# **Visualizing Huge Image Databases by Formal Concept Analysis**

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**Abstract.** Based on formal concept analysis we propose a novel lattice visualization system for huge image databases as a realization of the important paradigm of humancentered information processing based on granular computing. From a given cross table of objects (images) and attributes (image features) the proposed system first constructs a concept lattice. Then the Hasse diagram of this lattice is visualized. The information granules in the proposed system correspond to the elements of the concept lattice. All the important components of granular computing are shown to be present in the proposed system, such as: abstraction of data, derivation of knowledge and empirical verification of the abstraction. Since formal concept analysis generates an order relation, we obtain a hierarchical structure of concepts. This structure is shown to be also strongly related to the granular computing, since this is how the lattice visualization system implements the zoom in and zoom out capability of granular computing systems. Using the proposed system, a user can freely analyze the perspective and detailed structure of a large image database in the setting of granular computing. Furthermore, through an interaction function, the potential user can adjust the quantization of features, being able in this way, to select the attributes which allow him to obtain a suitable concept lattice. Therefore, the proposed system can be regarded as a promising human-centric information processing algorithm, based on granular computing.

### **1 Introduction**

Due to the spread of high performance computers, digital input devices and high capacity memory devices, information explosion is becoming nowadays a more and more acute problem raised by the society to the information processing community [\[26\]](#page-22-0). Especially the amount of digital images has been rapidly increasing in accordance with prevailing digital cameras for individual use.

IDC [\[25\]](#page-22-1) reports that the digital universe was in 2007, at the level of 281 exabytes or 281 billion gigabytes. Meanwhile, the ability of human-beings to process the information has not dramatically improved in the last several hundreds of years. Due to this information explosion, we need novel methods to visualize information contained in image databases or video sequences (see e.g., [\[5\]](#page-21-0) [\[6\]](#page-21-1) [\[8\]](#page-21-2) [\[20\]](#page-22-3) [\[27\]](#page-22-4)).

In order to extract information with understandable contents and suitable size from such a huge volume of data, we need new, intelligent, human-centric information processing techniques and information visualization schemes. The users need to be able to recognize and understand comprehensive information about several images at once.

#### **1.1 Information Visualization**

In the topic of information visualization there is a very intense, ongoing research. Almost all the research efforts in the topic of information visualization [\[5\]](#page-21-0) [\[6\]](#page-21-1) [\[8\]](#page-21-2) [\[20\]](#page-22-3) [\[27\]](#page-22-4), with target objects either text or images are directed towards a metric spaces based approach, where the similarity between objects is obtained based on a distance between them. From the mathematical point of view, this research can be sought as metric structure based visualization.

This chapter introduces a novel idea in this research field, by proposing a visualization method for huge image databases based on *granular computing* [\[1\]](#page-21-3) [\[23\]](#page-22-5), and *formal concept analysis* [\[9\]](#page-21-4).

#### **1.2 Granular Computing**

Human centric information processing traces back to the celebrated works of L. Zadeh [\[22\]](#page-22-6), [\[23\]](#page-22-5). Indeed, Information Granulation appears for the first time in the paper [\[23\]](#page-22-5), but as Zadeh himself underlines in [\[23\]](#page-22-5), the basic ideas related to this concept can be rooted back to the very beginning of Fuzzy Sets Theory, [\[22\]](#page-22-6). Human-centric information processing relates to the concept of information granulation since this is one of the three basic tasks underlying human cognition [\[23\]](#page-22-5). Information Granulation is the conceptual framework for granular computing. The Granular Computing paradigm was first considered by T.Y. Lin in [\[13\]](#page-22-7) and the first monograph in this direction was written by A. Bargiela and W. Pedrycz, [\[1\]](#page-21-3).

Information granulation (see e.g., [\[21\]](#page-22-8), [\[23\]](#page-22-5), [\[24\]](#page-22-9) ) is the process of grouping elements based on their indistinguishability, similarity, proximity or functionality. According to [\[1\]](#page-21-3), information granules are complex entities that arise in the process of abstraction of data and derivation of knowledge from information. Also, as it is shown in [\[1\]](#page-21-3), information granules typically have a hierarchical structure. From a mathematical point of view, this is shown in the very recent paper [\[2\]](#page-21-5), to be the Information Sciences analogue to the axiomatization of the Classical Set Theory using classes (Von Neumann–Bernays–Gödel set theory). This idea and the mathematical formulation of the Von Neumann–Bernays–Gödel set theory leads to a higher level of abstraction and generalization in this area, which further emphasizes the hierarchical nature of granular computing. Following this definition for information granules, granular computing can be defined as (see [\[2\]](#page-21-5)) a "structured combination of algorithmic abstraction of data and nonalgorithmic empirical verification of the semantics of abstractions". This definition allows a higher level of generality since there are no prescribed mechanisms to perform the tasks of abstraction of data and derivation of knowledge (see also [\[14\]](#page-22-10)) and it is consistent with the hierarchical approach of [\[1\]](#page-21-3).

#### **1.3 Granular Computing in Image Processing**

Human beings have the gift that looking at an image or a set of images, they are able immediately to perform the granulation of data and derivation of knowledge. For machines this is not at all a trivial task and the way how information granulation is performed is usually application oriented, despite the trend towards generality in what regards this issue. In the Image Processing field the idea of human centered information processing and granularity of information was present tacitly for a long time  $([1])$  $([1])$  $([1])$ , since the pattern recognition field has its central issue the recognition, classification and abstraction of image data. It appeared recently also explicitly in e.g.  $([1], [15], [16], [19])$  $([1], [15], [16], [19])$  $([1], [15], [16], [19])$  $([1], [15], [16], [19])$  $([1], [15], [16], [19])$  $([1], [15], [16], [19])$  $([1], [15], [16], [19])$  $([1], [15], [16], [19])$  $([1], [15], [16], [19])$ .

Information in Image Processing is represented by images. Machines represent them as matrices with integer elements. Human beings, as mentioned earlier have the ability to see in an image beyond the numbers a "flower" or a "dinosaur". Our point is that an information granule that humans use for the representation of a "flower" is a collection of images gathered by some image features and attributes. This is also consistent with the ideas and the formalism in [\[2\]](#page-21-5). Indeed we can regard images as information (set) and collections of several images, together with their features as information granules (classes).

#### **1.4 Ordered Structure-Based Image Processing**

Furthermore, our point is that in the visualization problem, using only the idea of "distance" is not precisely reflecting the human way of thinking. Indeed, human beings can recognize a flower in an image that contains also buildings and animals. This gift of human beings translates in mathematical terms into an order relation, "inclusion" relation between objects, in some sense. In terms of mathematical structures this pushes our thoughts from a purely metric spaces based approach, towards an ordered structure based approach for image processing.

Most of the literature in information visualization focuses on a metric space approach so conventional image database visualization systems are unable to show the perspective nature of the whole database. The present chapter, in contrast, focuses on a different kind of basic mathematical structure, i.e., the ordered structure [\[7\]](#page-21-6) [\[11\]](#page-21-7). This approach allows us to develop and to implement a more comprehensive information visualization strategy, namely, we visualize the lattice structure of an image database. It is easy to see that the ordered structure is very suitable for human perception and intuition. Partially this is the reason why fuzzy sets theory, based on a lattice structure as well, is able to interpret and to implement expert's knowledge [\[12\]](#page-22-14) [\[17\]](#page-22-15) [\[22\]](#page-22-6). Consequently, the ordered structure has a great potential ability to fit our intent to perform information processing tasks in a more and more human-centered manner. This approach allows us to deal with objects and relations in terms of an *ordered* *structure* [\[7\]](#page-21-6) [\[11\]](#page-21-7), which is different from the mathematical structure used by the conventional ongoing research.

### **1.5 Visualizing Huge Image Databases by Formal Concept Analysis**

Formal concept analysis [\[9\]](#page-21-4) constructs a concept lattice (which is a complete lattice) from the context table (information table) which contains objects and their attributes (feature vectors). The concept lattice has an ordered structure induced by the order of different concepts in a given context. This structure is very intuitive and so it is suitable for human perception. The proposed system is able to visualize a complete lattice obtained from a huge image database or a long video sequence, and users can easily recognize, understand and moreover, they can further process comprehensive information about the images in the database at once. In the present paper we discuss in detail the construction process which generates the context table, the quantization of attributes, and finally the generation of the concept lattice itself.

Finally the Hasse diagram of the concept lattice is visualized, and it is easy to see that it supports the empirical verification of the semantics of abstractions. Also, the knowledge obtained automatically is intuitively communicated to the user. So, a huge image database is visualized by the proposed method at once. Surely the hierarchical structure of the granular information makes it possible to the user to zoom in and out at any information granule.

The proposed system was realized using the JAVA based programming language *Processing* [\[18\]](#page-22-16), on an ordinal platform  $(CPU = 2.13 \text{GHz}, \text{MM} = 2 \text{GB})$ . As visualization experiments, we perform lattice visualization for the Corel Image Gallery and the Ubiquitous Home Image Database.

Since concept lattices are strongly related to the processes of discretization and quantization, we can see that the proposed visualization system perfectly fits in the setting of granular computing [\[1\]](#page-21-3). Indeed, we can see in the proposed system all the important elements of granular computing. The link between the proposed system and granular computing based human-centric information processing can be summarized as follows:

- Abstraction of data is performed through the discretization and quantization of features and attributes. Then a relation which relates objects to attributes is constructed. This way we obtain a huge number of abstract data as objects and very complex relations.
- Derivation of knowledge, is performed through construction of a complete lattice by formal concept analysis.
- Formal concept analysis is based on an ordered structure given by the set inclusion between attributes as order relation. This hierarchical structure of concepts in the proposed structure corresponds exactly to the hierarchical representation of granular elements. Using this ordered structure, we have realized the zoom in and zoom out capability of granular computing.
- The binary relation used in the formal concept analysis can be extended to a fuzzy relation, i. e., the proposed system can naturally work in the context of human subjectivity. [\[3\]](#page-21-8)

• The ideas underlying the visualization platform and the interaction function of the proposed system give a great ability to support the empirical verification of the results. Moreover, the potential user can adjust the quantization of features, being able in this way, to select the attributes which allow the him to obtain a suitable concept lattice. This interaction between user and machine through the formal concept analysis can be regarded as a promising human-centric, granular computing-based information processing method.

In Sec. 2 give some preliminaries about formal concept analysis. In Sec. 3, we present the proposed formal concept analysis-based lattice visualization system, aimed to allow users to understand and extract information from large image databases or video sequences. Section 4 shows two visualization experiments of two huge image databases: the Corel Image Gallery and Ubiquitous Home Database produced by National Institute of Information and Communication Technology (NICT). In Sec. 5, we explain the relationship between the proposed lattice visualization system and the important paradigms of granular computing and in Sec. 6 we conclude the paper.

### **2 Formal Concept Analysis**

### **2.1 Formal Concept Analysis – An Overview**

Formal concept analysis [\[7\]](#page-21-6) [\[9\]](#page-21-4) is a powerful mathematical tool, which helps us to construct a complete lattice from a cross table (relation) (e.g. Tab. [1\)](#page-4-0) between of objects and attributes. Let us illustrate first on an example, how we can use formal concept analysis in our problem. We construct in the followings a complete lattice from objects and attributes corresponding respectively to images and features.

In Tab. [1,](#page-4-0) the fact that the *i*th object possesses the *j*th attribute is indicated by the symbol  $\times$  in the *i*<sub>1</sub>-position of the table, and Figs. [1](#page-5-0) - [4](#page-5-1) correspond to the images 1 - 8.

<span id="page-4-0"></span>In this context, a concept will be an ordered pair  $(A, B)$ , where A (the extent) is a subset of a set consisting of eight images and  $B$  (the intent) is a subset of the

|         |   |          |          | Human Animals RED GREEN BLUE WHITE |          |  |
|---------|---|----------|----------|------------------------------------|----------|--|
| Image 1 | X |          |          |                                    | $\times$ |  |
| Image 2 |   |          |          |                                    | $\times$ |  |
| Image 3 |   | $\times$ |          | $\times$                           |          |  |
| Image 4 |   | ×        |          |                                    |          |  |
| Image 5 |   |          | ×        | $\times$                           |          |  |
| Image 6 |   |          | $\times$ | $\times$                           |          |  |
| Image 7 |   | $\times$ |          | $\times$                           |          |  |
| Image 8 |   |          |          |                                    |          |  |

**Table 1.** An example of an information table



**Fig. 1.** Example: Images 1 and 2

<span id="page-5-0"></span>

**Fig. 2.** Example: Images 3 and 4



**Fig. 3.** Example: Images 5 and 6

<span id="page-5-1"></span>

**Fig. 4.** Example: Images 7 and 8

six types of image features, e.g., "Human" means that there are human beings in the image. To demand that the concept is determined by its extent and by its intent, means that  $B$  should contain just the properties shared by all the images in  $A$  and, similarly, the images in  $A$  should be precisely those sharing all the properties in B. A simple procedure for finding a concept is as follows: take an object, say the "Image 2", and let  $B$  be the set of attributes which it possesses, in this case

$$
B = \{ \text{BLUE}, \text{WHITE} \}. \tag{1}
$$

Let A be the set of all frames possessing all the attributes in  $B_i$ , i.e., in our case

$$
A = \{\text{Image 2, Image 8}\}.
$$
 (2)

Then  $(A, B)$  is a concept, and in this context, we can interpret this concept

$$
(A, B) = (\{ \text{Image 2, Image 8} \}, \{ \text{BLE, WHICH} \}),
$$
\n
$$
(3)
$$

as a "resort", since the images "Image 2" and "Image 8" are representing 'beach' and 'mountain', respectively. In other words, the concept 'resort' always contains the attributes 'BLUE' and 'WHITE'. If we will have any images with attributes 'BLUE' and 'WHITE', we can guess that the image is related to 'resorts' based on this relationship (knowledge). More generally, we may begin with a set of objects rather than a single object. Concepts may also be obtained via a similar process commencing with a set of attributes.

It is usual to regard a concept  $(A_1, B_1)$  as being 'less general' than a concept  $(A_2, B_2)$  if the extent  $A_1$  of  $(A_1, B_1)$  is contained in the extent  $A_2$  of  $(A_2, B_2)$ . Thus an order is defined on the set of concepts by

$$
(A_1, B_1) \le (A_2, B_2) \quad \Leftrightarrow \quad A_1 \subseteq A_2. \tag{4}
$$



<span id="page-6-0"></span>**Fig. 5.** Concept Lattice with respect to Table [1](#page-4-0)

The asymmetry in this definition is illusory since  $A_1 \subseteq A_2$  is equivalent to  $B_1 \supseteq B_2$ . The resulting ordered set of concepts for the image database is the lattice given in Fig. [5.](#page-6-0)

#### **2.2 An Intuitive Algorithm to Construct the Concept Lattice**

We show in the present section how to construct a concept lattice from any cross table, based on the following precise definitions. A context is a triple  $(G, M, I)$ where G and M are two sets and  $I \subseteq G \times M$  is a relation. The elements of G and M are called **objects** and **attributes** respectively. As it is usual, instead of writing  $(g, m) \in I$ , we write  $gIm$  and we say that 'the object g has the attribute m'. For  $A \subseteq G$  and  $B \subseteq M$ , we define

$$
A' = \{ m \in M \mid (\forall g \in A) \; gIm \},\tag{5}
$$

$$
B' = \{ g \in G \mid (\forall m \in B) \text{ } gIm \},\tag{6}
$$

<span id="page-7-0"></span>so A' is the set of attributes common to all the objects in A while B' is the set of objects possessing the attributes in B. Then a **concept** of the context  $(G, M, I)$ objects possessing the attributes in B. Then a **concept** of the context  $(G, M, I)$ is defined to be a pair  $(A, B)$  where  $A \subseteq G$ ,  $B \subseteq M$ ,  $A' = B$  and  $B' = A$ . The **set** of the concept  $(A, B)$  is A while its intent is B. The set of all concepts **extent** of the concept  $(A, B)$  is A while its intent is B. The set of all concepts of the context  $(G, M, I)$  is denoted by  $\mathbf{B}(G, M, I)$ .

Let  $(G, M, I)$  be a context. For concepts  $(A_1, B_1)$  and  $(A_2, B_2)$  in  $\mathbf{B}(G, M, I)$ we write  $(A_1, B_1) \leq (A_2, B_2)$ , if  $A_1 \subseteq A_2$ . Also,  $A_1 \subseteq A_2$  implies that  $A'_1 \supseteq A'_2$ ,<br>and the reverse implication is valid too because  $A'' = A_1$  and  $A'' = A_2$ . We and the reverse implication is valid too, because  $A_1'' = A_1$  and  $A_2'' = A_2$ . We have therefore have therefore,

$$
(A_1, B_1) \le (A_2, B_2) \Leftrightarrow A_1 \subseteq A_2 \Leftrightarrow B_1 \supseteq B_2. \tag{7}
$$

We can easily see that the relation  $\leq$  is an order relation on  $\mathbf{B}(G, M, I)$ , and  $\langle \mathbf{B}(G, M, I); \leq \rangle$  is a complete lattice, i.e., it is the **concept lattice** of the context  $(G, M, I).$ 

We consider in what follows an intuitive algorithm for the construction of the concept lattice. First, we select an object of the context  $(G, M, I)$ , and then using Eqs. [\(5\)](#page-7-0) and [\(6\)](#page-7-0), we can successively find the corresponding concepts.

The detailed procedure is as follows:

### 1. **Find all the extents of the concepts in the context**  $(G, M, I)$

- a) Draw a table with two columns headed **attributes** and **extents**. Leave the first cell of the attributes column empty and write  $G$  in the first cell of the extents column.
- b) Find a maximal attribute-extent, say  $m'$ .<br>i If the set  $m'$  is not already in the
	- i. If the set  $m'$  is not already in the extents' column, add the row<br> $[m|m']$  to the attribute-extent table. Intersect the set  $m'$  with all  $[m|m']$  to the attribute-extent table. Intersect the set m' with all<br>previous extents in the Extents column. Add these intersections to previous extents in the Extents column. Add these intersections to the Extents' column (unless they are already in the list), and leave the corresponding cells in the Attribute column empty.
- ii. If the set  $m'$  is already in the Extents column, add the label  $m$  to the attribute cell of the row where  $m'$  previously occurred the attribute cell of the row where  $m'$  previously occurred.<br>
ste the column below m from the table
- c) Delete the column below  $m$  from the table.
- d) If the last column has been deleted, stop, otherwise return to 1-(b).
- 2. **Draw the diagram with** m **and** m' **labels.** Start at the top of the diagram with one point labeled  $G$ . Work down the list of Extents in the table from 1. with one point labeled G. Work down the list of Extents in the table from 1. For each set  $S$  in the list, add an appropriately positioned new point to the diagram. Below the point corresponding to  $S$  list the elements in  $S$ . If  $S$  is an attribute-extent, say  $S = m'$ , add the label m above the point corresponding to  $S$ to S.

### 3. **Redraw the diagram with** g **and** m **labels**

- a) Redraw the diagram. Add the m labels as in the first diagram.
- b) For each object  $g$  in  $G$ , add a label  $g$  below the point on the diagram which has the smallest extent containing the object  $g$  (this point can be found from the first diagram). Alternatively, the point  $g$  to be labeled can be obtained by finding the point  $\Lambda\{m \mid gIm\}$ .

### 4. **Check the answer**.

- a) Check that every joint-irreducible element has a label  $q \in G$ .
- b) Check that every meet-irreducible element has a label  $m \in M$ .
- c) Check that

$$
(\forall g \in G)(\forall m \in M)gIm \Leftrightarrow g \le m \tag{8}
$$

by checking that, for all  $m \in M$ , the set of object labels in  $\downarrow m$  is exactly the attribute-extent  $m'$ , where  $\downarrow m$  stand for the down set of m i.e., the set of all elements less than  $m$ set of all elements less than m.

### **[Example]**

<span id="page-8-0"></span>Using the cross table shown in Table [1,](#page-4-0) we present in this particular case the way how we can construct the concept lattice. In order to improve the visualization of the lattice, we convert Table [1](#page-4-0) into the following table [2.](#page-8-0) With respect to Table [2,](#page-8-0) we apply the intuitive algorithm shown above, and we obtain the attributesextents table given in Tab. [3.](#page-9-0)

|                | Hu       | Ān       | Re       | Gr       | B <sub>l</sub> | Wh       |
|----------------|----------|----------|----------|----------|----------------|----------|
| 1              | $\times$ |          |          |          | $\times$       | $\times$ |
| $\overline{2}$ |          |          |          |          | $\times$       | $\times$ |
| 3              |          | $\times$ |          | $\times$ |                |          |
| $\,4\,$        |          | $\times$ |          |          |                | $\times$ |
| $\overline{5}$ |          |          | $\times$ | $\times$ |                |          |
| 6              |          |          | $\times$ | $\times$ |                |          |
|                |          | $\times$ |          | $\times$ |                |          |
| 8              |          |          |          |          | $\times$       | $\times$ |

**Table 2.** A cross table

| Attributes | Extents          |
|------------|------------------|
|            | G                |
| Gr         | $\{3, 5, 6, 7\}$ |
| Wh         | $\{1, 2, 4, 8\}$ |
| Αn         | $\{3,4,7\}$      |
|            | ${3, 7}$         |
|            | {4}              |
| ĒП         | $\{1, 2, 8\}$    |
| Re         | $\{5,6\}$        |
| Hu         | 1                |
|            |                  |

<span id="page-9-0"></span>**Table 3.** Attributes and Extents Table

We observe the inclusions in Tab. [3,](#page-9-0) and so we obtain Fig. [5,](#page-6-0) where the overlaps are eliminated.

### **2.3 Next Closure Algorithm**

The intuitive algorithm shown in the previous subsection includes some redundant computations, therefore in the present chapter we will use the **next closure algorithm** to perform the formal concept analysis step [\[9\]](#page-21-4). We will briefly describe the next closure algorithm in the followings:

### **[Def. Lexicographic Order]**

$$
A, B \subseteq G, i \in G
$$
  
\n
$$
A < B \iff \exists i \in B \setminus A,
$$
  
\n
$$
A \cap \{1, 2, \dots, i - 1\} = B \cap \{1, 2, \dots, i - 1\}.
$$
  
\n
$$
(9)
$$

ii)

$$
A \leq_i B \quad \Leftrightarrow \quad \in B \setminus A,
$$
  
\n
$$
A \cap \{1, 2, \dots, i - 1\} = B \cap \{1, 2, \dots, i - 1\}.
$$
  
\n(10)

**[Def. Next Closure]**

$$
A \oplus i = ((A \cap \{1, ..., i - 1\}) \cap \{i\}).
$$
\n(11)

According to the above definitions, the next closure algorithm will work as follows:

### **[Next Closure Algorithm]**



## **3 A Lattice Visualization System for Huge Image Databases**

As shown in the previous section, we can construct concept lattices from any cross tables having texts, images, video or sound files as objects, provided that they have precisely defined attributes. The main, still not completely solved question is, how to quantize the feature vectors to obtain relevant but not redundant attributes. These can contain multiple values as e.g., intensities of pixels in  $\{0, 1, \ldots, 255\}$  while, in contrast, the attributes should be binary values. Of course, we can assign the digit of each feature value to an attribute in the cross table, however, the concept lattice will be too complex if we adopt such a strategy. The quantization should be performed carefully, however, we currently do not have any systematic way to quantize the feature vectors into attributes. This is apparently a drawback but it can be turned into an advantage allowing a more human-centered processing. Therefore, we have developed a lattice structure visualization system with an interaction function, where the user can adjust the

```
Next Closure
  begin
    i = maximum element of G
    i= succ(i)success = falsei = \text{pred}(i)if (i \notin A){
     B = (A \cap \{1, 2, \ldots, i-1\}) \cap \{i\}C = B''if(A < i)A=Csuccess = true}
     }
  until success or i = minimum element of G
```


<span id="page-11-0"></span>**Fig. 6.** Overview of the proposed visualization system

way how the features of images (color, intensity information) are quantized into attributes (Fig. [6\)](#page-11-0).

The proposed system defines the features of images according to the following approach. We denote the images by  $P_k$   $(k = 1, 2, \ldots, P_{max})$   $(P_{max} =$  the number of images in database) as the objects  $G = \{P_k | k = 1, 2, ..., P_{max}\}\$ of the cross table. Each image of the size  $m \times n$  is represented by three image planes, i.e., read, green, blue, therefore, the definition of an object is as follows:

$$
P = \{ P_k^c \mid P_k^c \in \{0, \dots, 255\}^{m \times n}, c \in \{R, G, B\} \}.
$$
 (12)

In order to acquire a suitable and at the same time understandable lattice structure for the proposed human visualization system, HSB color space is employed. Furthermore, as attributes used in formal concept analysis, we propose new image features considering the effects of saturation and brightness.

The Hue (H), Saturation (S), and Brightness (B) are respectively defined as

$$
H_k(i,j) = \tan^{-1} \frac{\gamma}{\beta - \alpha},\tag{13}
$$

$$
S_k(i,j) = \frac{\alpha^2 + \beta^2 + \gamma^2}{3}
$$
 (14)

and

$$
B_k(i,j) = P_k^R(i,j) + P_k^G(i,j) + P_k^B(i,j),
$$
\n(15)

where

$$
\begin{cases}\n\alpha = P_k^B(i,j) - P_k^R(i,j), \\
\beta = P_k^B(i,j) - P_k^G(i,j), \\
\gamma = P_k^G(i,j) - P_k^B(i,j).\n\end{cases}
$$
\n(16)

In the framework of the new image features that we are currently using in formal concept analysis, the weights of Saturation, and Brightness are given by

$$
S_w(P_k(i,j)) = \frac{S(P_k(i,j))}{S_{max}},
$$
\n(17)

and

$$
B_w(P_k(i,j)) = 1 - \frac{|B(P_k(i,j)) - \frac{B_{max}}{2}|}{\frac{B_{max}}{2}},\tag{18}
$$

where <sup>S</sup>*max* and <sup>B</sup>*max* are the maximum values of the Saturation and Brightness respectively.

Finally we define a modified Hue as a new image feature as

$$
H'(P_k(i,j)) = \sum_{0 \le i \le m, 0 \le j \le n} H(P_k(i,j)) S_w(P_k(i,j)) B_w(P_k(i,j)). \tag{19}
$$

The attribute sets used in the framework of formal concept analysis are defined as

$$
M = \{A_1, A_2, \dots, A_{Max}\},\tag{20}
$$

where

$$
A_k = \begin{cases} 0 & otherwise, \\ 1 \ T_{k-1} \le H' < T_k, \end{cases} \tag{21}
$$

and T is a threshold value. The index 'max' corresponds to the quantization number of our color space, and the proposed system can adjust this value through computer-user interaction.

### **4 Visualization Experiments by the Proposed System**

The proposed visualization system is developed on a usual platform  $(CPU =$ 2.13GHz, MM = 2GB) using the JAVA based programming language 'Processing' [\[18\]](#page-22-16).

In the first step of the algorithm, the proposed system successively loads all the images from a given database. Then, with respect to each image we perform the image feature extraction and so we generate the candidates for the attributes used in the formal concept analysis step. In the next step, the user sets the desired attributes by the interaction function proposed, and the concept lattice will be generated based on the selected attributes.

#### **4.1 Visualization of Corel Image Database**

For our first experiment we use the Corel Image Gallery which consists of 1,000 images, shown in Fig. [7,](#page-13-0) in the categories of *Human, Dinosaur, Flower, Elephant, Bus, Mountain, Horse, Dish, Building* and *Sea*. This visualization experiment



**Fig. 7.** Examples of Corel Image Database

<span id="page-13-0"></span>

<span id="page-13-1"></span>**Fig. 8.** Concept lattice obtained by first attributes set (Corel Image Database)



**Fig. 9.** Concept lattice obtained by second attributes set (Corel Image Database)

<span id="page-14-0"></span>

<span id="page-14-1"></span>**Fig. 10.** Concept 'flower', attributes : Hue = Red, Saturation = Low

uses two fixed attribute sets, this means that the proposed system generates two attribute sets by the interaction function. The first set is composed of 15 attributes (12 color quantized regions, and 3 brightness levels). The second set



**Fig. 11.** Concept 'dinosaur', the attributes : Saturation = High

<span id="page-15-0"></span>

Fig. 12. Concept 'elephant', RGB color space = Red and Green

<span id="page-15-1"></span>of attributes is composed of 24 attributes (20 color quantized regions, and 4 brightness levels).

The concept lattices obtained are shown in Fig. [8](#page-13-1) (first attribute set) and Fig. [9](#page-14-0) (second attribute set), respectively. In these figures, the size of an element is proportional to the number of images which belong to each concept. The color of an element of the lattice represents the average color of the images it contains.



**Fig. 13.** Example Images, Ubiquitous Home Database

<span id="page-16-0"></span>Therefore, using the proposed visualization platform of the concept lattice, we can recognize a perspective structure of the whole image database.

The proposed system can focus on each of the elements of the concept lattice. The user can zoom in at the level of any element and he can view the images included in each concept as shown in Figs. [10,](#page-14-1) [11,](#page-15-0) and [12.](#page-15-1) These can



Fig. 14. Concept lattice obtained by first attributes set (Ubiquitous Home Database)

<span id="page-17-0"></span>

<span id="page-17-1"></span>**Fig. 15.** Concept lattice obtained by second attributes set (Ubiquitous Home Database)



**Fig. 16.** Obtained scene 1 by proposed system

<span id="page-18-0"></span>

**Fig. 17.** Obtained scene 2 by proposed system

be recognized as belonging to the 'flower', 'dinosaur' or 'elephant' categories, respectively. Of course, the attributes of these concepts express the features in the 'flower', 'dinosaur' and 'elephant' concepts.

By visualization based on formal concept analysis, we found the following knowledge with respect to Corel Image Gallery:

- The 'flower' concept is composed of the attributes  $:$  Hue  $=$  Red, Saturation  $=$  Low
- The 'Dinosaur' concept is composed of the attributes : Saturation = High
- The 'elephant' concept has the following attributes : RGB color space = Red and Green



**Fig. 18.** Obtained scene 3 by proposed system

<span id="page-19-0"></span>

**Fig. 19.** Obtained scene 4 by proposed system

### **4.2 Ubiquitous Home Database Visualization**

In this subsection, we perform a lattice visualization experiment with the *ubiquitous home database* produced by NICT (National Institute of Information and Communication Technology). Some example images are shown in Fig. [13.](#page-16-0) As it can be seen from these images, they are obtained by fixed cameras. Unlike the previous section's data, images of the ubiquitous home database have almost the same picture composition. In this visualization experiment, we confirm the effectiveness of the proposed system for scene extraction and recognition. As attribute sets, we employ the same two sets as in the previous section.

The concept lattices obtained are shown in Fig. [14](#page-17-0) (first attribute set) and Fig. [15](#page-17-1) (second attribute set), respectively. Figures [16](#page-18-0) - [19](#page-19-0) show some extracted scenes. As it can be seen from these results, the proposed visualization system is useful for scene recognition.

# **5 Proposed Lattice Structure Visualization and Granular Computing Based Human Centric Information Processing**

At the beginning of the manuscript we have pointed out the conceptual relationships between the proposed system and the granular computing paradigm. In the present section we compare in view of the experimental results shown, the proposed visualization system with the basic concepts of granular computing.

These results show a strong relationship between granular computing and the proposed, formal concept analysis-based lattice structure visualization system. Indeed, we can see a correspondence between our system and basic concepts of Granular Computing.

- 1. The proposed system's information granules correspond to the elements of the concept lattice while the partitioning corresponds to the quantization of features.
- 2. It is easy to see that the concept lattice is a realization of the perspective, hierarchical nature of granular computing. In the concept lattice, upper elements stand for big information granules, and by duality, lower elements represent small granules.
- 3. The ability of the proposed system for the derivation of knowledge was shown in the proposed examples. It is interesting to remark here that the proposed system also caries the property to be highly intuitive, and so it allows the empirical verification of the results.
- 4. Another notable result of this manuscript is that the proposed system has interaction functions to adjust the quantization number, and to select the attributes for obtaining a suitable concept lattice.

We can now conclude by pointing out that altogether, the features of the proposed system show the characteristics of *Human-Centric Information Processing based on Granular Computing*.

### **6 Conclusions**

In order to extract understandable information with suitable sizes from huge volumes of image data, a lattice visualization system based on granular computing and formal concept analysis has been proposed. The proposed system generates the information granules as elements in the concept lattice, from the cross table composed of objects and attributes. The objects and attributes are images and features respectively. The interaction function proposed here, allows users to adjust the quantization of features and to select desired attributes. In the visualized lattice, users can easily recognize the perspective structure of a whole image database.

Since we can recognize in this approach information granules as elements of our lattice and partitioning of granular elements as the quantization of attributes, the proposed lattice visualization system is a realization of the important, granular computing paradigm.

The proposed system has been developed on an ordinal computer (CPU = 2.13GHz, MM = 2GB) based on *Processing* language, and two visualization experiments have been performed. In these experiments, we have used the Corel Image Gallery and the Ubiquitous Home Database. Through these experiments, we have confirmed that the proposed system is able to visualize huge image databases in an intuitive way, and also it becomes suitable for scene recognition from a huge amount of data.

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