Chapter 11
Goal Programming: From Constrained Regression to Bounded Rationality Theories

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Abstract The purpose of the paper is to provide a critical overview of the decision-making approach known as Goal Programming (GP). The paper starts by tracing the origins of GP back to work by Charnes and Cooper at the end of the 1950s in fields like non-parametric regression, and the analysis of contradictions in non-solvable linear programming problems. After chronicling its evolution from its original form into a powerful decision-making method, the GP approach is linked with the Simonian bounded rationality theories based upon the "satisficing" concept. In this way, several GP models are presented as fruitful vehicles for implementing this kind of "satisficing" philosophy. The last part of the paper presents some critical issues and extensions of the GP approach. The chapter ends by discussing potential extensions, as well as GP's role for solving complex real-world problems in the near future.

11.1 A Historical Sketch

The original idea of Goal Programming (GP) appears in a paper by Charnes, Cooper and Ferguson published in *Management Science* in 1955 [19]. The paper was aimed at developing an executive compensation formula for a division of a major company (*General Electric*). The need to introduce *a priori* requirements and sign conditions for the coefficients of some variables into the model made it impossible to solve by using classic regression analysis techniques. Given the insufficiency of classic statistical techniques, they formulated a "constrained regression" model, minimizing the sum of the absolute deviations. Since absolute deviation is a non-linear form that cannot be straightforwardly optimised, they linearised the model by introducing

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negative and positive deviation variables for the first time in the literature. The seminal value of this paper is enormous for at least two reasons. First, it is the embryo of the future GP methodology, although the term GP is not explicitly used. Second, it is the outset of non-parametric regression methods.

Charnes and Cooper used the term GP explicitly for the first time six years later in Appendix B of their classic book *Management Models and Industrial Applications of Linear Programming* , under the heading of "Basic Existence Theorems and Goal Programming". Paradoxically, the two fathers of GP do not address a proper decision-making problem with multiple goals but analyse the contradictions in non-solvable linear programming problems. In other words, they use GP as an approach for determining a compromise solution to an infeasible linear programming problem.

In the early 1960s, Ignizio [32] had to solve a complex problem in the field of engineering design with the deployment of the antenna system for the Saturn/Apollo moon landing mission. This problem comprehended multiple goals, non-linear functions, as well as integer variables. With the help of a GP formulation, he was able to obtain implementable solutions.

Charnes et al. [20] demonstrated the potential of GP in financial and accounting problems. Ijiri [39] developed mathematical techniques, like the inverse generalized matrix to deal with preemptive priorities. Charnes et al. [18] formulated GP models to plan a company's advertising campaign. Finally, in 1969 Jaaskelainen [40] proposed a GP model for scheduling production, employment and inventories.

Two books published in the seventies and solely devoted to GP [33, 46] had a seminal influence on the development of this approach. In the 1970s other key papers appeared introducing refinements and extensions of the GP approach like: interactive GP, fuzzy GP, interval GP, multidimensional dual, algorithmic improvements, computer codes, etc. All these theoretical efforts fed a real explosion of applied papers, successfully addressing key decision-making problems through GP.

Some surveys and expository presentations of GP are, in chronological order, Ignizio [34], Zanakis and Gupta [67], Romero [53], Schniederjans [58], Tamiz et al. [63], Lee and Olson [47], Aouni and Kettani [3], Jones and Tamiz [43], Ignizio and Romero [38], and Caballero et al. [7].

11.2 Goal Programming and Bounded Rationality Theories

Modern GP is philosophically underpinned by the Simonian concept of satisficing that leads to a bounded rationality theory deeply rooted in psychology. This marks a clear departure from the classical theories based upon a perfect rationality paradigm. It should be noted that the term satisficing does not appear in English dictionaries. "Satisficing" is a Northumbrian term, chosen by Simon, to indicate the DM's desire to get "satisfying" and "sufficient" solutions to many real-world problems [60]. In short, "satisficing" is a merger of the words "satisfying" and "sufficing".

Let us develop the satisficing argument. Simon [59] conjectured that in today's complex organizations the environment is defined by incomplete information, limited resources, conflicts of interests, etc. In such an environment the decision-maker (DM) does not try to maximize anything, much less a well-defined objective function, as is assumed by classic theories of perfect rationality. Quite the contrary, within this kind of realistic environment, the DM tries to get as close as possible to a set of goals as determined by a set of satisficing targets, where satisficing means established figures that are perhaps not the "best" but are satisfactory and sufficient for the decision-making problem under consideration.

Although the Simonian satisficing solution seems to be the most fruitful philosophical groundwork for GP, it is not the only one. Section 11.4 of this chapter illustrates how GP can also be interpreted in terms of classic utility theory.

According to the satisficing philosophy, GP can be defined as an analytical approach devised to address decision-making problems where targets have been assigned to all the attributes and the DM is interested in minimizing, in one way or another, the non-achievement of the respective goals. As a consequence of the satisficing philosophy, the "goodness" of any solution to a decision-making problem is represented by an achievement function rather than a utility function or similar construct. This type of function measures the degree of non-achievement of the defined goals [38]. González-Pachón and Romero [27] give a formal derivation of the link between satisficing logic and GP, attacking the problem axiomatically.

11.3 Some Basic Goal Programming Models

Let us consider a decision-making problem involving goals. The structure of the generic ith goal reads as follows:

$$(g_i) \quad f_i(\mathbf{x}) + n_i - p_i = t_i \tag{11.1}$$

where:

$f_i(\mathbf{x})$ = mathematical expression for the ith attribute (i.e. a function of the vector $\mathbf{x}$ of decision variables).
t_i = target value for the ith attribute; i.e. the achievement level that the DM considers as satisficing for the ith attribute.
n_i = negative deviation variable; i.e. quantification of the under-achievement of the ith goal.
p_i = positive deviation variable; i.e. quantification of the over-achievement of the ith goal.

Once the goals have been formulated, the next step is to detect the unwanted deviation variables. These variables are unwanted in the sense that they are the ones a DM wants to minimize. To illustrate this idea, let us consider the following cases:

1. The goal derives from a "more is better" attribute (i.e., *satisfice* $f_i(\mathbf{x}) \geq t_i$). In this case, the DM does not want under-achievements with respect to target t_i. Consequently, the unwanted deviation variable would be the negative one (n_i) and would have to be minimized.
2. The goal derives from a "less is better" attribute (i.e., *satisfice* $f_i(\mathbf{x}) \leq t_i$). In this case, the DM does not want over-achievements with respect to target t_i. Consequently, the unwanted deviation variable would be the positive one (p_i) and would have to be minimized.
3. The goal derives from an attribute that needs to be achieved exactly (i.e., *satisfice* $f_i(\mathbf{x}) = t_i$). In this case, the DM wants neither over-achievements nor under-achievements with respect to target t_i. Hence, both the negative variable n_i and the positive one p_i are equally unwanted, making it necessary to minimize both deviation variables.

Let us assume that the unwanted deviation variables for a given problem are

$$p_1, n_2, \ldots, n_i, p_i, \ldots, p_q$$

The formulation of a GP model implies the minimization of a function of the former unwanted deviation variables:

$$\text{Min} \quad g(p_1, n_2, \ldots, n_i, p_i, \ldots, p_q) \tag{11.2}$$

The above function has a typical "less is better behaviour" and receives the name of achievement function. The arguments (i.e. the unwanted deviation variables) of (11.2) must be normalized. This type of normalization is required for two different types of reasons. First, the goals are generally measured in different units. Therefore, it makes no sense to apply a mathematical operator like the sum (e.g., to add together kilos of potatoes and pints of beer). Second, the value of the targets might be very different. Hence the minimization of (11.2) can lead to solutions biased towards goal with higher values being held for their targets. Finally, it is also necessary to introduce into (11.2) parameters reflecting the relative importance the DM attaches to the achievement of the different goals. Therefore, the achievement function (11.2) should read as follows:

$$\text{Min} \quad g\left(\frac{W_1 p_1}{K_1}, \frac{W_2 n_2}{K_2}, \ldots, \frac{(W_i n_i, W_i p_i)}{K_i}, \ldots, \frac{W_q p_q}{K_q}\right) \tag{11.3}$$

where W_i and K_i are the preferential weights and the normalizing factor attached to the generic ith goal, respectively. A suitable normalization factor is the target value of each goal; that is, $K_i = t_i$. Thus, all deviations are measured on a percentage scale. However, this normalization system is not applicable when any of the goals has a target value of zero. In this case, it is possible to resort to other normalization systems. See Tamiz et al. [64] (pages 572–573) and Kettani et al. [44] for a discussion of the different normalization techniques within a GP context.

Different methods can be used to minimize the achievement function, each one leading to a different GP variant. Let us introduce first the variant known as weighted

GP (WGP). The achievement function of a WGP model comprises the unwanted deviation variables, each weighted according to their importance [33]. Thus, we have:

$$\begin{aligned}&\textbf{Achievement function:}\\&\text{Min}\quad \sum_{i=1}^{q}(\alpha_i n_i+\beta_i p_i)\\&\textbf{Goals and constraints:}\\&f_i(\mathbf{x})+n_i=t_i,\quad i\in\{1,\ldots,q\}\\&\mathbf{x}\in\mathbf{F},\ \mathbf{n}\geq\mathbf{0},\ \mathbf{p}\geq\mathbf{0}\end{aligned}\tag{11.4}$$

where $\alpha_i = W_i/K_i$ if n_i is unwanted, otherwise $\alpha_i = 0$, $\beta_i = W_i/K_i$ if p_i is unwanted, otherwise $\beta_i = 0$.

Let us now introduce the variant known as lexicographic GP (LGP). The achievement function of a LGP model is made up of an ordered vector whose dimension is equal to the Q number of preemptive priority levels defined in the model. Each component of this vector comprises the unwanted deviation variables of the goals placed at the corresponding priority level [37, 46]. Thus, we have:

$$\begin{aligned}&\textbf{Achievement function:}\\&\text{Lex min } a=\left[\sum_{i\in h_1}(\alpha_1 n_i+\beta_i p_i),\ldots.\sum_{i\in h_r}(\alpha_1 n_i+\beta_i p_i),\ldots.,\sum_{i\in h_Q}(\alpha_1 n_i+\beta_i p_i)\right]\\&\textbf{Goals and constraints:}\\&f_i(\mathbf{x})+n_i-p_i=t_i,\quad i\in\{1,\ldots,q\},\ i\in h_r,\ r\in\{1,\ldots,Q\}\\&\mathbf{x}\in\mathbf{F},\ \mathbf{n}\geq\mathbf{0},\ \mathbf{p}\geq\mathbf{0}\end{aligned}\tag{11.5}$$

where h_r means the index set of goals placed at the rth priority level.

Finally, let us introduce the third classic variant called MINMAX (Chebyshev) GP (MGP). The achievement function of a MGP implies the minimization of the maximum deviation from any single goal [23]. Thus, we have:

$$\begin{aligned}&\textbf{Achievement function:}\\&\text{Min } D\\&\textbf{Goals and constraints:}\\&(\alpha_i n_i+\beta_i p_i)-D\leq 0\\&f_i(\mathbf{x})+n_i-p_i=t_i,\quad i\in\{1,\ldots,q\}\\&\mathbf{x}\in\mathbf{F},\ \mathbf{n}\geq\mathbf{0},\ \mathbf{p}\geq\mathbf{0}\end{aligned}\tag{11.6}$$

where the variable D represents the maximum weighted and normalized deviation.

11.4 A Utility Interpretation of a Goal Programming Model

GP, like any other decision-making approach, can be based on different philosophies or rationality theories. The primary philosophy underpinning GP is the Simonian concept of "satisficing" that leads to a bounded rationality theory, as the preceding sections show. However, this is not the only possible interpretation of GP. In this section GP will be analysed from the point of view of utility theory.

Let us start with LGP, where the non-compatibility between lexicographic orderings and utility functions is well known. In order to properly assess the effect of this property it is necessary to comprehend that the reason for this non-compatibility is exclusively due to the non-continuity of preferences underlying lexicographic orderings [54] (pp. 43–46).

Rather than disqualifying LGP because it implicitly assumes a non-continuous system of preferences, it would be worthwhile discussing whether or not the characteristics of the problem situation justify a system of continuous preferences. Hence, the possible problem associated with the use of the lexicographic variant lies not in its incompatibility with utility functions, but in the careless use of this approach. In contexts where the DM's preferences are clearly continuous, a compensatory GP model with utility support should be used.

It is relatively straightforward to demonstrate that a WGP model implies the maximization of a separable additive utility function in the attributes considered. This solution provides the maximum aggregated achievement between the different goals (e.g. [55]).

A MGP model implies the optimisation of a MINMAX utility function where the maximum deviation D is minimized. This type of function provides a solution that attaches the maximum importance to the most displaced goal with respect to its target. This achieves the most balanced solution for the achievement across the different goals (see [55, 64]).

11.5 Some Extensions of the Traditional Achievement Functions

Looking at GP models from the utility perspective discussed in the preceding section, we can say that from a preferential point of view, the WGP and the MGP solutions represent two opposite poles. Because the preferences underlying the weighted option are assumed to be separable, this variant can produce extremely biased results against one of the goals under consideration. On the other hand, because one of the goals is dominant, the MGP (Chebyshev) model sometimes provides results with poor aggregate performance across different goals. In short, the WGP solution implies the maximum aggregate achievement, while the MGP (Chebyshev) option provides the most balanced solution for achievement across the different goals. The extremity of both solutions can, in some cases, lead to solutions that are unacceptable for the DM. A possible solution for modelling this type of problem is a combination of the WGP and MGP models. This strikes a balance between the maximum aggregated achievement of the solution provided by the WGP model with the maximum balancedness of the solution provided by the MGP model. The result is the following Extended GP (EGP) model [11, 64]:

$$\begin{aligned} &\textbf{Achievement function:} \\ &\text{Min } (1-\lambda)D + \lambda \sum_{i=1}^{q} (\alpha_i n_i + \beta_i p_i) \\ &\textbf{Goals and constraints:} \\ &(\alpha_i n_i + \beta_i p_i) - D \leq 0 \\ &f_i(\mathbf{x}) + n_i - p_i = t_i, \quad i \in \{1,\ldots,q\} \\ &\mathbf{x} \in \mathbf{F},\ \mathbf{n} \geq \mathbf{0},\ \mathbf{p} \geq \mathbf{0},\ \lambda \in [0,1] \end{aligned} \tag{11.7}$$

where the different variables were previously defined and λ is a control parameter. For $\lambda = 0$, we have the MGP achievement function, and for $\lambda = 1$ the WGP achievement function. For other values of parameter λ belonging to the open interval $(0,1)$, the weighted combination of these two GP options can provide intermediate solutions, if they exist.

Two basic assumptions underlie all the achievement functions presented in the preceding sections:

1. The DM associates a precise target to each attribute.
2. Any unwanted deviation with respect to its target is penalized according to a constant marginal penalty; in other words, any marginal change is of equal importance no matter how distant it is from the target.

It is rather obvious that these assumptions are very strong and although they can suitably represent the preferences of certain DMs, they do not apply generally. Indeed, many DMs are not able to or are not interested in associating specific targets to certain attributes. Additionally, they may consider that the importance of marginal changes in the achievement of the goal depends upon its distance to the target.

Different achievement functions have been proposed in order to weaken the above assumptions. Chronologically, the first idea to address this problem was to conjecture that the DM feels satisfied when the achievement of a goal lies within the limits of a certain target interval $[a_i, b_i]$. This type of penalty function provides what is known as "Interval GP" (IGP) [16] or "goal range programming" [30]. The corresponding model implies a WGP formulation with a U-shaped penalty function with (1+1) sides. The analytical structure of the corresponding IGP is:

Achievement function:

$$\text{Min} \sum_{i=1}^{q} (\alpha_i \eta_i + \beta_i p_i)$$

Goals and constraints: (11.8)

$$f_i(\mathbf{x}) + n_i - p_i = a_i$$

$$f_i(\mathbf{x}) + \eta_i - \rho_i = b_i, \quad i \in \{1, \ldots, q\}$$

$$\mathbf{x} \in \mathbf{F},\ \mathbf{n} \geq \mathbf{0},\ \mathbf{p} \geq \mathbf{0},\ \boldsymbol{\eta} \geq \mathbf{0},\ \boldsymbol{\rho} \geq \mathbf{0}$$

Jones and Tamiz [42] suggested an efficient way of incorporating this type of penalty function into the achievement function of a GP model. For this type of modelling, however, all the achievement functions built with penalty systems underlie an assumption of separability of the DM's preferences It was argued above that this type of preference structure can produce biased results towards the achievement of some of the goals. For this reason, the DM might not interested in maximizing the aggregated achievement, but in getting the most balanced solution. To do this, some extensions and refinements of Jones and Tamiz's procedure have been proposed, like MINMAX (Chebyshev), Interval GP model [66] and Extended Interval GP [56].

Finally, note that refinements have recently been proposed in this direction. Thus, by using in part a computing procedure proposed by Li [48] for solving WGP problems, Chang [12, 13] introduced models that represent significant reductions in the number of auxiliary variables, as well as in the number of auxiliary constraints required to build the respective interval GP models.

11.6 Some Critical Issues and Extensions

This section addresses some key issues in GP for avoiding poor modelling practices, as well as for properly understanding the actual role of GP within a general MCDM context

11.6.1 Paretian Efficiency and GP

Within a GP context, efficiency is a condition that must hold any solution. In fact, if a GP solution is inefficient, then the achievement of at least one of the goals can be improved without impoverishing the achievement of the others. However, a standard GP formulation can produce inefficient solutions for all its variants. In the 1980s, this led to serious arguments against this approach. However, these criticisms were simply making a mountain out of a molehill. In fact, it has been demonstrated how GP models can, through minor refinements of the approach, assure the generation of efficient solutions.

Thus, Hannan [31] proposed a test to check whether or not a GP solution is efficient. The method can also establish the whole set of GP efficient solutions. Masud and Hwang [50] demonstrated that, in order to assure efficiency, it is enough to add an additional priority level to the GP model's achievement function, maximizing the sum of the wanted deviation variables. Tamiz and Jones [62] proposed a very general procedure for distinguishing the efficient from the non-efficient goals. This procedure can also restore the efficiency of the goals detected as non-efficient. Caballero et al. [8] developed procedures for generating efficient GP solutions for non-linear and convex models. Finally, Tamiz et al. [65] extended the issue of efficiency to integer and binary GP models.

In conclusion, the GP model's potential for generating an inefficient solution is not a real problem nowadays, since modern GP approaches can quite easily find a way around this prospective problem. In some areas, like engineering design, the efficient solutions can be very unstable. This high instability of the efficient solutions can make it sensible "to disregard" the issue of efficiency in some cases and to concentrate on the issue of GP solution stability (see [36]).

11.6.2 The Selection of Preferential Weights in GP

The preceding section introduced weights W_i reflecting the DM's preferences with respect to the generic ith goal. An important question is how to derive this type of weights in real applications. The following appear to be interesting procedures:

1. Establishing links between the Analytic Hierarchy Process (AHP) [57] and GP, as was suggested by Gass [24]. In this way, the weights derived from "pairwise" comparison matrices can be incorporated into a GP model.
2. Eliciting the preferential weights W_i through an interactive MCDM method. Lara and Romero [45] incorporate into a GP model preferential weights W_i, previously elicited using the Zionts-Wallenius interactive MCDM method [69].
3. Implementing a sensitivity analysis with the values of the preferential weights W_i in order to test the robustness of the GP solution to possible changes of value.

11.6.3 Redundancy in LGP

Let us now address another critical issue in GP: naive prioritization and redundancy in lexicographic models. It holds in all the algorithms solving LGP problems that if the mathematical programming problem corresponding to the ith component of the achievement function has no alternative optimal solutions, then the goals placed at priorities lower than the ith would be redundant. In other words, these goals do not play any real role in the optimisation process but become mere ornaments for the lexicographic model!

When the LGP model has a lot of priority levels, then lower priority goals are very likely to be redundant and will therefore be of no real use in the optimisation process. Such prioritisation is naive and should be avoided.

There being too many priority levels is not the only reason for the redundancy of goals in LGP. In fact, if the target values associated with the goals are very high (e.g. near their ideal values), then the likelihood of alternative optimal solutions is very small. Another possibility of redundancy is that there are many goals for which both deviational variables are unwanted. The exact achievement of a goal makes it much harder for there to be alternative optimal solutions and, consequently, the probability of redundant goals is high.

Goal redundancy is not just a theoretical possibility; it has important practical implications. Noteworthy in this sense is a research by Amador and Romero [1], testing more than twenty LGP applications reported in the literature for redundant goals. In all but one of the analysed cases at least one of the priority levels was redundant. In about 50% of the analysed cases, the number of redundant priorities was greater than or equal to two. Finally, in terms of aggregated results, more than a quarter of the goals considered were redundant.

11.6.4 Links Between GP and Other MCDM Approaches

It is common practice within the Multiple Criteria Decision Making (MCDM) field to present its different approaches separately, giving the impression that each approach is completely independent. However, this is not the case. In fact, there are significant connections between many MCDM methods. In this sense, the MULTIPLEX approach proposed by Ignizio [35] is a good example of a GP structure encompassing several single and multi-objective optimisation methods. Following on in this unifying direction, Romero [55] proposed a theoretical structure with the name of Extended Lexicographic Goal Programming (ELGP). If this structure is considered the *primary model*, then it is easy to demonstrate that a great many multi-criteria methods are just *secondary models* of ELGP. Thus, most of the multi-criteria methods can be straightforwardly deduced just by applying different parameter specifications to the above model.

The use of GP as a unifying framework looks interesting for at least the following reasons. The ELGP model stresses similarities between MCDM methods that can help reduce gaps between the advocates of different approaches. This unifying approach can become a useful teaching tool for introducing MCDM, thus avoiding the common presentation based upon a disconnected "pigeonhole" system of MCDM methods.

11.7 Other Topics

This section briefly presents some GP topics that are of clear theoretical and applied interest but have not been covered in the chapter for reasons of space and/or because some of these topics are still under development.

11.7.1 Interactive GP

The link between the interactive MCDM philosophy and GP can make GP more flexible, as well as increase the DM's level of involvement in the decision-making process. In this way, it could be easier to find a vector of target values leading to acceptable solutions for the DM. Some interesting GP proposals are: Spronk [61], Masud and Hwang [50] and Caballero et al. [9]. The introduction of the concept of meta-GP and its development and linkage within an interactive framework [52] is of interest in this respect. From a meta-GP perspective, then, the DM can establish targets on several achievement functions and use an interactive procedure to update these values. This alleviates the problem of selecting a suitable achievement function [10].

11.7.2 GP and Artificial Intelligence

The use of methods from the field of artificial intelligence (AI) for solving GP models with complex structures (non-linear goals, non-convexities, etc.) is an area of growing interest. Within the AI field, approaches like genetic algorithms, TABU search and neuronal networks look especially applicable. Jones et al. [41] is an extensive survey of applications of AI methods to the MCDM field and, particularly, to GP.

11.7.3 GP and the Aggregation of Individual Preferences

GP has proved to be a useful analytical tool for inducing models to aggregate individual preferences into a collective one. The basic idea underlying this type of approach is to define a consensus by minimizing a distance function. This function measures the distances between the information provided by the individual DMs and the unknown consensus. Different GP models have been formulated and resolved as a result of the distance function minimization process. Some results have been obtained for the complete ordinal case [25], the partial ordinal case [26], the cardinal case based upon utility functions [28] and the cardinal case based upon "pairwise" comparison matrices [29].

11.7.4 Stochastic GP

When any of the GP model parameters (target values, preferential weights, etc.) are not precisely known, the model moves from a certainty context to a stochastic/uncertainty context. Some attempts in the stochastic direction are Liu [49], Ballestero [5], and Aouni et al. [2]. Liu [49] proposes a procedure for solving stochastic GP problems with the help of genetic algorithms; Ballestero [5], presents a stochastic GP formulation within a mean-variance format; and Aouni et al. [2], model the DM's preferences within a stochastic GP. In the uncertainty direction, Rehman and Romero [51] proposed a procedure merging games against nature and GP, and Chang [14, 15] introduced the concept of Multi-Choice GP for working with target vectors instead of single figures.

11.7.5 Fuzzy GP

The basic idea underlying fuzzy GP is to represent some of the model parameters not as precise crisp numbers but as imprecise fuzzy numbers. The parameters that are usually fuzzyfied are the target values and the coefficients of the different goals. Several mathematical structures have been used to characterize the fuzzy parameters. The most widely used are triangular and trapezoidal fuzzy numbers. In this way, fuzzy GP aims to add the imprecision usually inherent in the information available into the models. Zimmerman [68] pioneered the work on fuzzy GP. Nowadays, all the different crisp GP variants have been successfully adapted to a fuzzy context (see [4, 6]).

11.7.6 GP and Data Envelopment Analysis

Data Envelopment Analysis (DEA) [21] is a linear programming-based, non-parametric approach widely used to analyse the efficiency of a set of organizational units, like branches of a bank or farms in an agricultural district. As Cooper [22] indicates, GP addresses management problems, while DEA targets problems related to the control and evaluation of activities. Even though GP and DEA have very different purposes, there are clear mathematical links between both approaches as Cooper [22] (pages 6–7) clearly demonstrate. These links are especially strong when the weighted GP variant is compared with the additive version of DEA.

11.8 Conclusions and Areas for Future Research

The enormous complexity of modern organizations make it very difficult to model, solve and analyse their real decision-making problems with methods of optimisation underpinned by classic theories of perfect rationality. In this context, the GP approach, underpinned by a bounded rationality theory, has represented an effective approach for solving decision-making problems in complex organizations. It is not bold to conjecture that the complexity of organizations will not drop in the near future and, consequently, GP will likely retain its prominent role for realistically addressing decision-making problems.

The following areas of research in GP look promising:

1. The combination of GP with AI approaches is an effective means to develop good solutions to very complex problems. In short, in many applied fields, the GP modelling effort leads to complex highly non-linear problems with high dimensions. This type of problems is not solvable by resorting to precise optimisation techniques. However, experience shows how metaheuristic procedures can output "good enough" solutions.
2. Connecting GP with other MCDM approaches. Good examples in this direction are the relationship between GP and AHP. Thus, preferential weights derived from the AHP approach can be fruitfully incorporated into a GP model. In the same way, GP can be a useful tool for deriving the weights from "pairwise" comparison matrices, as well as for dealing with inconsistencies within "pairwise" comparison scenarios.
3. Developing realistic and pragmatic interactive methods. This area can be a major aid in improving GP's inherent flexibility, allowing the DM to become involved in the problem-solving process.
4. Using GP to induce models in the field of social choice. Some initial results in this direction clearly show that this approach has enormous potential for determining social consensus.
5. The reliance on a single GP variant is not generally justified. It would be interesting, therefore, to research new achievement function forms by hybridising different variants. The recent idea of meta-goal programming offers an attractive prospective for addressing this type of problems.

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Appendix: The Algebra Underlying Charnes and Cooper's Ideas

The idea behind this appendix is to state the algebra underlying the two basic pillars upon which GP has been built over the last fifty years.

Let us start with the first pillar in the paper by Charnes et al. [19]. The authors deal with a side constraint regression problem, with the following mathematical structure:

$$\begin{aligned} &\text{Min } \sum_{i=1}^{n} d_i^2 \\ &\text{s.t. } f_i(\mathbf{x}) + d_i = t_i, \quad i = 1,2,\ldots,n \\ &\quad\;\; f(\mathbf{x}) \in \mathbf{F} \end{aligned} \tag{11.9}$$

where $f_i(\mathbf{x})$ is the function to be statistically fitted to the n observations $(t_1, t_2, \ldots, t_n)$ and $\mathbf{F}$ represents the set of side conditions defined over the parameters characterizing the function $f(\mathbf{x})$ to be fitted. Model (11.9) is a quadratic programming problem that was unsolvable in the mid-1950s. For that reason, instead of minimizing the sum of the square deviations, Charnes et al. [19] proposed minimizing the sum of the absolute deviations, as follows:

$$\begin{aligned} &\text{Min } \sum_{i=1}^{n} |t_i - f_i(\mathbf{x})| \\ &\text{s.t. } f(\mathbf{x}) \in \mathbf{F} \end{aligned} \tag{11.10}$$

However, as (11.10) implies the minimization of an absolute deviation, that is a non-linear form that it was impossible to compute at that time, Charnes et al. proposed linearizing the objective function of (11.10) by introducing the following change of variables:

$$n_i = \frac{1}{2}\left[|t_i - f_i(\mathbf{x})| + (t_i - f_i(\mathbf{x}))\right] \tag{11.11}$$

$$p_i = \frac{1}{2}\left[|t_i - f_i(\mathbf{x})| - (t_i - f_i(\mathbf{x}))\right] \tag{11.12}$$

By adding (11.11) and (11.12), and by subtracting (11.12) from (11.11), we have:

$$n_i + p_i = |t_i - f_i(\mathbf{x})| \tag{11.13}$$

$$n_i - p_i = t_i - f_i(\mathbf{x}) \tag{11.14}$$

Therefore, according to (11.13) and (11.14), the non-linear model (11.10) turns into the following LP model:

$$\begin{aligned} &\text{Min } \sum_{i=1}^{n} (n_i + p_i) \\ &\text{s.t. } f_i(\mathbf{x}) + n_i - p_i = t_i, \quad i = 1,2,\ldots,n \\ &\quad\;\; f(\mathbf{x}) \in \mathbf{F} \end{aligned} \tag{11.15}$$

Let us move now to Appendix B of the classic book by Charnes and Cooper [17]. In section 5 of Appendix B under the heading **Goal Programming** they address the analysis of contradictions in non-solvable problems within a linear programming context. They illustrate the basic idea with the help of the following illustrative machine-loading problem:

$$
\begin{aligned}
3x_1 + 2x_2 &\leq 12 \\
5x_1 \qquad &\leq 10 \\
x_1 + \ \ x_2 &\geq 8 \\
-x_1 + \ \ x_2 &\geq 4 \\
x_1, x_2 &\geq 0
\end{aligned} \tag{11.16}
$$

It is easy to check that there is no feasible solution to problem (11.16). Charnes and Cooper suggest considering the first two equations of (11.16) as proper constraints, and the last two equations as goals to be attained as closely as possible. Thus, model (11.16) turns into the following model:

$$
\begin{aligned}
\text{Min } & |8-(x_1+x_2)| + |4-(-x_1+x_2)| \\
\text{s.t. } & 3x_1 + 2x_2 \leq 12 \\
& 5x_1 \qquad \leq 10 \\
& x_1, x_2 \geq 0
\end{aligned} \tag{11.17}
$$

Again by introducing the deviation variables (11.11) and (11.12), and by implementing the above arithmetic operations, the non-linear model (11.17) turns into the following linear structure:

$$
\begin{aligned}
\text{Min } & n_1 + n_2 \\
\text{s.t. } & 3x_1 + 2x_2 \leq 12 \\
& 5x_1 \leq 10 \\
& x_1 + x_2 + n_1 - p_1 = 8 \\
& -x_1 + x_2 + n_2 - p_2 = 4 \\
& x_1, x_2 \geq 0
\end{aligned} \tag{11.18}
$$

Model (11.18) is a linear WGP model, for which the unwanted deviation variables appearing in the achievement function have not been normalized. This model was proposed by Charnes and Cooper almost fifty years ago!

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