Type-2 Fuzzy Logic and the Modelling of Uncertainty in Applications

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Abstract. Most real world applications contain high levels of uncertainty and imprecision. Sources of the imprecision include sensor noise; variation in actuator performance; linguistic variation between people; temporal modification of expert opinion; and disagreement between experts. Type-2 fuzzy logic is now accepted as a mature technology for coping with this wide variety of sources of uncertainty. This Chapter provides an overview of type-2 fuzzy logic systems providing the reader with an insight into how the various algorithms provide different approaches to modelling uncertainty. We place in context these issues by discussing a number of real world applications that have successfully deployed type-2 fuzzy logic.

1 Introduction

Recently there has been significant growth in interest in type-2 fuzzy logic. Type-2 fuzzy logic is an extension of type-1 (regular) fuzzy logic where the membership grade in a fuzzy set is itself measured as a fuzzy number.

Fuzzy sets (Zadeh [57]) have, over the past forty years, laid the basis for a successful method of modelling uncertainty, vagueness and imprecision in a way that no other technique has been able. The use of fuzzy sets in real computer systems is extensive, particularly in consumer products and control applications.

Zadeh [62] presents a powerful argument for the use of fuzzy logic for manipulating perceptions. As has been discussed, his argument is that perceptions (for example, perceptions of size, safety, health and comfort) cannot be modelled by traditional mathematical techniques and that fuzzy logic is more suitable. The discussion about perception modelling is both new and exciting. We argue that type-2 fuzzy sets, since they have non-crisp fuzzy membership functions (that is they are not exact), can model these perceptions more effectively than type-1 fuzzy sets where the membership grades are crisp in nature.

So, we take the position that although fuzzy logic has many successful applications there are a number of problems with the 'traditional' fuzzy logic approach that require a different set of fuzzy tools and techniques for modelling high levels of uncertainty. In particular the argument presented here is that fuzzy logic, as it is commonly used, is essentially **precise** in nature and that for many applications it is unable to model knowledge from an expert adequately. We argued that the modelling of imprecision can be enhanced by the use of type-2 fuzzy sets - providing a higher level of imprecision.

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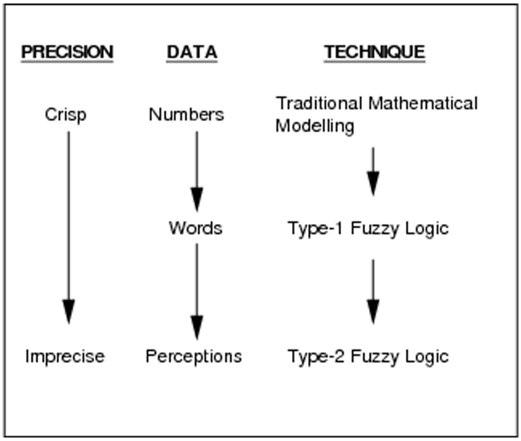

Fig. 1. Relationships between imprecision, data and fuzzy technique

Indeed, the tenet of this work is that the success of fuzzy logic can be built on by type-2 fuzzy sets and taken into the next generation of (type-2) fuzzy systems. The use of type-2 fuzzy sets allows for a better representation of uncertainty and imprecision in particular applications and domains. This argument is presented with the use of a mobile robot control application.

The more imprecise or vague the data is, then type-2 fuzzy sets offer a significant improvement on type-1 fuzzy sets. Figure 1 shows the view taken in this work of the relationships between levels of imprecision, data and technique. As the level of imprecision increases then type-2 fuzzy logic provides a powerful paradigm for potentially tackling the problem. Problems that contain crisp, precise data do not, in reality, exist. However some problems can be tackled effectively using mathematical techniques where the assumption is that the data is precise. Other problems (for example, in control) use imprecise terminology that can often be effectively modelled using type-1 fuzzy sets. Perceptions, it is argued here, are at a higher level of imprecision and type-2 fuzzy sets can effectively model this imprecision.

Section 2 provides an overview of type-2 fuzzy sets and type-2 fuzzy logic. The next Section in this chapter, Section 3, presents the history of the field of type-2 fuzzy logic including the recent emergence of generalised type-2 fuzzy system as a viable technology. Section 4 presents the application of type-1, type-2 interval and generalised type-2 fuzzy logic to a mobile robot control application. This example application demonstrates the potential of generalised type-2 fuzzy logic to give an improved performance over type-2 interval fuzzy logic. Section 5 draws conclusions from this work. We also note that some of this material is contained in John and Coupland [15].

2 Type-2 Fuzzy Sets and Type-2 Fuzzy Logic

Type-2 fuzzy sets (originally introduced by Zadeh [59]) have membership grades that are fuzzy. That is, instead of being in [0,1] the membership grades are themselves (type-1) fuzzy sets. Karnik and Mendel [25][page 2] provide this definition of a type-2 fuzzy set:

A type-2 fuzzy set is characterised by a fuzzy membership function, i.e. the membership value (or membership grade) for each element of this set is a fuzzy set in [0,1], unlike a type-1 fuzzy set where the membership grade is a crisp number in [0,1].

The characterisation in this definition of type-2 fuzzy sets uses the notion that type-1 fuzzy sets can be thought of as a first order approximation to uncertainty and, therefore, type-2 fuzzy sets provide a second order approximation. They play an important role in modelling uncertainties that exist in fuzzy logic systems[21] and are becoming increasingly important in the goal of 'Computing with Words'[61] and the 'Computational Theory of Perceptions'[62].

We now define various terms that relate to type-2 fuzzy sets and state the Representation Theorem (for a detailed discussion and proof of the Representation Theorem the reader is referred to [40]). The first definition we give is a formal definition of a type-2 fuzzy set.

Definition 1. *A type-2 fuzzy set, $\tilde{A}$, is characterised by a type-2 membership function $\mu_{\tilde{A}}(x,u)$, where $x \in X$ and $u \in J_x \subseteq [0,1]$*

$$\tilde{A} = \{((x,u), \mu_{\tilde{A}}(x,u)) \mid \forall x \in X, \forall u \in J_x \subseteq [0,1]\} \tag{1}$$

For any given x the $\mu_{\tilde{A}}(x,u)$, $\forall u \in J_x$, is a type-1 membership function as discussed in the introduction. It is this 'extra fuzziness' that is the attraction of type-2 fuzzy sets[21].

It can be seen from this definition that a type-2 membership function is three dimensional. To illustrate this Figure 2 provides an example of type-2 fuzzy set.

We have a three dimensional figure with the axes being x, u and $\mu_{\tilde{A}}(x,u)$. The 'spikes' are in [0,1] and represent $\mu_{\tilde{A}}(x,u)$ for a given (x,u). For a given x we have a vertical slice that we call a *secondary membership function*.

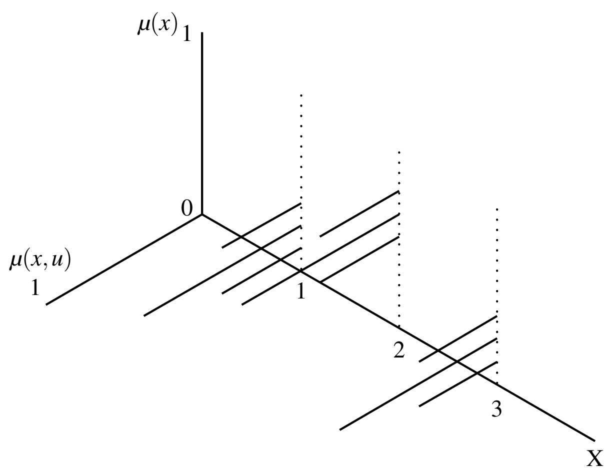

Fig. 2. A Type-2 Fuzzy Set

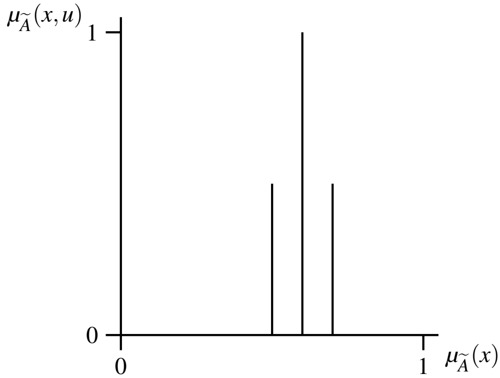

Fig. 3. A Secondary Membership Function

Definition 2. *At each value x (say x') then $\mu_{\tilde{A}}(x',u)$ is a secondary membership function of $\mu_{\tilde{A}}(x,u)$. We also know this as a vertical slice.*

An example secondary membership function of the type-2 fuzzy set in Figure 2 is given in Figure 3. In this case the secondary membership function is for $x' = 2$ with $\mu_{\tilde{A}}(2) = 0.5/0.5 + 1.0/0.6 + 0.5/0.7$. Note, then, that the type-2 fuzzy set $\tilde{A}$ is the union of all the secondary membership functions. The Representation Theorem lays the basis for the proof of the extended sup-star composition and this, in turn, relies on the notion of an embedded type-2 fuzzy set [40].

Definition 3. *For discrete universes of discourse X and U, an embedded type-2 fuzzy set $\tilde{A}_e$ has N elements, where $\tilde{A}_e$ contains exactly one element from $J_{x_1}, J_{x_2}, \ldots, J_{x_N}$, namely $u_1, u_2, \ldots, u_N$, each with its associated secondary grade $f_{x_i}(u_i)$ ($i = 1, \ldots, N$), i.e.*

$$\tilde{A}_e = \sum_{i=1}^{N} [f_{x_i}(u_i)/u_i]/x_i \qquad u_i \in J_{x_i} \subseteq U = [0,1] \tag{2}$$

Figure 4 gives an example of an embedded type-2 fuzzy set. As can be seen we now have what we might call a 'wavy slice' where we have one element (only) from each vertical slice contained in the embedded type-2 fuzzy set.

2.1 The Representation Theorem

The definitions so far in this chapter provide enough detail to understand the Representation Theorem[40] needed for this new proof of the extended sup-star composition. We give the Theorem without proof.

Theorem. *Let $\tilde{A}_e^j$ denote the jth type-2 embedded fuzzy set for type-2 fuzzy set $\tilde{A}$.*

$$\tilde{A}_e^j \equiv \{(u_i^j, f_{x_i}(u_i^j)), i = 1, \ldots, N\} \tag{3}$$

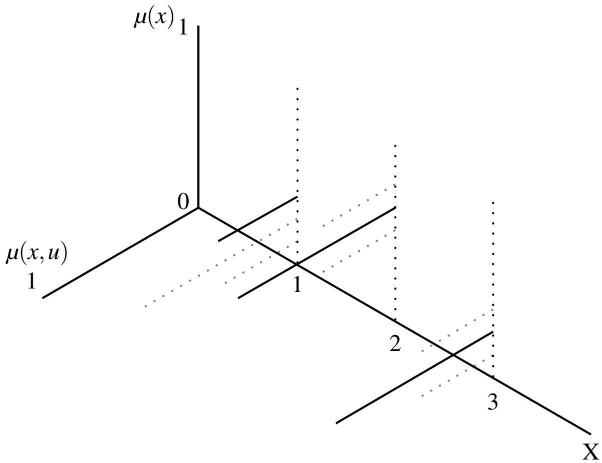

Fig. 4. An Embedded Type-2 Fuzzy Set

where

$$u_i^j \in \{u_{ik}, k = 1, \dots, M_i\} \tag{4}$$

$\tilde{A}$ can be represented as the union of all its type-2 embedded fuzzy sets. That is:

$$\tilde{A} = \sum_{j=1}^{n} \tilde{A}_e^j \tag{5}$$

where

$$n \equiv \prod_{i=1}^{N} M_i \tag{6}$$

We are able to show, then, that a type-2 fuzzy set $\tilde{A}$ is the union of all its type-2 embedded fuzzy sets. This Theorem has allowed for the derivation of union, intersection and complement of type-2 fuzzy sets without use of the extension principle [40]. The union and intersection of embedded type-2 fuzzy sets are as follows. Suppose we have two embedded type-2 fuzzy sets $\tilde{A}_e^j$ and $\tilde{B}_e^i$. The secondary grades at x_l are denoted as $f_{x_l}(u_l^j)$ and $g_{x_l}(w_l^j)$ respectively then

$$\tilde{A}_e^j \cup \tilde{B}_e^i \equiv [F_{x_1}(u_1^j, w_1^i)/u_1^j \vee w_1^i]/x_1 + \ldots + [F_{x_N}(u_N^j, w_N^i)/u_N^j \vee w_N^i]/x_N \tag{7}$$

where, for each $l = 1, \dots, N$,

$$F_{x_l}(u_l^j, w_l^i) = h[f_{x_l}(u_l^j), g_{x_l}(w_l^i)]$$

and h is a t-tnorm. This also known as the join ($\sqcup$). So that

$$\mu_{\tilde{A}_e^j} \sqcup \mu_{\tilde{B}_e^i} \equiv [F_{x_1}(u_1^j, w_1^i)/u_1^j \vee w_1^i]/x_1 + \ldots + [F_{x_N}(u_N^j, w_N^i)/u_N^j \vee w_N^i]/x_N \tag{8}$$

The intersection is given by

$$\tilde{A}_e^j \cap \tilde{B}_e^i \equiv [F_{x_1}(u_1^j, w_1^i)/u_1^j \wedge w_1^i]/x_1 + \ldots + [F_{x_N}(u_N^j, w_N^i)/u_N^j \wedge w_N^i]/x_N \tag{9}$$

This also known as the meet ($\sqcap$). So that

$$\mu_{\tilde{A}_e^j} \sqcap \mu_{\tilde{B}_e^i} \equiv [F_{x_1}(u_1^j, w_1^i)/u_1^j \wedge w_1^i]/x_1 + \ldots + [F_{x_N}(u_N^j, w_N^i)/u_N^j \wedge w_N^i]/x_N \tag{10}$$

3 The Historical Development of Type-2 Fuzzy Logic

Type-2 fuzzy logic is a growing research topic. In this section we discuss the main themes in type-2 fuzzy logic and highlight some applications.

3.1 Type-2 Fuzzy Sets Appear

Type-2 fuzzy sets were first defined and discussed in a trilogy of papers by Zadeh [1975*a*, 1975*b*, 1975*c*]. These papers concentrated on the notion of a fuzzy set where the memberships grades of a fuzzy set are measured with linguistic terms such as *low*, *medium* and *high*. Logical connectives for such sets were also given, although the terms join and meet were not used. Zadeh only explored the use of the minimum and maximum operators t-norm and t-conorm when investigating the logical operations. Mizumoto and Tanaka [1976,1981] and Dubois and Prade [1980] both studied the logical connectives of what became known as secondary membership functions. Mizumoto and Tanaka were the first to use the terms join and meet for these logical connectives. Both Dubois and Prade and Mizumoto and Tanaka studied the join and meet under a variety of t-norm and t-conorm operators.

3.2 Type-2 Interval Fuzzy Sets Are Promoted

Turksen [1993,1993*a*,1995], Schwartz [48] and Klir and Folger [29] promoted the use of type-2 fuzzy sets, at that time called interval valued or IV fuzzy sets. Schwartz believes that type-2 interval fuzzy sets should be employed when the linguistic uncertainty of a term cannot be sufficiently modelled by the type-1 methods. Klir and Folger advocate the use of IV fuzzy sets when the membership functions of type-1 fuzzy sets could not be agreed upon. These arguments were explored in greater detail by Mendel [36]. Turksen put forward a collection of logical connectives for type-2 interval fuzzy sets noting that the expressive power of type-2 fuzzy reasoning lies in the ability to retain the uncertainty throughout the inferencing process.

3.3 Type-reduction Is Defined

Karnik and Mendel [1998,1998*b*,2001] defined type-reduction, the technique used for defuzzifing type-2 fuzzy sets, by applying the extension principle to a variety of type-1 defuzzifiers. The notion of an output processing stage of a type-2 fuzzy system was developed in these papers.

3.4 Type-2 Fuzzy Logic Systems Are Fully Defined

Karnik and Mendel [25, 28] gave a complete description of the fuzzy inferencing process. This allowed work on the application of type-2 fuzzy logic to proceed. Around this time John [1998*a*,1998*b*,1999,1999*a*] published a series of review papers on type-2 fuzzy systems. Early applications of the technology also began to appear (see for example John [19, 23] and Karnik and Mendel [27]).

3.5 The First Textbook on the Subject of Type-2 Fuzzy Logic Appears

Following the consolidation of the definitions and existing literature by John and Karnik and Mendel, the field was opened up to a wider potential audience with the publication of the first type-2 textbook. Uncertain Rule-Based Fuzzy Logic System: Introduction and New Directions was written by Mendel [2001*a*] and published in 2001. This textbook references a great deal of the work on type-2 fuzzy logic that had been published to date, bringing together many of Mendel's earlier publications.

3.6 The Representation Theorem Is Defined

Mendel and John [40] gave the representation theorem of type-2 fuzzy sets. By representing a type-2 fuzzy set as a collection of simpler type-2 embedded sets it is possible to define operations of type-2 fuzzy sets without the use of the extension principle. The motivation behind this work was that by eliminating the need to learn about the extension principle, the field would be more accessible to type-1 fuzzy practitioners. However, the representation theorem has its own learning curve, and is not significantly simpler to understand than the extension principle. One of the outcomes of the representation theorem has been the definition of arithmetic operators for type-2 fuzzy numbers by Coupland and John [1].

3.7 Issues of Computational Complexity Begin to Be Explored

The complexity of join and meet operations and type-reduction of a type-2 fuzzy set limit the applicability of type-2 methods. Although type-2 interval sets are simpler, type-reduction is still a problem, due to inherent complexity and redundancies. The iterative method (Karnik and Mendel [28]) and the Wu-Mendel [55, 56] approximation were developed to make the type-reduction of type-2 interval fuzzy sets more efficient. This has led to the majority of the publications in the field of type-2 only discussing type-2 interval methods. Indeed, many authors refer to type-2 interval fuzzy set as type-2 fuzzy sets and add the qualifying term 'generalised' when discussing actual type-2 fuzzy sets. The computational problems of join and meet were effectively resolved by Karnik and Mendel [28]. This work is also discussed by the author, along with some aspects of the geometric approach in Coupland *et al.* [2, 3]. Greenfield *et al.* [12] give an efficient method for approximating the type-reduced set of a type-2 fuzzy set using a stochastic approach.

3.8 Computing with Words Appears

Zadeh [61, 62] made the claim that fuzzy logic, approximately at least, equates to computing with words (CWW). In CWW numbers are replaced with words not only when reasoning, but also when solving calculations. Zadeh‘s examples use fuzzy granules to model words. A fuzzy granule is actually the Footprint Of Uncertainty of a type-2 interval fuzzy set. Both Mendel [37, 39] and Turksen [52] point out that CWW requires type-2 fuzzy sets, both opting to use the simpler type-2 interval representations. Mendel [36] re-emphasised this point by demonstrating that human models of words as obtained through a survey require at least interval representations.

3.9 Control Applications

With the iterative method and the Wu-Mendel approximation allowing fast execution of type-2 fuzzy systems, control applications began to emerge. Melin and Castillo [34, 35] used type-2 interval systems in the context of plant control. Hagras [13] demonstrated that a type-2 interval fuzzy logic controller could outperform a type-1 fuzzy controller under large uncertainties. Wu and Tan [54] applied type-2 interval systems to the control of a complex multi-variable liquid level process. Figueroa *et al.* [10] used a type-2 interval control for non-autonomous robots in the context of a robot football game. The authors’ have performed a comprehensive study of both general and type-2 interval fuzzy controllers for an autonomous mobile robot. Some aspects of these studies are presented in Section 4 of this work and in Coupland [3]. Doctor *et al.* [8] used a type-2 interval system to model and adapt to the behaviour of people in an intelligent dormitory room. Additional work on type-2 fuzzy logic hardware has also contributed great to the field. Lynch *et al.* [31, 32] have implemented an industrial type-2 interval control system for large marine diesel engines which has very good performance, both in control response and cycle times. Melgarejo *et al.* [33] have also developed a limited hardware implementation of a type-2 interval controller which has a lesser focus on industrial application. Coupland *et al.* [6] have implemented generalised type-2 fuzzy logic, complete with Integrated Development Environment with a focus on dissemination of the technology.

3.10 Medical Applications

Medical applications are one of the few areas where a generalised type-2 fuzzy logic has been used in preference to type-2 interval fuzzy logic. This is largely because such systems do not require fast execution times but do contain large uncertainties. John *et al.* [18, 23] used a type-2 fuzzy system for the pre-processing of tibia radiographic images. Garibaldi *et al.* [11, 46] have done extensive work on assessing the health of a new born baby using knowledge of acid-base balance in the blood from the umbilical cord. Innocent and John [14] proposed the use of fuzzy cognitive maps to aid the differential diagnosis of confusable diseases and suggest that type-2 cognitive maps may yield improved results. Di Lascio *et al.* [7] also used type-2 fuzzy sets to model differential diagnosis of diseases, modelling the compatibility of the symptom to a disease as a linguistic term. John *et al.* [2001,2001*a*] used type-2 fuzzy sets to model the perception of clinical opinions of nursing staff as linguistic terms.

3.11 Signal Processing

Signal processing, like control, has to date only used type-2 interval methods. Liang and Mendel [30] implemented a fuzzy adaptive filter for the equalization of non-linear time-varying channels. Mitchell [42] defined a similarity measure for use with type-2 fuzzy sets which was used in a radiographic image classifier. Karnik and Mendel [27] used a type-2 interval system to predict the next value in a chaotic time series. Musikasuwan *et al.* [45] investigated how the learning capabilities of type-1 and type-2 interval systems differ according to the number of learning parameters used. Both systems were designed to to predict a Mackey-Glass time series.

3.12 Generalised Type-2 Fuzzy Logic Emerges as a Viable Technology

Two very recent major advances in generalised type-2 fuzzy logic have had a significant impact on the usability of generalised type-2 fuzzy systems. Coupland‘s geometric model [4, 5] of type-2 fuzzy sets and systems have eliminated the historical problem of the computing the centroid of a type-2 fuzzy set. The work presented in the following Section of this chapter is only possible because of the reduction in computation provided by the geometric model. The simultaneous definition of alpha-planes (or z-slices) of type-2 fuzzy sets by Mendel and Liu [41], and Wagner and Hagras [53] give an approximation of the geometric method which is highly efficient and highly parallel. Definitions and investigations of these techniques are currently being undertaken by a number of researchers in the field.

3.13 Summary

This Section has given the major developments that have taken place in the field of type-2 fuzzy logic and places them in a historical context. Type-2 literature has become predominately concerned with type-2 interval methods. The likely reason for this is the elimination of the computational problems for type-2 interval methods. The authors' view is that generalised type-2 fuzzy logic has a great deal to offer as will be demonstrated in the following section.

4 Type-2 Fuzzy Logic Controllers

This section presents a comparison of three fuzzy logic controller which are given the task of navigating around a curved obstacle. Each of the three controllers is based on a different fuzzy technology:

- Controller 1 uses type-1 fuzzy logic;
- Controller 2 uses type-2 interval fuzzy logic, and
- Controller 3 uses hybrid type-2 fuzzy logic.

The type-1 controller was designed first and provides a basis for controllers 2 and 3. The hybrid type-2 controller makes use of geometric fuzzy logic in order to achieve the execution speeds requires by the robot control system.

4.1 Task Selection

There are currently no reported systems (except Coupland *et al.* [2006] which reports some aspects of this experiment) where generalised type-2 fuzzy logic has been applied to a control application. This Section describes the first such application which has been made possible with the introduction of geometric type-2 fuzzy logic. As discussed in earlier, type-2 fuzzy logic systems should be able to cope with the uncertainties inherent in control applications. To best evaluate geometric type-2 fuzzy logic in a control application a difficult mobile robot navigation problem was designed. Type-2 interval fuzzy logic has already been applied to such an application by Hagras [13]. The Hagras study demonstrated improved performance in navigation tasks when using type-2 interval rules rather than type-1 under environmental uncertainties. One of the limitations of the Hagras study was that the robot only performed eight runs and therefore, it is difficult to state the significance, if any, of this performance improvement. However, the Hagras study demonstrated that mobile robot navigation is a useful application area for exploring the potential of type-2 fuzzy logic in control applications.

The task of mobile robot navigation represents a significant challenge for a type-2 FLC. The control system has to operate in real time on limited hardware resources. The environment which the robot has to operate in is challenging. The sensors on the robot are operating in the real world and are prone to noise and error. For example the accuracy of a sonar sensor is likely to be reduced the further away an object is. Background noise in the environment may also effect the sonar reading. The level of traction between the wheels and the floor depends on the type of flooring, type pressures and the speed at which the wheels are moving. The task to be completed by the FLC presented in this chapter is to navigate a mobile robot around the curved edge of a wall like obstacle maintaining a distance of 0.5 metres between the centre of the robot and the obstacle at all times. A diagram of the robot, the obstacle and the ideal path that the robot should follow around the obstacle is given in Figure 5. The initial position of the robot puts the obstacle at a right angle to the left wheel of the robot. The initial distance between the obstacle and the centre of the robot is set at 0.5 metres. The robot is facing the correct direction to begin navigation of the obstacle. This start position places the robot just below the start point of the ideal path that should be taken by the robot around the obstacle. Once the centre of the robot crosses the dotted start line

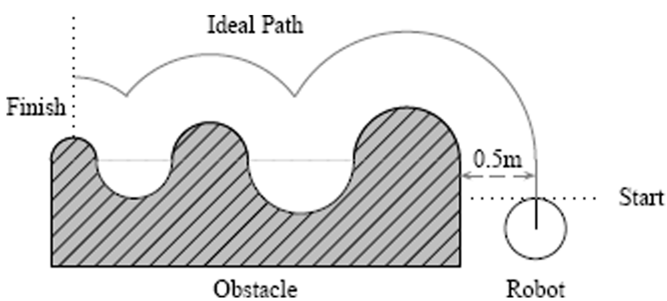

Fig. 5. Mobile Robot and Obstacle

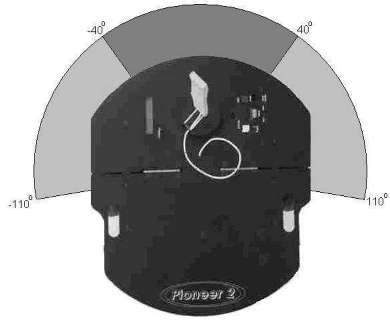

Fig. 6. The Pioneer 2 Mobile Robot

tracking begins. Once the centre of the robot crosses the dotted finish line tracking stops. All runs started from the same initial position. The task of the FLC is essentially to minimise the deviation from the ideal path between the start and finish lines.

The controllers were deployed on the commercially available pioneer 2 robot (depicted in Fig 6) built by ActivMedia. The robot has an on board personal computer which the software based FLCs were implemented on. This PC links to a microcontroller which is directly connected to the sensors and actuators. An array of eight sonar sensors each with a range of 3 metres provides sensory capability. Two independently driven wheels give mobility to the robot. The FLCs had four inputs, d_1, θ_1, d_2 and θ_2:

- The angle θ_1 is the angle to the closest object detected by all eight sensors. θ_1 is given as a value between -110^o and 110^o;
- the angle θ_2 is the angle to the closest object detected by the middle four sensors. θ_2 takes a value between -40^o and 40^o;
- the distance d_1 is the distance to the nearest object detected by all eight sensors, and
- the distance d_2 is the distance to the nearest object detected by the middle sensors.

The only output from the system is the change in direction (δh) of the heading of robot. Since only the direction of the robot is being altered the speed of the robot is kept constant at 0.1ms^{-1}. The robot travels at this speed when moving in a straight line. However, when turning a component of this speed is taken up as rotational velocity. The robot is always moving forwards and can never go backwards.

The Aria software library provided with the robot requires that control commands are executed within a tenth to a quarter of a second window. This is the definition of real time for this robot, command execution within a quarter of a second or the robot operations will shutdown. This is quite a low requirement by control standards. It is however a significant challenge to perform type-2 fuzzy inferencing on limited hardware within a quarter of a second.

4.2 Controller Design

At the heart of FLS is the rule base. We started with a type-1 fuzzy rule base and blurred the membership functions. In this work the control rules were based on the experience of a robot operator. Experience of how to drive the robot around the obstacle was gained by the author. A joystick was connected to the robot over a wireless network. This joystick was then used to manoeuvre the robot around the obstacle. This process was repeated until the author was competent at driving the robot around the obstacle. The rules were based on this experience of driving the robot around the obstacle manually with a joystick.

The FLC also used an idea from control theory, the change in error over time, the derivative δe. This added an element of proportionality to the controller (Reznik [47]). To obtain the value of δe the gradient of a best fit line placed through the last four error measurements e, where $e = 500 - d_1$ was taken. Taking δe is useful as it gives a measure of whether the robot is moving toward the ideal path or away from it. This is particularly useful with this configuration of pioneer robots as they do not have any sonar sensors at the rear to detect whether the robot is moving toward or away from an object.

4.3 Results

The path each robot FLC took around the obstacle was tracked fifty times. These tracked paths are depicted in Figures 7, 8 to 9. The error for each point in this tracked path relative to an ideal path was calculated. The RMSE for each tracked run around the obstacle was then calculated. The mean, median, standard deviation and coefficient of variance over the fifty runs was then calculated for each robot FLC. These results are given in table 1.

Table 1. The Mean, Median, Standard Deviation and Coefficient of Variance of Error for the Six Robot FLC Over Fifty Runs. All numbers quoted to 4 d.p.

Controller	Mean Error	Median Error	St Dev of Error	Co Var of Error
1	13.5852	13.4185	1.0995	0.0809
2	12.5394	11.9779	2.0543	0.1638
3	9.8171	9.7783	1.0185	0.1038

An initial visual comparison would suggest that the controller 3 performed most consistently. Controller 2 had a wide but consistent spread. Controller 1 had spread of paths somewhere between the two with a few paths quite far outside the main spread. It is difficult judge the error of the controllers visually, although the Controller 3 path appear more tightly packed than the other two.

The results from the experiment did not display either normality or equality of variance. Therefore the non-parametric Kruskal-Wallis test was used to assess whether or not there are any differences between the controllers' performance. The test gave a H statistic value of 97.01 and a p value of < 0.0005, suggesting strong evidence of differences between the controllers. The Kruskal-Wallis test works by ranking the data by median value. Table 2 gives the median values and the average ranking the three controllers. The median positions and mean rankings do point to the type-2 controller having the best performance, followed by the interval type-2 controller and then the

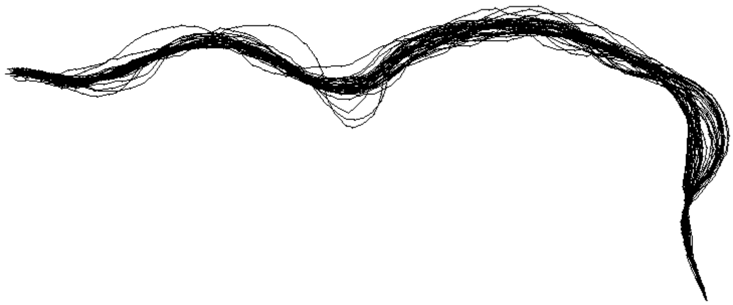

Fig. 7. Paths Taken By Controller 1

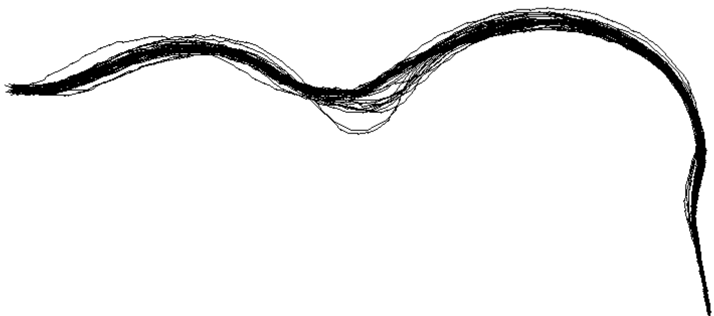

Fig. 8. Paths Taken By Controller 2

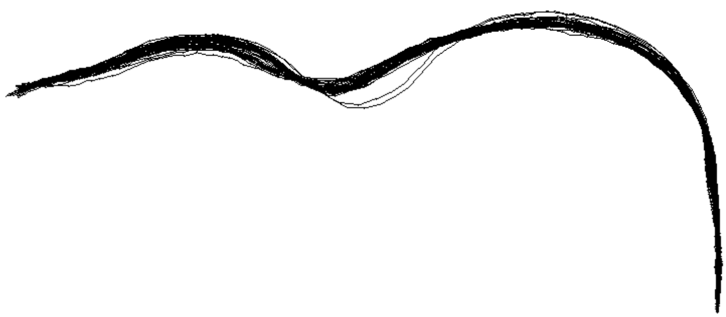

Fig. 9. Paths Taken By Controller 3

Table 2. The Median and Average Rank of the Three Controllers from the Kruskal-Wallis Test Procedure

Controller	1	2	3
Median	13.392	11.961	9.802
Average Rank	113.3	84.2	29.0

type-1 controller. This performance ranking is identical to the ordering of the $R\bar{M}SE$ of the FLC. Looking at consistency of performance both the test for equal variances and the values of $\sigma RMSE$ suggest that the type-1 and type-2 FLC were equally consistent. The interval type-2 FLC had a less consistent performance.

It is important to compare the outcomes that are suggested by the statistical comparison with those give by a visual comparison of the paths. The statistics suggest that FLC performance is ranked type-2, then interval type-2 and then type-1. The path depictions support this conclusion. The statistics suggest that the type-1 and type-2 FLC were equal in the consistency of performance. This is not immediately clear from the visual comparison. Take into account that the type-1 FLC gave the worst performance. A view can be taken that the type-1 FLC made more errors, however these errors were made consistently. The type-2 interval FLC gave a middling performance, but on occasionally made significant errors. This relates well to the visual paths. To summarise these points:

- The type-2 FLC performed consistently well.
- The interval type-2 FLC performed quite well, but was a little inconsistent.
- The type-1 FLC performed relatively badly, but was consistent in this level of error.

These findings are supported by a visual inspection of taken and by a statistical analysis of those paths.

5 Conclusion

This Chapter has presented an introduction to type-2 fuzzy sets, an overview of the history of type-2 fuzzy logic with emphasis on applications and a detailed description of a real application in control. There is much work still to be done on type-2 fuzzy logic and we believe the applications and theoretical results will continue to grow.

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