Chapter 6
New Trends in Aggregation-Disaggregation Approaches

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Abstract The aggregation-disaggregation approaches as an important field of multicriteria decision-aid systems aim to infer global preference models from preference structures, as directly expressed by one or more decision-makers. The main objective of this chapter is to present new research developments of aggregation-disaggregation models and discuss related research topics. These recent developments cover a wide variety of topics, like post-optimality analysis, robustness analysis, group and collective decision-making. They focus mainly on the UTA family of models and highlight their most important advantages: they are flexible in the modeling process of a decision problem, they may provide analytical results that are able to analyze the behavior of the decision-maker, and they can offer alternative ways to reduce the preferential inconsistencies between the decision-maker and the results of the disaggregation model. Finally, future research topics in the context of preference disaggregation approaches are outlined in this chapter.

6.1 Introduction

Preference disaggregation constitutes an important Multiple Criteria Decision Aid (MCDA) philosophy aiming to assess/infer global preference models from given preference structures and to address decision-aiding activities through operational models within the aforementioned framework. In other words, the preference disaggregation approach refers to the analysis (disaggregation) of the global preferences (judgment policy) of the Decision-Maker (DM) in order to identify the criteria ag-

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gregation model that underlies the preference result. From the previous it is clear that contrary to the traditional aggregation paradigm, where the criteria aggregation model is known a priori and the global preference is unknown, the philosophy of preference disaggregation aims to infer the preference models from given global preferences.

Although several approaches have developed in the context of aggregation-disaggregation paradigm, UTA methods [14, 40] may be considered as the main initiative and the most representative example of preference disaggregation theory. UTA methods are regression-based approaches that have been developed as an alternative to multiattribute utility theory (MAUT).

The philosophy of aggregation-disaggregation is explicitly presented in Figure 6.1, where the emphasis on the analysis of the behavior and the cognitive style of the DM is clear. In the context of UTA methods, special iterative interactive procedures are used, where the components of the problem and the DM's global judgment policy are analyzed and then they are aggregated into a value system. The goal of this approach is to aid the DM to improve his/her knowledge about the decision situation and his/her way of preferring that entails a consistent decision to be achieved.

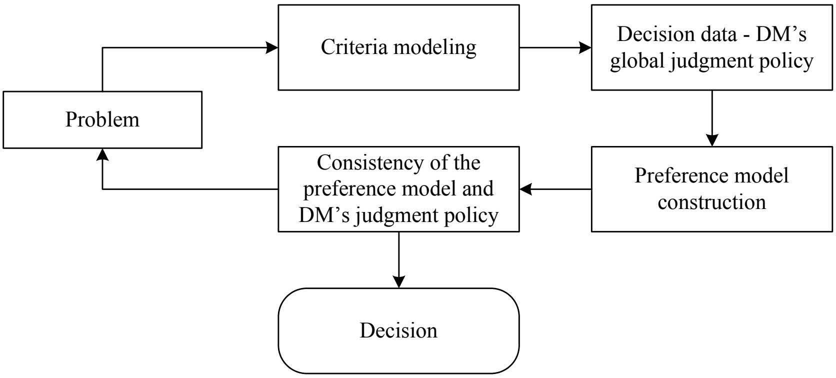

Fig. 6.1 The aggregation-disaggregation approach [39]

As known, in the general MCDA context the main objective is usually to analyze a set A of potential actions (or objects, alternatives, decisions) in terms of multiple criteria in order to model all the possible impacts, consequences or attributes related to this set A. However, in the aggregation-disaggregation approach, it is often necessary to use a set reference actions A_R in order to clarify the DM's global preference [15]. This reference set may be a set of past decision alternatives, a subset of decision actions, especially when A is large ($A_R \subset A$), or a set of fictitious actions, consisting of performances on the criteria, which can be easily judged by the DM to perform global comparisons [38].

Moreover, following the general modeling methodology of MCDA problems, a consistent family of criteria $\{g_1, g_2, ..., g_n\}$ should be assessed [26]. Each criterion is a non-decreasing real valued function defined on A, as follows:

$$g_i : A \rightarrow [g_{i*}, g_i^*] \subset \mathbb{R}/a \rightarrow g(a) \in \mathbb{R} \tag{6.1}$$

where $[g_{i*}, g_i^*]$ is the criterion evaluation scale, g_{i*} and g_i^* are the worst and the best level of the i-th criterion respectively, $g_i(a)$ is the evaluation or performance of action a on the i-th criterion and $\mathbf{g}(a)$ is the vector of performances of action a on the n criteria.

Therefore, the preference structure on a set of actions, which is necessary in a preference disaggregation approach, may have the following form, based on the aforementioned definitions:

$$\begin{cases} g_i(a) > g_i(b) \Leftrightarrow a \succ b \text{ (a is preferred to b)} \\ g_i(a) = g_i(b) \Leftrightarrow a \sim b \text{ (a is indifferent to b)} \end{cases} \tag{6.2}$$

This preference structure has a form of a weak-order, although alternative aggregation-disaggregation approaches may adopt different problem statements. In any case, the DM is asked to externalize and/or confirm his/her global preferences on the set A_R taking into account the performances of the reference actions on all criteria. So, in the UTA family of models, the problem is to adjust additive value or utility functions based on multiple criteria, in such a way that the resulting structure would be as consistent as possible with the initial structure.

Goal programming techniques have always played an important role in the development of preference disaggregation models. In fact, the history of the disaggregation principle in multidimensional/multicriteria analyses begins with the use of this special form of linear programming. Between mid 50s and mid 70's the most important research efforts refer to the development of linear or non-linear multidimensional regression analyses [4, 18, 20, 42, 45], while later works studied the case of ordinal criteria in order to assess/infer preference/aggregation models [14, 31, 40, 46]. Jacquet-Lagrèze and Siskos [15] and Siskos et al. [38] present an analytical review of the history of aggregation-disaggregation principle.

The main objective of this chapter is to present new research developments of aggregation-disaggregation approaches. Although these recent research efforts cover a wide variety of research topics, the chapter focuses on particular issues that have recently drawn significant attention in the literature, like post-optimality analysis, robustness analysis, group and collective decision-making.

The chapter is organized into 5 sections. Section 6.2 presents briefly the UTA model, as well as its significant proposed extensions. Post-optimality and robustness analysis in UTA-type models is discussed in section 6.3, while section 6.4 refers to the presentation of UTA-based group and collective decision models. Finally, section 6.5 summarizes some concluding remarks and outlines future research topics in the context of preference disaggregation approaches.

6.2 The UTA Family of Models

6.2.1 UTA and UTASTAR Methods

The UTA method initially proposed by Jacquet-Lagrèze and Siskos [14] originates an entire family of preference disaggregation models during the last thirty years. As already noted, the main objective of the method is to infer one or more additive value functions from a given ranking on a reference set A_R.

The original UTA method assumes an additive value function of the following form:

$$u(\mathbf{g}) = \sum_{i=1}^{n} u_i(g_i) \tag{6.3}$$

subject to normalization constraints:

$$\begin{cases} \sum_{i=1}^{n} u_i(g_i^*) = 1 \\ u_i(g_{i*}) = 0 \quad \forall i = 1,2,\ldots,n \end{cases} \tag{6.4}$$

where u_i $(i = 1,2,...,n)$ are the marginal value functions.

As discussed by Siskos et al. [38], this additive formula satisfies the axioms of comparability, reflexivity, transitivity of choices, continuity, and strict dominance, since $u_i(g_i) \geq 0$ holds and $du_i/dg_i > 0$ is assumed (see [19] for a detailed discussion about the properties of an additive utility model).

The method estimates the aforementioned value functions using linear goal programming techniques so that the ranking(s) obtained through these functions on A_R is (are) as consistent as possible with the one given by the DM. Thus, introducing a potential error $\sigma(a)$ relative to A_R, the value of each action may be written as:

$$u'[\mathbf{g}(a)] = \sum_{i=1}^{n} u_i[g_i(a)] + \sigma(a) \quad \forall a \in A_R \tag{6.5}$$

The implementation of the UTA algorithm requires the use of linear interpolation in order to approximate the marginal value functions u_i in a piecewise linear form. Moreover, taking into account the DM's ranking on $A_R = \{a_1, a_2, ..., a_m\}$, the reference actions are "rearranged" from the best (a_1) to the worst action (a_m).

As emphasized by Jacquet-Lagrèze and Siskos [14], DM's ranking has the form of weak order R, and thus, given the transitivity of R, it is possible to avoid unnecessary comparisons on A_R. Thus, assuming that

$$\Delta(a_k, a_{k+1}) = u'[(\mathbf{g}(a_k)] - u'[(\mathbf{g}(a_{k+1})] \tag{6.6}$$

the comparison of every pair of consecutive actions (a_k, a_{k+1}) gives the following conditions:

$$\begin{cases} \Delta(a_k, a_{k+1}) \geq \delta \ \text{ iff } a_k \succ a_{k+1} \\ \Delta(a_k, a_{k+1}) = 0 \ \text{ iff } a_k \sim a_{k+1} \end{cases} \tag{6.7}$$

where δ is a small positive number so as to discriminate significantly two successive equivalence classes of R.

Taking into account the previous assumptions and notations, the following Linear Program (LP) is used in order to estimate the marginal value functions:

$$\begin{cases} [\min]\ F = \sum_{a \in A_R} \sigma(a) \\ \text{s.t.} \quad \left.\begin{array}{l} \Delta(a_k, a_{k+1}) \geq \delta \text{ if } a_k \succ a_{k+1} \\ \Delta(a_k, a_{k+1}) = 0 \text{ if } a_k \sim a_{k+1} \end{array}\right\} \quad \forall k \\ \quad u_i(g_i^{j+1}) - u_i(g_i^j) \geq s_i \qquad \forall i \text{ and } j \\ \quad \sum_{i=1}^{n} u_i(g_i^*) = 1 \\ \quad u_i(g_{i*}) = 0,\ u_i(g_i^j) \geq 0,\ \sigma(a) \geq 0\ \forall a \in A_R,\ \forall i \text{ and } j \end{cases} \tag{6.8}$$

where the last two constraints refer to the monotonicity and normalization constraints of u_i, respectively (with $s_i \geq 0$ indifference thresholds defined on each criterion).

On the other hand, the UTASTAR method proposed by Siskos and Yannacopoulos [40] may be considered as an improved version of the original UTA model, since it proposes two important modifications in the UTA algorithm:

- Double error function: the single error $\sigma(a)$ is replaced by a double positive error term (i.e. $\sigma^+(a)$ and $\sigma^-(a)$ being the overestimation and the underestimation error, respectively) in order to assure the minimization of the objective function of LP (6.8).
- Transformation of the variables: the original $u_i(g_i^j)$ variables of LP (6.8) are replaced by the new transformation variables w_{ij}, which represent the successive steps of the marginal value functions u_i, in order to reduce the size of LP (6.8) by removing the monotonicity constraints of u_i.

Siskos and Yannacopoulos [40] note that the UTASTAR algorithm perform better compared to the original UTA method based on a variety of experimental data and taking into account a number of comparison indicators (e.g. number of the necessary simplex iterations for arriving at the optimal solution, Kendall's τ between the initial weak order and the one produced by the estimated model, and the total sum of errors as the indicator of dispersion of the observations).

6.2.2 *Extensions of the UTA Method*

There are several variants and extensions of the UTA/UTASTAR method that try to model different forms of DM's preferences, apply different optimality criteria in the aforementioned LP formulation, or adopt the method in different decision problems.

As presented in the previous section, the LP formulation is a simple but powerful modeling approach that allows considering alternative types of global preference expressed by the DM. For example, Jacquet-Lagrèze and Siskos [14] propose to infer $u[\mathbf{g}(a)]$ from pairwise comparisons among the actions of the A_R. The intensity of the DM's preferences may also be considered in the formulation of LP (6.8) by adding a series of constraints of the following type [38]:

$$\Delta(a,b) - \Delta(b,c) \geq \varphi \tag{6.9}$$

where φ is a measure of preference intensity, which implies that the preference of alternative a over alternative b is stronger than the preference of b over c.

Similar modeling approaches have been proposed by Despotis and Zopounidis [6] and Oral and Ketanni [23], where a ratio scale is used to express intensity of preferences.

Regarding the different optimality criteria, it should be emphasized that this problem is discussed by Jacquet-Lagrèze and Siskos [14] in the development of the original UTA method. They propose, for example, the following alternatives:

1. Maximize the Kendall's τ between the ranking provided by the DM and the ranking given by the model (i.e. minimize the number of violated pairs between these rankings).
2. Weight the potential errors in F taking into account a different degree of confidence in each ranked action.

Alternative desired properties of u_i may also lead to different variations of the UTA/UTASTAR method. In this context, Despotis and Zopounidis [6] present extensions of the method in the case of non-monotonic marginal value functions or other additional properties of the assessed value functions (e.g. concavity).

An analytical presentation and discussion of other variants of the UTA and UTASTAR methods may be found in Jacquet-Lagrèze and Siskos [15] and Siskos et al. [38]. The most important of these extensions include:

- The stochastic UTA method developed in the framework of multicriteria decision-aid under uncertainty, where the adopted aggregation model has the form of a von Neumann-Morgenstern additive utility function [33, 34].
- The UTA-type sorting methods, which are developed in the context of problem statement β (sorting the actions into predefined and preference-ordered categories), like the UTADIS family of models [7, 47, 49] and the MHDIS method [48].
- The incorporation of the UTA method in the solution process of multiobjective programming problems (see for example the works of Stewart [43], Jacquet-Lagrèze et al. [12], Siskos and Despotis [35]).
- The MACBETH method (Measuring Attractiveness by a Categorical Based Evaluation Technique) proposed by Bana e Costa and Vansnick [1], which infers a single value function from pairwise comparisons externalized from the DM on a single criterion in terms of criterion attractiveness. The same procedure is repeated for each criterion and, finally for the whole set of criteria in order to infer the criteria weights. The overall evaluation model is an additive value model.

Finally, it should be emphasized that the main principles of the aggregation-disaggregation approach may be adopted in other MCDA fields (e.g. outranking relation methods, ordinal regression analysis), where the problem is to extract DM's preferences (value system, model parameters, etc) in a consistent way. In general, as mentioned by Siskos et al. [38] this philosophy is also employed in other non-classical MCDA approaches, like rough sets, machine learning, and neural networks, in order to infer some form of a decision model (a set of decision rules or a network) from given decision results involving assignment examples, ordinal or measurable judgments.

6.3 Post-optimality Analysis and Robustness

6.3.1 Post-optimality Analysis

The stability analysis is considered as an important part of the algorithm of the UTA methods, since all these approaches are based on a LP modeling and thus often the problem of multiple or near optimal solutions appears.

In the classical approach of the UTA/UTASTAR method the stability analysis is considered as a post-optimality analysis problem, based on a heuristic method for near optimal solutions search [37]. These solutions have some desired properties, while the heuristic technique is based on the following:

- In several cases, the optimal solutions are not the most interesting, given the uncertainty of the model parameters and the preferences of the decision-maker [44].
- The number of the optimal or near optimal solutions is often huge. Therefore an exhaustive search method (reverse simplex, Manas-Nedoma algorithms) requires a lot of computational effort.

In particular, if the optimum $F^* = 0$, the polyhedron of admissible solutions for u_i is not empty and many value functions lead to a perfect representation of the weak order R. Even when the optimal value $F^* > 0$, other solutions, less good for F, can improve other satisfactory criteria, like Kendall's τ. In any case, as emphasized by Jacquet-Lagrèze and Siskos [14], it is crucial to explore the post-optimal solutions space defined by the polyhedron:

$$\begin{cases} F \leq F^* + k(F^*) \\ \text{all the constraints of LP (6.8)} \end{cases} \tag{6.10}$$

where $k(F^*)$ is a positive threshold, which is a small proportion of F^*.

The previous polyhedron is partially explored in the original UTA method by solving the following LPs:

$$\begin{cases} [\min]\ u_i(g_i^*) \\ \text{in} \\ \text{polyhedron (6.10)} \end{cases} \text{and} \begin{cases} [\max]\ u_i(g_i^*) \\ \text{in} \\ \text{polyhedron (6.10)} \end{cases} \quad \forall i = 1, 2, \ldots, n \qquad (6.11)$$

As noted by Siskos et al. [38], the solutions of the above LPs give the internal variation of the weight of all criteria g_i, and consequently give an idea of the importance of these criteria in the DM's preference system. The final solution of the problem is calculated as the average of the previous LPs; this average solution is less representative, if a large variation of the provided solutions appears.

However, the efficiency of the aforementioned optimization procedure is based on the number and the meaning of the criteria introduced in the model. As mentioned by [32], when the number of criteria is small, the previous $2n$ LPs may be solved, otherwise it is possible to solve only n LPs (maximization of $u_i(g_i^*)$). Nevertheless, if there is an a priori typology of the criteria, i.e. when the criteria can be grouped into different classes determining different policies of the DM, then it is possible to minimize or maximize the sum of the weighting factors of the criteria for each policy. In this way, the value systems obtained by the post-optimality analysis are able to show the strengths and weaknesses of these policies, in relation to the global policy and to the DM's behavior.

Other approaches in the post-optimality analysis process of the UTA methods may be also found in the literature. These approaches propose the use of alternative optimality criteria during the exploration of the polyhedron (6.10), like the minimization of the errors' dispersion, i.e. Tchebycheff criterion [5], or the optimal assessment of the δ and s parameters in the context of the UTAMP models [2, 3].

6.3.2 Robustness in UTA-type Models

Roy [27] recently considers the robustness as a tool of resistance of decision analysts against the phenomena of approximations and ignorance zones. In fact, robustness appears as a tool to analyze the gap between the "true" DM's model and the one resulting from a computational mechanism. It is important to note that the robustness analysis should be distinguished from the sensitivity analysis, which is marginal and depends each time on the changes of one or more parameters. Moreover, it should be emphasized that robustness refers mainly to the decision model, in the light of the assertion "robust models produce a fortiori robust results". However, robustness should also refer to the results and the decision support activities (e.g. conclusions, argumentation).

From the previous comments, it is clear that robustness should be measured and controlled in any decision-aid activity. However, this need poses a number of new problems referring to the measurement of the robustness of a decision model, the development of appropriate robustness indicators, and the potential improvement of robustness. Moreover, this measurement process should always take into account the different perspectives of robustness:

1. Analyst's point of view (is a decision model reliable?)
2. DM's point of view (is a decision model acceptable?)

As already mentioned, in the UTA family of models, robustness deals with LP, which is the main mechanism to infer decision models. In particular robustness refers to the post/near-optimality analysis, as presented in the previous section.

The general methodological framework for applying robustness analysis in the context of preference disaggregation approaches is presented in Figure 6.2 and consists of the following main steps:

1. The applied preference disaggregation method is used to infer a representative additive value model based on A_R. This step is discussed in the previous sections of this chapter.
2. The inconsistencies between the DM's preferences and the results of the disaggregation method are identified and removed using interactive techniques with the DM. An example methodological approach for this is given in Figure 6.3, while further details may be found in [36, 41].
3. A robustness measure is established.
4. If the robustness measure, established in step 3, is judged satisfactory by the analyst, the model is proposed to the DM for application on the set A and the process is terminated. Otherwise, the process goes to step 5.
5. Alternative rules of robustness analysis are examined and the process goes back to step 3.

Particularly for the assessment of the robustness measures (step 3), it should be noted that the robustness of the decision model depends on the post-optimality analysis results, and especially on the form and the extent of the polyhedron of multiple/near optimal value functions. In order to handle this polyhedron, the following heuristic is applied: during post-optimality analysis $2n$ LPs are formulated and solved, which maximize and minimize repeatedly $u_i(g_i^*)$. The observed variance in the post-optimality matrix indicates the degree of instability of the results. Thus, following the approach of Grigoroudis and Siskos [11] an Average Stability Index (*ASI*) may be assessed as the mean value of the normalized standard deviation of the estimated values $u_i(g_i^*)$. Alternatively, instead of exploring only the extreme values of $u_i(g_i^*)$, the post-optimality analysis may investigate every value of each criterion $u_i(g_i^j)$. In this case, during the post-optimality stage, $T = 2\sum_i(\alpha_i - 1)$ LPs are formulated and solved, which maximize and minimize repeatedly $u_i(g_i^j)$ and the *ASI* for the i-th criterion is assessed as follows:

$$ASI(i) = 1 - \frac{1}{\alpha_i - 1}\sum_{j=1}^{\alpha_i - 1}\frac{\sqrt{T\sum_{k=1}^{T}\left(u_i^{jk}\right)^2 - \left(\sum_{k=1}^{T}u_i^{jk}\right)^2}}{\frac{T}{\alpha_i - 1}\sqrt{\alpha_i - 2}} \tag{6.12}$$

where α_i is the number of points that are estimated in the interval $[g_{i*}, g_i^*]$ and u_i^{jk} is the estimated value of $u_i(g_i^j)$ in the k-th post-optimality analysis LP ($j = 1, 2, \ldots, \alpha_i$).

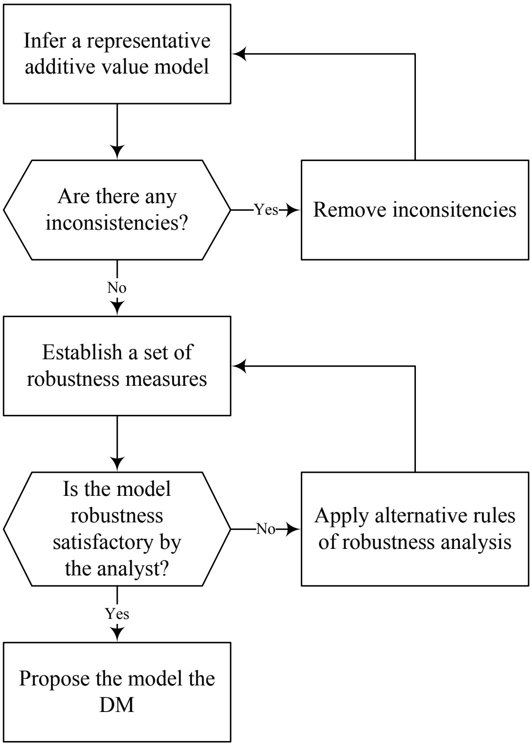

Fig. 6.2 Robustness analysis in preference disaggregation approaches

Using formula (6.12), the global robustness measure may be assessed as the average of the individual $ASI(i)$ values:

$$ASI = \frac{1}{n}\sum_{i=1}^{n} ASI(i) \tag{6.13}$$

It should be noted that all the previous ASI measures are normalized in the interval $[0,1]$, and thus high levels of robustness are achieved when ASI is close to 1.

On the other hand, the alternative rules that should be applied if the analyst is not satisfied with the value of the ASI measures (step 4), may include the following:

- Addition of new global preference judgments (e.g. pairwise comparisons, preference intensities as mentioned in section 6.2.2, or even new reference actions).
- Visualization of the observed value variations to support the DM in choosing his/her own model (see the example below).
- Enumeration and management of the hyperpolyhedron vertices (Manas-Nedoma algorithm, Tarry's method, etc.) in post-optimality analysis.

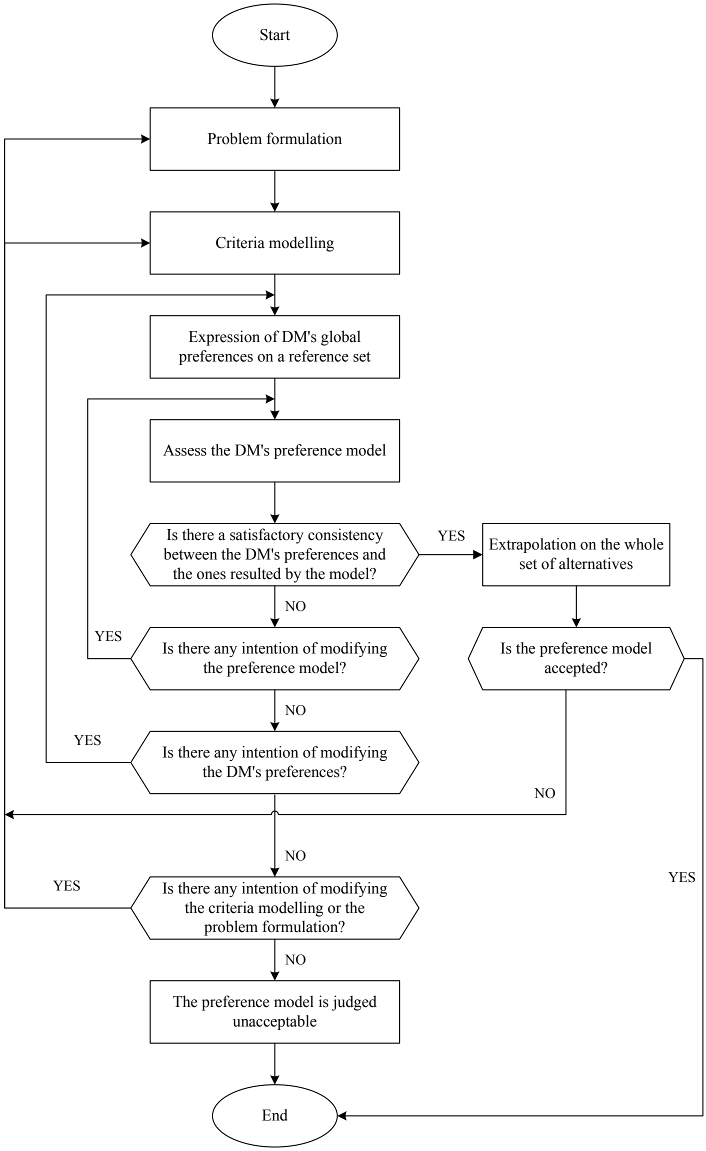

Fig. 6.3 Removal of inconsistencies in UTA methods [15]

- Building new preference relations on the set A during the extrapolation phase (see the next section).
- Computation of the barycentre as a representative model.

In order to illustrate the aforementioned methodological approach, let us consider a simple example where an ordinal evaluation scale is used in a single criterion case. Suppose that the evaluation scale has the following form: b=Bad, m=Medium, g=Good, and e=Excellent. The problem is to estimate the DM's value function u on the scale $\{$b, m, g, e$\}$. Of course, this naïve example is not realistic, but it is presented here in order to illustrate the proposed robustness approach.

Taking into account the normalization constraints of u, we have $u(\mathrm{b}) = 0$ and $u(\mathrm{e}) = 1$, while $u(\mathrm{m})$ and $u(\mathrm{g})$ are the model parameters that should be estimated. Suppose also, that the DM's preferences may be modeled according to the following relations:

1. The value of $u(\mathrm{m})$ should not be more than 20% and less than 10%:

$$0.1 \leq u(\mathrm{m}) \leq 0.2$$

2. The difference between "good" to "excellent" is at least 2 times more important than the difference between "medium" and "good":

$$\frac{u(\mathrm{e}) - u(\mathrm{g})}{u(\mathrm{g}) - u(\mathrm{m})} \geq 2 \Leftrightarrow 1 - u(\mathrm{g}) \geq 2u(\mathrm{g}) - 2u(\mathrm{m}) \Leftrightarrow 3u(\mathrm{g}) - 2u(\mathrm{m}) \leq 1$$

3. The indifference threshold on g is selected so as $s = 0.01$, thus:

$$u(g^{j+1}) - u(g^j) \geq s \Rightarrow \begin{cases} u(\mathrm{m}) - u(\mathrm{b}) \geq 0.01 \\ u(\mathrm{g}) - u(\mathrm{m}) \geq 0.01 \\ u(\mathrm{e}) - u(\mathrm{g}) \geq 0.01 \end{cases} \Rightarrow \begin{cases} u(\mathrm{m}) \geq 0.01 \text{ (redundant)} \\ u(\mathrm{g}) - u(\mathrm{m}) \geq 0.01 \\ 1 - u(\mathrm{g}) \geq 0.01 \text{ (redundant)} \end{cases}$$

$$\Rightarrow u(\mathrm{g}) - u(\mathrm{m}) \geq 0.01$$

Using the previous constraints and introducing the error variables, the preference disaggregation LP has the following form:

$$\begin{array}{llllll}
\min z = z_1 + z_2 + z_3 & & & & & \\
\text{s.t.} \quad -u(\mathrm{m}) + & u(\mathrm{g}) & & & & \geq 0.01 \\
\quad u(\mathrm{m}) & & + z_1 & & & \geq 0.1 \\
\quad u(\mathrm{m}) & & & - z_2 & & \leq 0.2 \\
\quad -2u(\mathrm{m}) + & 3u(\mathrm{g}) & & & - z_3 & \leq 1 \\
\quad u(\mathrm{m}) \ , & u(\mathrm{g}) \ , & z_1 \ , & z_2 \ , & z_3 & \geq 0
\end{array}$$

The previous LP gives $z^* = 0$, while it should be emphasized the existence of optimal solutions, as indicated by the polygon ABCD in Figure 6.4. The solutions obtained during the post-optimality analysis, where 4 LPs are formulated and solved, are presented in Table 6.1. Also, the barycentral solution, along with the variation of the value function in post-optimality analysis is given in Figure 6.5. Finally, using formula (6.12), the proposed robustness measure for the examined criterion is calculated as $ASI = 0.8059$.

Although this value of ASI may be considered as acceptable by the analyst, suppose that the DM is able to give new global preference judgments. For example, if the value of "good" is assumed to be no less than 40%, the new constraint

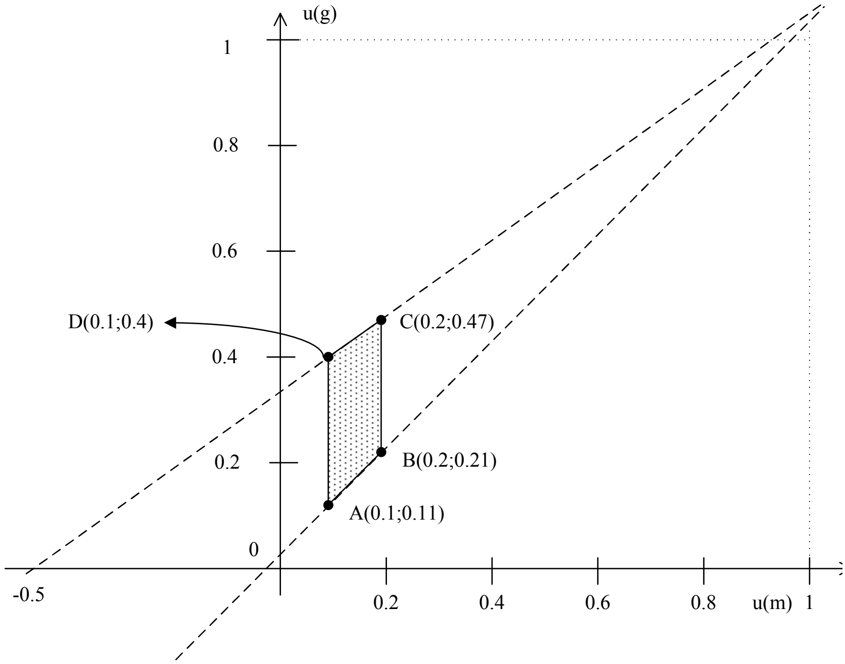

Fig. 6.4 Multiple optimal solutions for the numerical example

Table 6.1 Post-optimality analysis results

	u(m)	u(g)
[min] u(m)	0.10	0.40
[max] u(m)	0.20	0.21
[min] u(g)	0.10	0.11
[max] u(g)	0.20	0.47

$u(\mathrm{m}) \geq 0.4$ should be added in the previous LP. The new optimal solution is also $z^* = 0$, but the optimal solution space, as shown in Figure 6.6 is now defined by the triangle CDE. In this case, the *ASI* increases from 80.59% to 91.97%. Figure 6.7 presents the revised barycentral solution for the simple numerical example, which is more robust than the initial one. In any case, this Figure is able to visualize the variability of the value function in order to support the DM in choosing his/her own model.

Consequently, robustness may be considered as a gap between the "true" DM's model and the one resulting from a computational mechanism. Robustness is also a decision support tool to decide about:

- the decision model and
- the answers to the decision problematic.

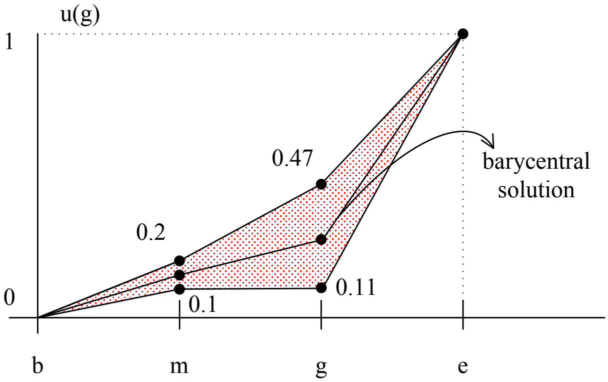

Fig. 6.5 Visualization of multiple decision models for the numerical example

Although research about robustness could continue taking into account the methodological issues highlighted in this chapter, the proposed stability index (*ASI*) may provide a helpful robustness measure from post-optimality analysis stage in any preference disaggregation method.

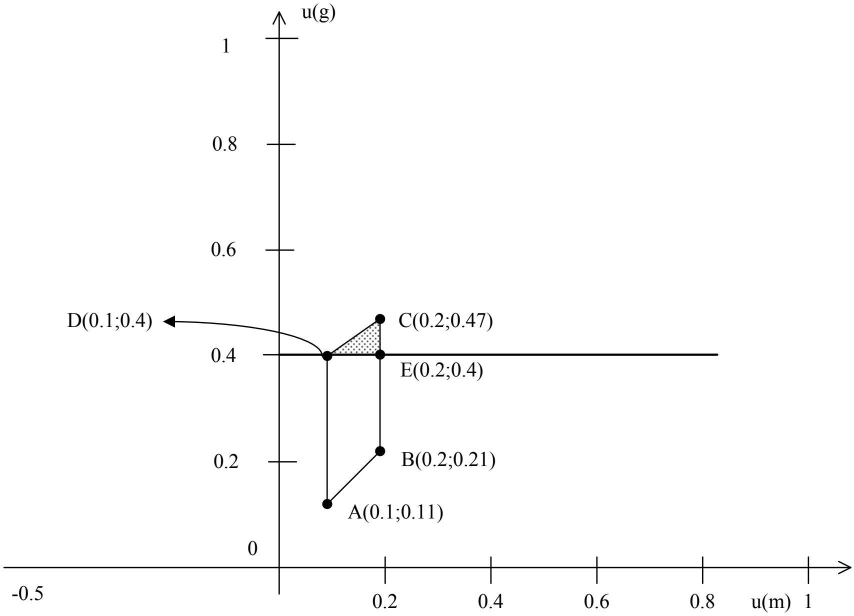

Fig. 6.6 Revised optimal solution space for the numerical example

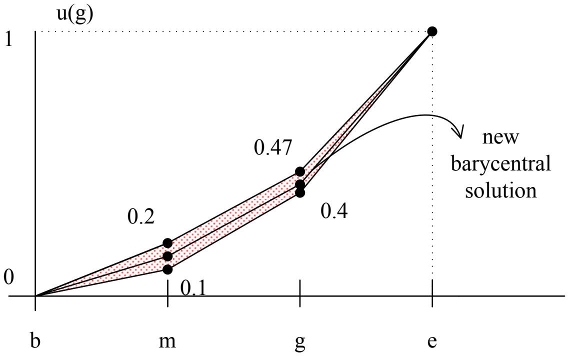

Fig. 6.7 Revised decision models' space for the numerical example

6.3.3 Building Preference Relations

An important alternative rule applicable in the proposed robustness analysis of aggregation-disaggregation approaches refers to the assessment of preference relations by exploiting the results of post-optimality analysis. Thus, such a rule is able to deal with the problem of multiple optimal solutions.

In this context, Siskos [32] proposes the assessment of fuzzy outranking relations based on the results of the UTA models. In this approach, the term "fuzzy" refers to the fuzzy aspect of the DM's preferences. In particular, these fuzzy relations analyze the preference relation R and they are based on a system of additive value functions estimated by a UTA model. It should be mentioned that a fuzzy outranking relation is a fuzzy subset of the set of all pairs of actions, i.e. a fuzzy subset of the set $A \times A$, characterized by a membership function $d(A \times A)$ called the degree of credibility of the outranking [25]. The main objective of the approach is to evaluate this degree of superiority of one action to another, according to the information given by the criteria and the a priori preferences of the DM.

As already noted, post-optimality analysis offers different possibilities to restitute the weak order R by additive value functions. Thereby, Siskos [32] introduces the notion of an additive value system, which is represented by a set of indexed value functions $\mathscr{U} = \{u^1, u^2, \ldots, u^i, \ldots\}$ and must satisfy the following rules:

- u_i must give a satisfactory degree of consistency between the weak order that it defines over A_R and the initial weak order R.
- The value functions of the system must be as characteristic as possible of the polyhedron (6.10).

It should be emphasized that using the heuristic approach of exploring the multiple/near optimal solutions space presented in the previous section (see LPs (6.11)), the convex set of value functions defined by the polyhedron (6.10) becomes a discrete and finite set.

So, the problem is to synthesize the results on the set A, taking into account this system of additive value functions $\mathscr{U}$ in order to obtain a general rule of decision. According to Siskos [32], this synthesis may be done in a simple way, considering the majority or concordance rule on the value functions of the system. In particular, he suggests calculating the percentage of value functions for which an action is better or equivalent than another.

The degree of credibility, which is actually the membership function of the fuzzy relation in $A \times A$, is defined by the following formula [32]:

$$d(a,b) = \frac{\left|u/u^i[\mathbf{g}(a)] - u^i[\mathbf{g}(b)] \geq 0\right|}{|\mathscr{U}|} \tag{6.14}$$

where $\left|u/u^i[\mathbf{g}(a)] - u^i[\mathbf{g}(b)] \geq 0\right|$ is the number of value functions for which $a \succ b$ or $a \sim b$ and $|\mathscr{U}|$ is the number of value functions of the system $\mathscr{U}$.

It is easy to see that $0 \leq d(a,b) \leq 1$, while it should be mentioned that the previous fuzzy relation enables to measure the outranking degree of one action by another, using only the ordinal structure determined over A by the additive value functions of the system $\mathscr{U}$.

Fuzzy preference relations have been extensively studied in the literature in order to analyze their properties and examine the conditions under which they can be applied in decision-making process. So, taking into account the aforementioned definitions and assumptions, the following fuzzy preference relations may be introduced [24]:

1. Fuzzy indifference relation

$$\mu^e(a,b) = \min\{d(a,b), d(b,a)\} \tag{6.15}$$

2. Fuzzy strict preference relation

$$\mu^s(a,b) = \begin{cases} d(a,b) - d(b,a) & \text{if } d(a,b) \geq d(b,a) \\ 0 & \text{otherwise} \end{cases} \tag{6.16}$$

Using the definition of μ^s, the fuzzy nondomination degree of an action is given by:

$$\begin{aligned} \mu^{ND}(a) &= \min_{b \in A}\{1 - \mu^s(b,a)\} = 1 - \max_{b \in A}\{\mu^s(b,a)\} \\ &= 1 - \max_{b \in A}\{d(b,a) - d(a,b)\} \end{aligned} \tag{6.17}$$

The value $\mu^{ND}(a)$ represents the degree to which the action a is dominated by no one of the other actions in A [24].

So, under problematic α, the best action a^* may be selected by maximizing $\mu^{ND}(a)$ on A. If $\mu^{ND}(a)^* = 1$, then a full robustness is achieved, otherwise the robustness of a^* is characterized by a value between 0 and 1. Similarly, under problematic γ, the ranking of actions from A can be made according to the values of the $\mu^{ND}(a)$ indicator.

The previous methodological approach is also adopted by the UTA $^{\rm GMS}$ and GRIP models, which are also based on the whole set of additive value functions "compatible" with the given preference information.

In particular, the UTA$^{\rm GMS}$ method requires a set of pairwise comparisons on A_R as the main input preference information. Using LP techniques, the method extrapolates the results on A by assessing two relations:

- The necessary weak preference relation which holds for any two alternatives $a,b \in A$, if and only if all compatible value functions give to a a value greater than the value given to b.
- The possible weak preference relation which holds for this pair $a,b \in A$, if and only if at least one compatible value function give to a a value greater than the value given to b.

The previous preference relations follow the main principles of the aforementioned fuzzy outranking relations proposed by Siskos [32]. In fact they may be considered as two special cases of the this fuzzy preference relation, with $d(a,b) = 1$ for the necessary relation and $0 < d(a,b) \leq 1$ for the possible relation.

These preference relations are used the UTA$^{\rm GMS}$ method in order to produce two rankings on the set A, such that for any pair of solutions $a,b \in A$: (1) in the necessary ranking, a is ranked at least as good as b, if and only if, $u(a) \geq u(b)$ for all value functions compatible with the preference information and (2) in the possible ranking, a is ranked at least as good as b, if and only if, $u(a) \geq u(b)$ for at least one value function compatible with the preference information.

As noted by Greco et al. [9] the necessary ranking can be considered as robust with respect to the preference information: any pair of solutions is compared in the same way whatever the additive value function compatible with the preference information is. So, when no preference information is given, the necessary ranking boils down to the dominance relation, and the possible ranking is a complete relation. The addition of new pairwise comparisons on A_R is able to enrich the necessary ranking and impoverish the possible ranking, so that they converge with the incorporation of this preference information. Thus, the method is intended to be used interactively, with an increasing reference set A_R and a progressive statement of pairwise comparisons.

In the same context, Figueira et al. [8] propose the GRIP (Generalized Regression with Intensities of Preference) method as an extension of the UTA$^{\rm GMS}$ approach, which infers this set of compatible additive value functions, taking into account not only a preorder on a set of alternatives, but also the intensities of preference among alternatives. These comparisons may be expressed comprehensively (on all criteria) and/or partially (on each criterion).

The previous methods, although able to deal with the robustness problem, cannot always provide a "final solution" to the DM. For this reason, Greco et al. [10] propose a procedure to explore the set of compatible value functions and identify the "most representative" one. Their idea is to select among compatible value functions the one that better highlights the necessary ranking (maximize the difference of evaluations between actions for which there is a preference in the necessary ranking). Alternatively, they propose to minimize the difference of evaluations between actions for which there is not a preference in the necessary ranking.

6.4 Group and Collective Decision Approaches

Most of the real-world decision-making problems involve multiple actors having different viewpoints on the way the problem should be handled and the decision to be made [17]. In these situations it is common to encounter conflict between the opinions and desires of the group members. This conflict may arise because multiple DMs have different value and informational systems (objectives, criteria, preference relations, communication support, etc). This is also noted by Roy [26] as “distinct value systems”, e.g. different ethical and or ideological beliefs, different specific objectives, or different roles within an organization. In this context, MCDA methods have been used in numerous previous studies in order to represent the multiple viewpoints of the problem, to aggregate the preferences of the multiple DMs, or to organize the decision process (see [22] for a detailed review of MCDA methods in group decision support systems).

The family of the UTA methods has been also used in several studies of conflict resolution in multi-actor decision situations [15]. These studies refer to the development and application of group decision or negotiation support systems [16, 28, 29, 30], or conflict resolution approaches for single actors [13]. Beside UTA methods, Matsatsinis and Samaras [22] review several other aggregation-disaggregation approaches incorporated in group decision support systems.

While group decision approaches aim to achieve consensus among the group of DMs or at least attempt to reduce the amount of conflict by compensation, collective decision methods focus on the aggregation of the DMs’ preferences. Therefore, in the latter case, the collective results are able to determine preferential inconsistencies among the DMs, and to define potential interactions (trade-off process) that may achieve a higher group and/or individual consistency level.

The UTA method may be extended in the case of multiple DMs, taking into account different input information (criteria values) and preferences for a group of DMs. Two alternative approaches may be found in the literature:

1. Application of the UTA/UTASTAR methods in order to optimally infer marginal value functions of individual DMs; the approach enables each DM to analyze his/her behavior according to the general framework of preference disaggregation.
2. Application of the UTA/UTASTAR methods in order to assess a set of collective additive value functions; these value functions are as consistent as possible with the preferences of the whole set of DMs, and thus they are able to aggregate individual value systems.

In the context of the first approach, Matsatsinis et al. [21] propose a general methodology for collective decision-making combining different MCDA approaches. As shown in Figure 6.8, in the first step of the methodology, the UTASTAR algorithm is implemented in order to assess individual's preference systems. Then, the values of the alternatives are aggregated with some averaging operator (normalized relative utility values). However, such representations of group preferences were found no to guarantee neither a consensus nor a good compromise, since individual assessments may be considerably different. Therefore, Matsatsinis et al. [21] incorporate in their proposed methodology several criteria in order to measure the DMs' satisfaction over the aggregated rank-order of alternatives.

The UTA/UTASTAR method may also be applied in the problem of inferring collective preference systems. Consider for example the case of q DMs evaluating m alternatives (a_k with $k = 1,2,...,m$) according to a set of n criteria (g_i with $i = 1,2,...,n$) which are assessed in the interval $[g_i* = g_i^1, g_i^2, ..., g_i^* = g_i^{\alpha_i}]$. Furthermore, suppose that $g_i^r(a_k)$ is the evaluation of the r-th DM for the k-th alternative on the i-th criterion and $R^r(a_k)$ is the ranking of the the k-th alternative given by the r-th DM.

Using the previous notations, formula (6.5), which represents the global value of actions in terms of marginal values, may be rewritten as follows:

$$u'[\mathbf{g}^r(a)] = \sum_{i=1}^{n} u_i[g_i^r(a)] - \sigma_r^+(a) + \sigma_r^-(a) \forall a \in A_R \tag{6.18}$$

where a double error function is included, similar to the UTASTAR method.

Respectively, taking into account the multiple DMs, formula (6.6) becomes:

$$\Delta^r(a_k, a_{k+1}) = u'[(\mathbf{g}^r(a_k)] - u'[(\mathbf{g}^r(a_{k+1})] \tag{6.19}$$

and thus, the LP (6.8) may be written as follows:

$$\begin{cases} [\min]\ F_1 = \sum_{r=1}^{q} \sum_{k=1}^{m} \left[\sigma_r^+(a) + \sigma_r^-(a)\right] & \\ \text{s.t.} \quad \left.\begin{array}{l} \Delta^r(a_k, a_{k+1}) \geq \delta \text{ if } a_k \succ a_{k+1} \\ \Delta^r(a_k, a_{k+1}) = 0 \text{ if } a_k \sim a_{k+1} \end{array}\right\} & \forall k, r \\ u_i(g_i^{j+1}) - u_i(g_i^j) \geq s_i & \forall i \text{ and } j \\ \sum_{i=1}^{n} u_i(g_i^*) = 1 & \\ u_i(g_{i*}) = 0,\ u_i(g_i^j) \geq 0,\ \sigma_r^+(a),\ \sigma_r^-(a) \geq 0\ \forall i, j, k, r & \end{cases} \tag{6.20}$$

This LP minimizes the sum of (absolute) errors for all DMs, which in several cases may not provide a "compromise" solution. Therefore, different optimality criteria in the previous LP formulation may be considered, given that the main objective of such methodology is to minimize potential individual deviation from the inferred group preference system.

For example, the following LP minimizes the maximum sum of errors for every DM (i.e. variance of errors to DMs):

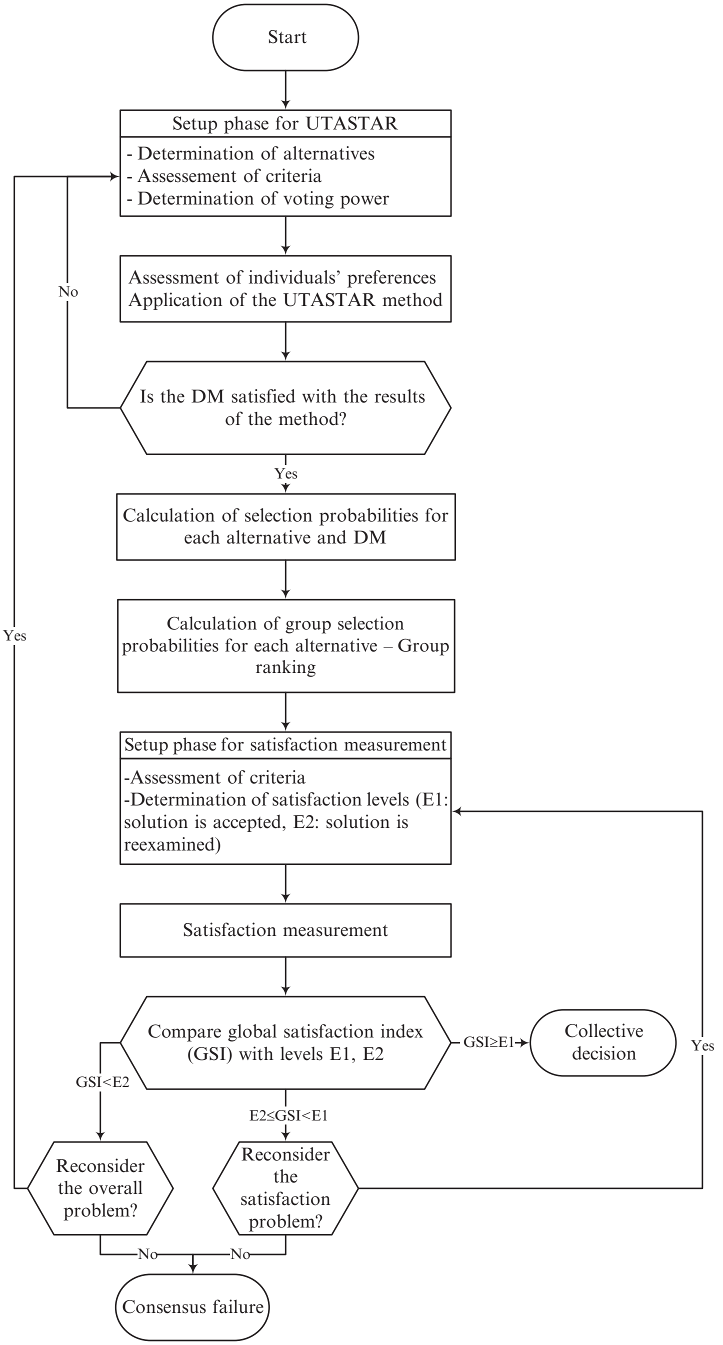

Fig. 6.8 Preference disaggregation approach for collective decision-making [21]

$$\begin{cases} [\min]\ F_2 = \sigma_{\max} \\ \text{s.t.} \quad \text{all the constraints of LP (6.20)} \\ \qquad \sum_{k=1}^{m} [\sigma_r^+(a_k) + \sigma_r^-(a_k)] \leq \sigma_{\max} \quad \forall r \end{cases} \tag{6.21}$$

Another approach is to consider the number of violated pairs of R^r (equivalent to Kendall's τ). In this case, similarly to Jacquet-Lagrèze and Siskos [14], a set of new binary variables should be introduced:

$$\gamma_{ab}^r = \begin{cases} 0 \text{ if } u[\mathbf{g}^r(a)] - u[\mathbf{g}^r(b)] \geq \delta & \text{(the judgment is respected)} \\ 1 \text{ otherwise} & \text{(the judgment is violated)} \end{cases} \tag{6.22}$$

Thus the following LP minimizes the sum of violated judgments for all DMs:

$$\begin{cases} [\min]\ F_3 = \sum_{r=1}^{q} \sum_{(a,b)\in R^r} \gamma_{ab}^r \\ \text{s.t.} \quad \left. \begin{array}{ll} \sum_{i=1}^{n} \{u_i[g_i^r(a_k)] - u_i[g_i^r(a_{k+1})]\} + M\gamma_{ab}^r \geq \delta & \text{if } a_k^r \succ a_{k+1}^r \\ \left. \begin{array}{l} \sum_{i=1}^{n} \{u_i[g_i^r(a_k)] - u_i[g_i^r(a_{k+1})]\} + M\gamma_{ab}^r \geq 0 \\ \sum_{i=1}^{n} \{u_i[g_i^r(a_k)] - u_i[g_i^r(a_{k+1})]\} + M\gamma_{ba}^r \leq 0 \end{array} \right\} & \text{if } a_k^r \sim a_{k+1}^r \end{array} \right\} \forall k, r \\ u_i(g_i^{j+1}) - u_i(g_i^j) \geq s_i \qquad \forall i \text{ and } j \\ \sum_{i=1}^{n} u_i(g_i^*) = 1 \\ u_i(g_{i*}) = 0,\ u_i(g_i^j) \geq 0,\ \gamma_{ab}^r \in \{0,1\} \qquad \forall i, j, r, (a,b) \in R^r \end{cases} \tag{6.23}$$

where M is a large number.

Finally, similarly to LP (6.21), the minimization of the maximum sum of violated judgments for every DM may considered by the following LP:

$$\begin{cases} [\min]\ F_4 = \gamma_{\max} \\ \text{s.t.} \quad \text{all the constraints of LP (6.23)} \\ \qquad \sum_{(a,b)\in R^r} \gamma_{ab}^r \leq \gamma_{\max} \quad \forall r \end{cases} \tag{6.24}$$

These alternative LP formulations may provide different collective results, and thus provide alternative bases for the compensation process among the DM's, which is usually applied in order to achieve consensus. In order to illustrate the previous modeling approaches, consider the simple example of Table 6.2, where 3 DMs evaluate a set of 7 alternatives (cars) using 6 criteria. As shown, the criteria evaluations are the same for all DMs, except for the "design" criterion, where the DMs express different preferences (this particular criterion is evaluated using a 5-point ordinal scale). Also, Table 6.3 shows the different rankings given by the set of DMs.

Table 6.2 DMs' evaluations

Alternatives	Horse power (CV)	Max Speed (km/h)	Acceleration (0-100km/h)	Consumption (lt/100km)	Design			Price (€)
					DM1	DM2	DM3	
Daewoo Matiz	75	152	16	7	****	*	**	9000
Opel Agila	80	155	13	7	****	****	****	10500
Hyundai Atos	55	142	15	6.5	**	*****	*****	8400
Daihatsu Cuore	60	140	13	5	*	**	*	7500
Ford CA	70	155	15	6	*****	***	****	8600
Suzuki Wagon	50	145	19	6	*	****	**	9000
Fiat Seicento	55	150	14	6.5	***	***	***	8300

Table 6.3 DMs' rankings

Alternatives	Ranking		
	DM1	DM2	DM3
Daewoo Matiz	6	4	6
Opel Agila	1	5	3
Hyundai Atos	3	2	1
Daihatsu Cuore	4	1	2
Ford CA	5	2	4
Suzuki Wagon	2	4	3
Fiat Seicento	6	3	5

The previous alternative LP models give different optimization results and rankings for the set of DMs as shown in Tables 6.4–6.5. These results correspond to different collective solutions, and thus they are able to determine preferential inconsistencies among the DMs, and define potential interactions (trade-off process) that may achieve a higher group and/or individual consistency level. For example, Figure 6.9 presents the alternative estimated values for the set of alternatives for every DM.

Trade-off analysis can be used in order to reduce preferential inconsistencies among the DMs. For example, the DMs' preferences that may be modified include the criteria evaluations $g_i^r(a_k)$ or the ranking of alternatives $R^r(a_k)$. The process can be easily implemented using the following approach:

1. Search error variables $(\sigma_r^+, \sigma_r^-, \gamma_{ab}^r)$ with non zero values.
2. Find $g_i^r(a_k)$ and/or $R^r(a_k)$ that should be modified.
3. Propose changes that reduce inconsistencies, while creating new ones.

In any case, it should be emphasized that the UTA method can be easily extended in the case of multiple actors, taking advantage of the flexibility of the LP modeling. Moreover, the different optimality criteria may lead to different results and thus they may offer alternative solutions to start a negotiation dialogue.

Table 6.4 Optimization results for the alternative LPs

Optimality criteria	SAE (F_1)	MAE (F_2)	SVJ (F_3)	MVJ (F_4)
Sum of errors	0.892	1.05	2.206	1.90
Maximum error per DM	0.392	0.35	1.277	1.15
Sum of violated judgments	14	12	6	6
Maximum violated judgments per DM	5	4	3	2

Table 6.5 Rankings from the alternative LPs

	Ranking (SAE)			Ranking (MAE)			Ranking (SVJ)			Ranking (MVJ)		
Alternatives	DM1	DM2	DM3	DM1	DM2	DM3	DM1	DM2	DM3	DM1	DM2	DM3
Daewoo Matiz	7	7	7	7	7	7	2	6	5	2	2	2
Opel Agila	2	3	3	4	4	4	6	7	7	7	7	7
Hyundai Atos	4	4	4	3	3	3	5	1	1	6	1	1
Daihatsu Cuore	1	1	1	2	2	2	4	2	4	4	5	5
Ford CA	5	5	5	5	5	5	1	3	2	1	4	3
Suzuki Wagon	3	2	2	1	1	1	7	5	6	3	3	4
Fiat Seicento	6	6	6	6	6	6	3	4	3	5	6	6

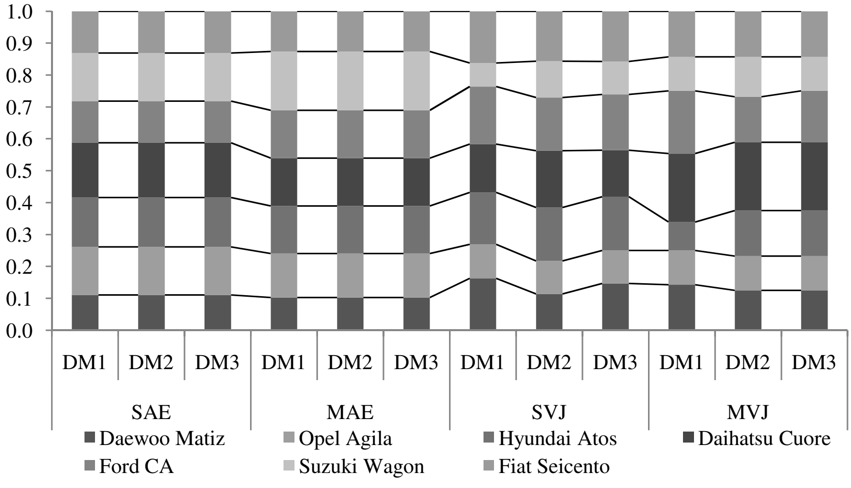

Fig. 6.9 Alternatives' values for every DM

6.5 Conclusions and Future Research

The aggregation-disaggregation philosophy is not only an important field of MCDA, but its principles can be found in other decision-making areas. The main aim of all these approaches is to infer global preference models from preference structures, as directly expressed by one or more DMs. In this context, the UTA methods not only adopt the preference disaggregation principles, but they may also be considered as the main initiative and the most representative example of preference disaggregation theory.

The new research developments of aggregation-disaggregation approaches presented in this chapter cover a variety of topics, like post-optimality analysis, robustness analysis, group and collective decision-making. They focus mainly on the UTA family of models and highlight their most important advantages (e.g. flexible modeling, analytical results, and alternative ways to reduce preferential inconsistencies).

Besides the numerous previous studies, additional research efforts are necessary in order to further exploit the potentials of the preference disaggregation philosophy within the context of MCDA. These efforts may include the development of more sophisticated aggregation models, the further exploitation of the provided results, or the adoption of aggregation-disaggregation philosophy in other decision-making approaches.

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