \cdot v(a),$$
$$lhs^{in_j}(\mathcal{R}_i)(a) = \sum_{r:u\rightarrow (v,in_j) \in R_i^\Pi} R_i(r) \cdot u(a);$$
$$rhs^{in_j}(\mathcal{R}_i)(a) = \sum_{r:u\rightarrow (v,in_j) \in R_i^\Pi} R_i(r) \cdot v(a).$$

We have the following properties:

- $lhs^{obj}(\mathcal{R}_i) = rhs^{obj}(\widetilde{\mathcal{R}_i})$ and $rhs^{obj}(\mathcal{R}_i) = lhs^{obj}(\widetilde{\mathcal{R}_i})$;
- $lhs^{out}(\mathcal{R}_i) = rhs^{in_i}(\widetilde{\mathcal{R}}_{parent(i)})$ and $rhs^{out}(\mathcal{R}_i) = lhs^{in_i}(\widetilde{\mathcal{R}}_{parent(i)})$;
- if $j \in children(i)$ then $lhs^{in_j}(\mathcal{R}_i) = rhs^{out}(\widetilde{\mathcal{R}_j})$, $rhs^{in_j}(\mathcal{R}_i) = lhs^{out}(\widetilde{\mathcal{R}_j})$;
- $lhs(\mathcal{R}_i) = lhs^{obj}(\mathcal{R}_i) + lhs^{out}(\mathcal{R}_i) + \sum_{j\in children(i)} lhs^{in_j}(\mathcal{R}_i)$.

Now we can prove the statements of this Proposition. We prove only the first one; the proof of the second one is similar. If $M \xrightarrow{\mathcal{R}}_{mpr} \rightarrow_{msg} N$ then there exists an intermediate configuration P such that $M \xrightarrow{\mathcal{R}}_{mpr} P$ and $P \rightarrow_{msg} N$. Then $\mathcal{R}_i$ are maximally valid in $w_i(M)$ and $w_i(P) = w_i(M) - lhs(\mathcal{R}_i) + rhs(\mathcal{R}_i)$. Since $w_i(M)$ is a multiset of objects, it follows that $obj(w_i(P)) = w_i(M) - lhs(\mathcal{R}_i) + rhs^{obj}(\mathcal{R}_i)$. If $j \in children(i)$ we have $in_j(w_i(P)) = rhs^{in_j}(\mathcal{R}_i)$ and moreover, $out(w_i(P)) = rhs^{out}(\mathcal{R}_i)$. Since $P \rightarrow_{msg} N$ we have $w_i(N) = obj(w_i(P)) +$

$in_i(w_{parent(i)}(P)) + \sum_{j \in children(i)} out(w_j(P))$. Replacing $w_i(P)$, $w_{parent(i)}(P)$ and $w_j(P)$ we obtain

$$w_i(N) = w_i(M) - lhs(\mathcal{R}_i) + rhs^{obj}(\mathcal{R}_i) + rhs^{in_i}(\mathcal{R}_{parent(i)}) + \sum_{j \in children(i)} rhs^{out}(\mathcal{R}_j)$$

which is equivalent to

$$w_i(\widetilde{N}) = w_i(M) - lhs(\mathcal{R}_i) + lhs^{obj}(\widetilde{\mathcal{R}_i}) + lhs^{out}(\widetilde{\mathcal{R}_i}) + \sum_{j \in children(i)} lhs^{in_j}(\widetilde{\mathcal{R}}_i)$$

i.e., $w_i(\widetilde{N}) = w_i(M) - lhs(\mathcal{R}_i) + lhs(\widetilde{\mathcal{R}_i})$. Therefore $\widetilde{\mathcal{R}_i}$ is valid in $w_i(\widetilde{N})$, $\forall i \in \{1, \dots, m\}$. Suppose that $\widetilde{\mathcal{R}}$ is not reversely valid in $\widetilde{N}$. Then we have three possibilities, given by Definition 7. First, if there is $i \in \{1, \dots, m\}$ and $\widetilde{r} : u \to v \in R_i^{\widetilde{\Pi}}$ such that $v \leq w_i(\widetilde{N}) - lhs(\widetilde{\mathcal{R}_i})$ it means that $lhs(r) \leq w_i(M) - lhs(\mathcal{R}_i)$, which contradicts the maximal validity of $\mathcal{R}_i$. Second, if there is $i \in \{1, \dots, m\}$ and $\widetilde{r} : u \to (v, in_i) \in R_{parent(i)}^{\widetilde{\Pi}}$ such that $v \leq w_i(\widetilde{N}) - lhs(\widetilde{\mathcal{R}_i})$ then again $lhs(r) \leq w_i(M) - lhs(\mathcal{R}_i)$ (contradiction). The third situation leads to the same contradiction. Thus, there exists an intermediate configuration Q in $\widetilde{\Pi}$ such that $\widetilde{N} \xrightarrow{\widetilde{\mathcal{R}}}_{\widetilde{mpr}} Q$. We have to show that $Q \to_{msg} \widetilde{M}$, i.e., to prove

$$w_i(\widetilde{M}) = obj(w_i(Q)) + in_i(w_{parent(i)}(Q)) + \sum_{j \in children(i)} out(w_j(Q))$$

Since $w_i(Q) = w_i(\widetilde{N}) - lhs(\widetilde{\mathcal{R}_i}) + rhs(\widetilde{\mathcal{R}_i})$ it follows that $obj(w_i(Q)) = w_i(M) - lhs(\mathcal{R}_i) + rhs^{obj}(\widetilde{\mathcal{R}_i})$. We also have that $in_i(w_{parent(i)}(Q)) = rhs^{in_i}(\widetilde{\mathcal{R}}_{parent(i)})$ and $out(w_j(Q)) = rhs^{out}(\widetilde{\mathcal{R}_j})$. So the relation we need to prove is equivalent to

$$w_i(\widetilde{M}) = w_i(M) - lhs(\mathcal{R}_i) + rhs^{obj}(\widetilde{\mathcal{R}_i}) + rhs^{in_i}(\widetilde{\mathcal{R}}_{parent(i)}) + \sum_{j \in children(i)} rhs^{out}(\widetilde{\mathcal{R}_j})$$

which is true because

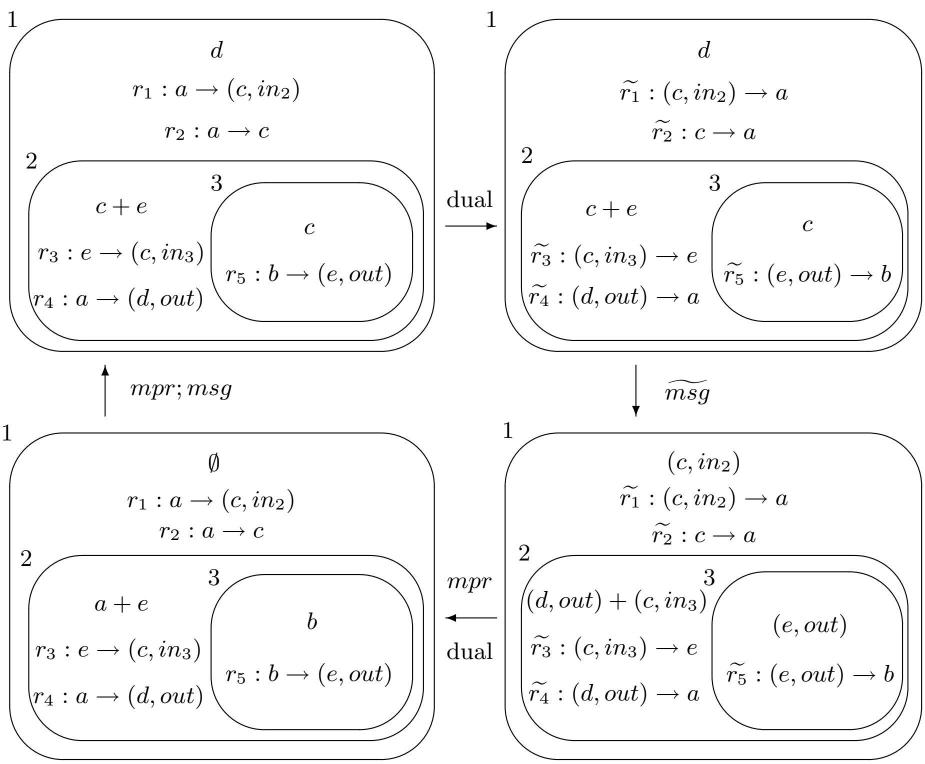
$$lhs(\mathcal{R}_i) = lhs^{obj}(\mathcal{R}_i) + lhs^{out}(\mathcal{R}_i) + \sum_{j \in children(i)} lhs^{in_j}(\mathcal{R}_i). \qquad \square$$

5 An Alternative Approach

Another way to reverse a computation $N \xrightarrow{\mathcal{R}}_{mpr} \to_{msg} M$ is to move objects instead of moving rules. We start by reversing all rules of the P system Π; since these rules can be communication rules, by their reversal we do not obtain another P system. For example, a rule $a \to (b, out)$ yields $(b, out) \to a$, whose left-hand side contains the message *out* and therefore is not a rule. However, we can consider a notion of extended P system in which we allow rules to also have messages in their left-hand side. We move objects present in the membranes and transform them from objects to messages according to the rules of the membrane system. The aim is to achieve a result of form

$$M \xrightarrow{\mathcal{R}}_{mpr} N \rightarrow_{msg} P \text{ if and only if } \widetilde{P} \rightarrow_{\widetilde{msg}} \widetilde{N} \xrightarrow{\widetilde{\mathcal{R}}}_{\widetilde{mpr}} \widetilde{M}$$

An example illustrating the movement of the objects is the following:



where the "dual" movement $\rightarrow_{\widetilde{msg}}$ of objects between membranes is:

- d in membrane 1 $\xrightarrow{\text{called by rule } \tilde{r_4}}$ (d, out) in membrane 2;
- c in membrane 2 $\xrightarrow{\text{called by rule } \tilde{r_1}}$ (c, in_2) in membrane 1;
- e in membrane 2 $\xrightarrow{\text{called by rule } \tilde{r_5}}$ (e, out) in membrane 3;
- c in membrane 3 $\xrightarrow{\text{called by rule } \tilde{r_3}}$ (c, in_3) in membrane 2.

By applying the dual rules, messages are consumed and turned into objects, thus performing a reversed computation to the initial membrane.

6 Conclusion

In this paper, we solve the problem of finding all the configurations N of a P system which evolve to a given configuration M in a single step by introducing dual P systems. The case of P systems without communication rules is used as a stepping stone towards the case of P systems with simple communication rules. In the latter case, two approaches are presented: one where the rules are reversed and moved between membranes, and the other where the rules are only reversed. On dual membranes we employ a semantics which is surprisingly close to the one giving the maximally parallel rewriting (and message sending, if any).

The dual P systems open new research opportunities. A problem directly related to the subject of this paper is the predecessor existence problem in dynamical systems [1]. Dual P systems provide a simple answer, namely that a predecessor for a configuration exists if and only if there exists a system of multisets of rules which is reversely valid.

Dualising a P system is closely related to reversible computation [3]. Reversible computing systems are those in which every configuration is obtained from at most one previous configuration (predecessor). A paper which concerns itself with reversible computation in energy-based P systems is [2].

Further development will include defining dual P systems for P systems with general communication rules. Other classes of P systems will also be studied.

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