Chapter 1
To Better Respond to the Robustness Concern in Decision Aiding: Four Proposals Based on a Twofold Observation

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Abstract After reviewing what the adjective "robust" means in decision aiding [1] (DA) and explaining why it is important to be concerned about robustness in DA, I present a twofold observation (section 1.2), which leads me to make four proposals in order to better respond to robustness concern in decision aiding. With the first two proposals (sections 1.3 and 1.4), I show that, in many cases, the vague approximations and the zones of ignorance against which robustness helps to prevent, must be considered in terms of substituting the concept of version for the usual concept of scenario and focusing on the diverse processing procedures that must be used to study the decision aiding problem as it was formulated. Next, I show (section 1.5) that the traditional responses formulated in terms of "robust solutions" limit the meaning of this concept. I briefly describe a certain number of avenues for research that could be explored further, not only in order to otherwise conceive the solutions that could be qualified as robust in another way, but also to better interact with decision-makers to make them aware that the adjective "robust" can be subjective. Finally, the fourth proposal is related to forms of responses that lead to stating "robust conclusions", which do not necessarily refer to solutions characterized as robust. After defining what I mean by robust conclusions and giving some examples, I mention the rare approaches that have been proposed for obtaining such conclusions.

1.1 The Robustness Concern in Decision Aiding

Robustness in decision aiding (DA) is a subject that has given rise to many publications over the last several years (see, [3, 4, 11, 12]). After reviewing what the adjective "robust" means in terms of decision aiding and explaining why it is im-

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[1] I use "decision aiding" rather than "decision support", "decision making" or "decision analysis" to avoid simplistic assimilations

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portant to be concerned about robustness in DA, I present a twofold observation, which leads me to make four proposals designed to facilitate a better response to the robustness concern in decision aiding.

When proposing solutions and making recommendations to a decision-maker, the analyst must base these solutions and recommendations on models. However, the inevitable presence of vague approximations and zones of ignorance (explained in more detail later on) affect the modeling. The models must try to take into account these vague approximations and zones of ignorance so that the solutions and the recommendations based on the modeling do not risk ending up in results that are far worse than was anticipated when they will be implemented. Paying attention to robustness means taking robustness concern into account throughout the decision aiding activity (see [15]).

I have proposed (see [38, 39]) that, by definition, paying attention to robustness in decision aiding means seeking to be able to **withstand "vague approximations" and "zones of ignorance", in order to prevent undesirable impacts**, such as goals attained that are much worse than those anticipated or degradation of some of the properties to be maintained.

With respect to the decisions in question, the presence of vague approximations and zones of ignorance intervene at two levels of the decision aiding process:

1. the model on which the decision aiding is based may not conform exactly to the context in which the decisions will be implemented; and
2. the value system used to conceive and process the model may not conform exactly with the value system that will be used to judge these decisions.

These are two clear-cut facts which are the source of the inevitable gaps that exist between what I called the formal representation and the real-life context (see [40]):

- *formal representation (RF)*: the model and the processing procedures that are applied to highlight the results on which decision aiding is based);
- *real-life context (RLC)*: the context in which decisions will be made, executed and judged. [2]

Assuming that FR conforms exactly to RLC may lead the analyst to make recommendations that are likely to have extremely undesirable impacts. I will refer here only to one example in the field of linear programming. Believing that this conformity, without being perfect, could be judged excellent would justify the analyst recommending that the decision-maker adopt the optimal solution. In this type of model, it is the extreme point of a polyhedron that characterizes the optimal solution. An extreme point corresponds to a point on the boundary that separates that which has been judged acceptable from that which has been judged unacceptable for each of the constraints that define the extreme point. If only one of these boundary constraints has been poorly apprehended, the recommended solution may be quite

[2] *State of nature* could be used instead of *real-life context*, but because the latter expression refers to real life, it is, in my opinion, more appropriate to decision aiding than the former, which refers to nature.

different from the solution that would really be optimal, and may even be unfeasible. The optimization criterion is often defined as a synthesis of the elementary partial criteria. If these criteria are taken into account in a way that differs from the way that they are taken into account in the RLC, the recommended solution could be much worse than was expected.

In a given decision-making context, the analyst must thus seek to identify the vague approximations and zones of ignorance because they can engender a lack of conformity between FR and RLC. If the analyst ignores them, it could lead to proposing solutions and/or to making recommendations that would not protect the decision-maker from the impacts judged undesirable.

These brief considerations[3] are sufficient to make understand that the analyst who is concerned about robustness must formulate and process the problem in a way that responds appropriately to the following two requirements:

- *Requirement 1*: Inventory carefully what I have proposed calling **frailty points** and appropriately take these points into account in FR (see [39]).
  By definition, these frailty points are the place in the FR where the vague approximations and zones of ignorance are situated. I will give further examples later in the text. For the moment, I just want to draw attention on the fact that these frailty points are not only located in the model; they can also affect the processing procedures.
- *Requirement 2*: Develop forms of responses that are capable of helping decision-makers protect him/herself from the undesirable impacts that can result from the presence of vague approximations and zones of ignorance.
  These forms of responses must take into account what the decision-maker expects from this protection: are there levels of impact that he/she is willing to tolerate in certain circumstances and others that he/she judges unacceptable with respect to what he/she wants to be protected from, regardless of what happens and regardless of the cost? Depending on the RLC, the solution adopted by the decision-maker could, due to the presence of vague approximations and zones of ignorance in FR, reconcile relatively well the desire for protection and the hope to optimize the performance criterion/criteria that serve to evaluate this solution. The forms of responses elaborated will be really useful to the decision-maker only if these forms give him/her enough information to be able, given his/her own subjectivity, to decide between two conflicting risks: being poorly protected with regard to very poor performances that could result from undesirable impacts and/or abandoning all hope of good, even very good, performances.

The two above requirements above are the starting point for the twofold observation presented below.

[3] For more details, see the references cited above, as well as Roy [35] and Roy and Bouyssou [41].

1.2 A Twofold Observation

In the literature (see the enclosed references, particularly the bibliographies in these references), I have observed that many researchers who study the robustness concern perceive the two requirements stated above in a way that leads them to concentrate their research in two "special channels". By referring to these channels, I do not mean to minimize the importance of this research. I simply want to emphasize the following point: if, at this point in time, most researchers conduct their research exclusively on these two favored channels, other paths risk being ignored, despite the fact that they are equally fruitful on both the theoretical and practical levels.

- *First observation (first channel)*: The frailty points generally taken into account concern solely the **data** whose purpose is to bring the RLC aspect qualified as **uncertainty** into play in the FR. This conception leads to defining a set of **scenarios** that are supposed to describe all the realities likely to occur, which the robustness concern requires be taken into account.
  Although appropriate for responding to the first of the two requirements presented above, this conception may, in some cases, prove insufficient for identifying all the frailty points. I will show in the following two sections that this conception may lead to ignoring some vague approximations and zones of ignorance that should be taken into consideration.
- *Second observation (second channel)* : Most researchers focus on only one particular form of response, which makes the concept of **robust solution** play a central role. To qualify a solution as robust, these researchers assign a privileged role to the solution's performance in the **worst scenario**(s).In addition, this performance is modeled in the FR with a single criterion.
  This form of response leads to proposing to the decision-maker one or more solutions, which, depending on the vague approximations and zones of ignorance taken into account, optimize the most undesirable impact that the decision-maker faces. This impact can be understood either in terms of absolute performance or in terms of regret with respect to the optimal solution in the scenario that will be implemented. Such a solution can very well not correspond to what the decision-maker means by robust solution. In any case, in order to adopt such a solution, the decision-maker may need more information, particularly about what could happen if the scenario that occurs is not the worst case. These brief considerations show that the second of the two requirements presented above may not be taken into account in a satisfactory manner by this form of response. In sections 1.5 and 1.6, I will show that the robustness of a solution can be defined otherwise, not only by referring to a single performance criterion, and that, more generally, other forms of responses deserve to be studied.

1.3 First Proposal: Move Beyond the Scenario Concept

In some cases, the concept of scenario can be too reductive to allow an appropriate response to the first requirement (see section 1.1). This can lead to the desire to move beyond this concept. This reductive nature is due to two reasons:

The first reason is semantic. For a decision-maker, “scenario” stands for a description of a potential future (sometimes present) situation. It is thus this meaning of the term that the decision-maker will understand when the analyst employs “scenario” when speaking of the robustness concern. Therefore, the analyst must be aware of the risk of misunderstanding.

The second reason is related to the fact that “scenario” is generally associated with the notion of **uncertainty**. This uncertainty can lead to a belief that, somewhere, certainty exists. This implicit belief can lead to thinking that there is an objective state of reality that needs to be reproduced as closely as possible, either in qualitative or quantitative terms, in order to make the FR conform as closely as possible to the RLC.

It is thus with the aspects of reality said to be “**first order**”[4],which by definition are linked to objectively significant physical properties of things. By nature, such aspects give rise to repeated experimental verifications. Thus, speaking of approximation, either in qualitative or quantitative terms, in relation to such aspects of reality makes sense. For example, this is the case with the length of a path in kilometers, the quantity of CO2 emitted by a motor, or the price of goods available today on the market.

However, in decision aiding, aspects of reality that are not of this nature are also present. These aspects of reality are said to be “**second order**”. By definition, these aspects of reality involve a connotation, the essentially subjective value that is assigned to the content of this reality. They can thus not give rise to repeated experimental verifications to reach an agreement on the way that they should be taken into account. It does not seem to me very appropriate to speak of uncertainty, and even less of approximation, when describing them. This is the case when speaking of, for example, objectives, standards, attitudes towards risk, preferences and understanding of a future situation.

It is thus not only uncertainty that leads to being concerned about robustness. More generally, it is the presence of vague approximations and zones of ignorance. Some of them cannot stem from the presence of uncertainties, as is shown in the following examples.

Vague approximations stemming from a part of arbitrariness in the modeling: It can come from the presence of a form of simplification that can be envisioned in two or three different ways, or from the introduction of two or three probability distributions representing a random phenomenon with the choice of these distributions being highly dependent on the ease of calculation.

Vague approximations stemming from the way that qualitative information is coded: : Consideration of such information (notably when modeling preferences)

[4] I borrow this terminology from Watzlawick [49].

can lead to several possible interpretations: several techniques, particularly normalization, can be envisioned to quantitatively process data that, in the form in which they are gathered, are not really quantitative.

Zones of ignorance stemming from imperfect knowledge about a complex phenomenon: This imperfect knowledge can lead to introducing different variants to describe the phenomenon.

Zones of ignorance stemming from the subjective nature of certain constraints: The boundary between possible and impossible or acceptable and unacceptable can remain very poorly defined and thus can lead to a set of options, which can be defined either as the set of all the real values that a parameter in a given interval can have or as a list of all the possible values in such an interval.

Zones of ignorance stemming from the need to differentiate the role that the criteria must play in a multicriteria aggregation model: This differentiation is most often made by means of parameters that are called, depending on the case, importance coefficients, scaling constants, substitution rates or weights, for example. Frequently, these parameters have no real existence in the mind of the decision-maker, who may even have a very superficial comprehension of the role that these parameters play in the model's functioning. Depending on the available information, the model's aggregation logic and the desired objectives, several not necessarily discrete set of values may be appropriate for assigning a value to these parameters.

Chosen among many others, these examples show that the notion of uncertainty covers rather imperfectly all the vague approximations and zones of ignorance that can cause frailty points. Consequently, using uncertainty to inventory these frailty points can lead to ignoring some that should be taken into account by the robustness concern.

It is important not to underestimate the impact of vocabulary. The words used often play a decisive role in the direction that research takes, as well as in the way that applications are conducted. In order to reduce the risk of decision-maker misunderstandings, and especially to avoid the often unconscious restrictive influence that vocabulary choices have on thought processes, at the 56th meeting of the European work group "Multiple Criteria Decision Aiding" in 2002 (see [36]), I proposed to replace the term "scenario", with the more general term **version**, The term "version" does not emphasize the notion of uncertainty. In addition, this term is more clearly associated than "scenario" with a decision aiding problem formulation (DAPF). A DAPF version is characterized by the selection of a specific option for each identified frailty point. This option is one of the items in the (finite or infinite) set of possible options associated with the frailty points under consideration. As the examples given above show, the objective of this option is different according to the nature of the vague approximations and zones of ignorance that have caused a given frailty point. This option can consist of, for example, setting the numerical value assigned to a parameter, determining the analytic form retained for a probability distribution or a utility function, choosing the coding technique, selecting the model variant adopted to describe a complex phenomenon or defining the way a trend is described.

In a given decisional context, the analyst must define a DAPF version set $\widehat{V}$ that is likely to be appropriate for responding to the robustness concern. To do so, this analyst must carefully inventory the frailty points connected to both the way the decision aiding problem was formulated and the way the problem was modeled. Questions about how this inventory can be completed and how $\widehat{V}$ can be defined so that it can be taken into account effectively without being so reduced that it no longer responds to robustness concern are not within the scope of this article. The interested reader can find some paths that can be considered to answer these questions in Roy [37, 38, 39]. Their level of difficulty is fairly similar regardless of whether version or scenario is used.

1.4 Second Proposal: Take the Way That the Different Versions are Processed Explicitly into Account

Whether the goal is finding robust solutions or developing other forms of responses related to the robustness concern, it is necessary to apply at least one processing procedure to each of the versions in $\widehat{V}$. It is also possible to apply several procedures that may, or may not, be part of the same method. Before explaining my second proposal, I need to explain briefly my meaning for the terms “procedures” and “methods”. (For more details, please consult Roy [37, 39].)

A **procedure** P designates a set of execution instructions for handling a problem that will produce a $R(P,V)$ result when applied to a version V of a DAPF. Depending on the case, this result takes diverse forms. It can be a solution (e.g., an optimal solution) or a bundle of solutions (e.g., all non-dominated solutions). It can also be a statement: “there is no feasible solution”, “according to the criterion g, the deviation from the optimal solution $\mathbf{x}$ is bounded by such-and-such value”, or “action a is outranked by all the other actions, with the exception of the following, ...”.

A **method** M designates a family $\widehat{P}$ of similar procedures that satisfy the following two requirements:

- the procedures in the family have enough similar features (e.g., structures, group of concepts, axiomatic corpus, types of approach) that considering them as belonging to the same class is justified; and
- the procedures in the family can only be differentiated by the options chosen with regard to certain frailty points in the method (e.g., concordance levels or cut thresholds in the ELECTRE methods, thresholds that make some inequalities strict in mathematical programming, the various parameters involved in a metaheuristic method, individual subjectivity in a process that calls for expert intervention at certain steps).

Wanting to apply one single processing procedure P to each of the versions in $\widehat{V}$ can, in many cases, prove to be insufficient for correctly understanding the links between FR and RLC. This is the case when distinct procedures warrant being taken into account. Even if a single procedure P may appear sufficiently appropriate, it is actually quite rare that this procedure will not have any frailty points at all. It can be the value assigned to a purely technical parameter or related to a second-order reality, the arbitrariness involved in the way certain arbitrages are conducted, especially between really similar solutions, the manner that some syntheses are conducted, particularly for taking into account the solutions that are close to the optimum in the case of a "flat" optimum.

Whatever the procedure P considered, it is necessary to inventory its frailty points and to identify the one that could significantly affect the result $R(P,V)$, For such frailty points, several options must be introduced. This leads to a multiplication of the procedures. Here again, it is important to choose wisely in order to not retain too many procedures, which would lead to processing times that are too long to adequately respond to the robustness concern.

In the following, $\widehat{P}$ designates the set of procedures retained. These procedures may, or may not, be part of a same method. Each one can be applied to each of the versions in $\widehat{V}$. It may also be that certain procedures will appear inappropriate for processing a specific version in $\widehat{V}$. In general, this leads to defining the set S of pairs (procedure P, version V) judged compatible: $S \subseteq \widehat{P} \times \widehat{V}$.

As the above observations show, basing the response on a single version set alone, or even more, on a scenario set, can in many cases not be enough to adequately respond to the robustness concern. I propose a larger conception that begins by defining the three sets - $\widehat{V}$, $\widehat{P}$ and S - which leads to basing the responses to robustness concern on the set of results: $R(P,V)$, $\forall (P,V) \in S$. Nevertheless, basing the responses on a subset $\widehat{S} \subset S$ is sometimes enough.

Before examining the different possible forms of response to the robustness concern (see sections 1.5 and 1.6), I need to make one thing clear. There are frailty points for which it is possible to wonder if the points are due to the way the problem was modeled or to certain characteristics of the processing procedures considered. This is the case, for example, with the parameters that define the role the different criteria must play in preference modeling. The way that this question is treated modifies the definition of the sets $\widehat{P}$ and $\widehat{V}$. The set of results $R(P,V)$ remains invariant facing such modifications. This comes from that the answer brought to the question does not modify the definition of S. Every element $s \in S$ is defined by a particular option retained for each of the frailty points, regardless of whether a given frailty point is assigned to procedures or to versions. To refer to this sequence of options that defines an element $s \in S$, I will use the term **variable setting**.

1.5 Third Proposal: Look for "Other" Definitions for *Robust Solutions*

In this section, I focus exclusively on the forms of response used to propose to the decision-maker one or more solutions qualified as robust. In these conditions, abandoning the second channel (see section 1.2) consists of proposing definitions for robust solutions that, one way or another, go beyond the usual schema to which most researchers currently conform. This schema seems to me to be characterized by the following three features:

Characteristic feature #1: In the FR model, preference is modeled using a single performance criterion (e.g., gain, cost); the robustness concern is not taken into account in any way in this single criterion.

Characteristic feature #2: A single robustness measure is defined to give meaning to the statements "solution **x** is at least as robust as solution **y**". This measure is the only criterion used to define robust solutions.

Characteristic feature #3: The single robustness measure assigns a preponderant role to the performance of the solution in the worst case(s), for example, the worst scenarios or the worst pairs (P,V) in S.

The basic and innovative work of Kouvelis and Yu [27] has probably contributed the most to the fact that research is mainly conducted within the framework of the above schema. A robust solution is a solution **x** that, by definition, optimizes a single robustness measure $r(\mathbf{x})$, which is one of the three measures on which Kouvelis and Yu primarily focused. Below, I remind the reader of their definitions.

These three measures insure that the optimization criterion v comes into play in a given FR model (Characteristic feature #1). This criterion assigns a value $v_s(\mathbf{x})$ to **x** in scenario s, assuming that "optimum" means "maximum".

Absolute robustness. The robustness measure that must be maximized is defined by the value of the solution in the worst scenario:$r(\mathbf{x}) = \min_s v_s(\mathbf{x})$.

Absolute deviation. The robustness measure that must be minimized is defined by the value of the absolute regret in the worst scenario, due to the fact that the solution differs from that which would be optimal in this scenario:$r(\mathbf{x}) = \max_s [v_s^* - v_s(\mathbf{x})]$, where v_s^* is the value taken by the optimal solution in scenario s.

Relative deviation. The robustness measure that must be minimized is defined by the value of the relative regret in the worst scenario, due to the fact that the solution is not optimal in this scenario:

$$r(\mathbf{x}) = \max_s \frac{v_\mathbf{x}^* - v_s(\mathbf{x})}{v_\mathbf{x}^*}$$

When, further on, I need to refer to these measures, I will speak of the "three measures of Kouvelis and Yu". I remind the reader that these measures correspond to the classic Wald and Savage criteria in decision under uncertainty.

In the sub-sections below, without any pretentions of exhaustivity, I evoke the directions that have begun to be explored but that I think deserve to be more explored. All aim to more or less free themselves from the schema that I have described above with its three characteristic features. This exploration should lead to new definitions of "robust solutions", which because they abandon the second channel (see section 1.2) will permit, at least in certain cases, to better respond to the robustness concern. To accomplish this aim, these new definitions must consider a double goal:

- take into account other conceptions of robustness than those taken into account in the schema described above;

- be appropriate for organizing a discussion with the decision-maker to help him/her understand the subjective aspect of robustness.

1.5.1 *Abandoning the Third Characteristic Feature Without Necessarily Abandoning the First Two*

Three directions at least seem to deserve more exploration:

a) Involve the neighborhood of the worst case

Once one of the three robustness measures of Kouvelis and Yu is chosen, it is possible to consider as robust not only the solutions that optimize this measure, but also all those that are in the neighborhood of the optimum. This neighborhood can be defined in several ways.

For example, let us assume that the measure $r(\mathbf{x})$ chosen is the absolute robustness of Kouvelis and Yu. It is then possible to qualify as robust every solution $\mathbf{x}$ that verifies $r(\mathbf{x}) \geq w$, where w is a value defined *a priori* by the decision-maker. When the criterion v represents a gain, the value w is interpreted as the minimum gain that the decision-maker wants to guarantee. The neighborhood of the optimum thus taken into account does not allow the robust solutions to be highlighted unless $w \leq w^* = \max_{\mathbf{x} \in X} r(\mathbf{x})$.

Among others, Aubry [6] and Rossi [33] have focused on this way of qualifying a robust solution. Clearly, the greater the difference $w^* - w$ the greater the number of solutions qualified as robust. Limiting the definition of the neighborhood is possible and allows a solution to be qualified as robust by retaining only the solutions that maximize the number or the proportion of scenarios that permit an objective b to be reached. This objective can be defined either in terms of absolute gain, or in terms of absolute regret, or even in terms of relative regret (again, with the meaning of the three criteria of Kouvelis and Yu). It was precisely this way of defining that led me to define three new measures: (b,w)-absolute robustness, (b,w)-absolute deviation, (b,w)-relative deviation (see [38, 40]). In these articles, I particularly emphasized the role that the analyst could assign to parameters b and w to help the decision maker determine the precise meaning that he/she wants to give to *robust solutions*. The choice of the values that he/she assigns to these parameters determines the compromise between guaranteed gain and the hope to reach a goal.

It is also possible to propose to qualify as robust the solutions that verify:

$$r(\mathbf{x}) \geq \left(1 - \frac{p}{100}\right) \max_{\mathbf{x} \in X} r(\mathbf{x}),$$

where $r(\mathbf{x})$ denotes absolute robustness and p, a percentage of deviation to the worst case that the decision-maker gives *a priori*. This is the p-robustness (see [1]; [45]). By proceeding in this manner, regardless of the value of the percentage p, the set of solutions qualified as robust is never empty. Diverse variants have been proposed to limit the set of solutions thus qualified as robust.

It is possible to propose similar definitions for robust solutions by adopting, for $r(\mathbf{x})$, the absolute regret or the relative regret of Kouvelis and Yu. However, it is not certain that such definitions would be so easily interpreted by the decision-maker.

b) Design a robustness measure that, while assigning a role to the worst case, weakens its influence

With the exception of research limited to very specific concrete cases, only one proposal seems to have been made in this direction: "lexicographic α-robustness" (see [5, 25]). This proposal is based on any of the three robustness measures of Kouvelis and Yu, and it assumes that a finite set S of scenarios has been defined. It takes into account all the gains or regrets lexicographically, from the worst case to the best case. This lexicographic ranking brings into play a reference point and an indifference threshold α, which according to the authors is supposed to reflect "the subjective dimension of robustness". The set $A(\alpha)$ of the solutions thus qualified as robust is such that $\alpha' > \alpha \Rightarrow A(\alpha) \subseteq A(\alpha')$. With the values of α, $A(\alpha) = \varnothing$ can be obtained. The conditions for solution $\mathbf{x}$ belonging to, or not belonging to, $A(\alpha)$ are highly complex. Thus, it seems to me that it would be difficult to make a decision-maker understand the general meaning of "lexicographic α-robustness". Still, as the authors have underlined, this approach can also be seen as a source for developing robust conclusions (see section 1.6).

c) Limit the option combinations appropriate for defining a worst case

Exploring this direction consists of seeking to benefit from the following empirical judgments. Let us assume that, for certain "critical" frailty points, the identity of the option that would most negatively affect the performance $v(\mathbf{x})$ of a solution $\mathbf{x}$ can be identified. In general, these options are "extreme". A scenario, or more generally a variable setting (in the sense given at the end of section 1.4), which combines a very large number or a very large proportion of such options, reflects a real-life context that could be judged highly unrealistic. Under these conditions, it can seem justifiable either to totally eliminate from S the scenarios or variable settings that are not very meaningful for defining the robustness $r(\mathbf{x})$ of a solution or to assign them a separate role that will lessen their impact.

To characterize the elements $s(S)$ judged not very meaningful, many different ways are possible. For example, each of the critical frailty points can be associated to a particular option, called "reference option", that must have a central position or must appear to be the most realistic. Then, it is possible to consider as not very meaningful the element $s(S)$ in which the number or proportion of options that differ from the reference option for which the critical frailty points, exceed a certain threshold. If, for each critical frailty point, a measure of the gap between a given option and the reference option can be defined, and if the aggregation of such a measure has meaning, the result of this aggregation can be used to define the relative meaningfulness of an element s not very meaningful.

The work of Bertsimas and Sim [8, 9] related to linear programming FR models goes in this direction, as does the work of Ben-Tal and Nemirovski [7], Chinneck and Ramadan [14], Gabrel and Murat [20], and Minoux [30].

These three directions are certainly not the only ones. Other directions have already begun to be explored. A non-exhaustive list would include the research of Lamboray [28], Liesiö et al. [29], Perny et al. [31], Salazar and Rocco [44], and Spanjaard [47]. The proposal of Aissi and Roy [3, 4] (sub-section 3.4) also goes in this direction. This proposal leads to (almost) ignoring the role played by the worst case. It takes into account only the position of solution $\mathbf{x}$ in each of the rankings defined by the decreasing performance of $v_s(\mathbf{x}), \forall s \in S$.The values assigned to the two parameters that fix two types of thresholds should help the decision-maker to find a good compromise between the performance desired and the risk tolerated.

1.5.2 Abandoning the Last Two Characteristic Features Without Necessarily Abandoning the Third

Here again, the case in which the preference has been modeled (in the FR model) by a single performance criterion that does not take robustness concern into account is considered. In such conditions, several criteria can be brought into play in order to qualify a solution as robust. Two possibilities can be considered.

a) Take only a single robustness measure into account

Along with the criterion defined by a single robustness measure, one or more other criteria can be used to define the precise conditions in which a solution warrants being qualified as robust. The role of this/these complementary criterion/criteria, which are not based on a robustness measure, is to provide additional information, essentially about performance, in order to help the decision-maker determine what "robust" means to him/her.

For example, a complementary criterion $w(\mathbf{x})$ can be defined in one of the two following ways:

- $w(\mathbf{x})$ is the median performance $v_s(\mathbf{x}), \forall s \in S$ (if S is probalilized, the median can be replaced by the expected value); and
- $w(\mathbf{x}) = v_{s_0}(\mathbf{x})$, where s_0 is a reference scenario or a reference variable setting defined by retaining, for each frailty point, the option that the decision-maker thinks is the most realistic.

Once a robustness measure $r(\mathbf{x})$ has been chosen (not necessarily one of the ones proposed by Kouvelis and Yu), let us denote by Ω the set of solutions that verifies:

$$r(\mathbf{x}) \geq \left(1 - \frac{p}{100}\right) \max_{s \in S} r(\mathbf{x}) \quad \text{if } r(\mathbf{x}) \text{ must be maximized,}$$
$$r(\mathbf{x}) \leq \left(1 + \frac{p}{100}\right) \min_{s \in S} r(\mathbf{x}) \quad \text{if } r(\mathbf{x}) \text{ must be minimized,}$$

where p is a percentage introduced to set the boundaries of the neighborhood Ω of the optimum; outside of this neighborhood, the solutions cannot be considered robust. This leads to associating each solution $\mathbf{x} \in \Omega$ to its robustness measure $r(\mathbf{x})$ and the value corresponding to the performance indicator $w(\mathbf{x})$. Let:

$$w_\rho = \min_{\mathbf{x} \in \Omega, r(\mathbf{x}) = \rho} w(\mathbf{x})$$

Showing the decision-maker the curve (efficient frontier) representing the way that the performance indicator w_ρ varies in relation to the robustness measure ρ can be very useful. This curve can help the decision-maker select the solutions that, in his/her eyes, achieve a suitable compromise between the acceptable risk and the expected performance.

The following are examples of publications that take this approach: Chen et al. [13], Ehrgott and Ryan [16], Kennington et al. [26], and Salazar and Rocco [44]. Interested readers can consult the brief analysis of these publications in Aissi and Roy [3, 4] (section 3.2).They will also find in this article (at the end of section 1.5.2) a proposal dealing with linear programming models in which only the coefficients of the constraint matrix are imperfectly known (the set S that allows the imperfect knowledge to be modeled being assumed finite). In addition, this proposal assumes that the decision-maker can tolerate a robust solution that does not verify all the constraints, if this is true only for a small number of elements $s(S)$ and leads to a low cost solution. It also assumes that a solution that is mathematically infeasible is preferable to a solution that satisfies all the constraints but costs a lot more. This proposal brings into play a robustness measure and a cost criterion. In fact, three possible measures are proposed. Consequently, this approach helps the decision-maker to find a compromise between the solution's performance (here, the cost) and the gaps that reflect the imperfect satisfaction of the constraints.

b) Take several robustness measures into account

It is clear that robustness can be apprehended from various perspectives. The undesirable impacts from which the decision-maker wants to protect him/herself can also vary, including for example objectives that remain unattained or properties that are not maintained. One robustness measure can be associated to each of these objectives and/or to each of these properties.

Let $R = \{r_1(\mathbf{x}), \ldots, r_k(\mathbf{x})\}$ be a set of k measures judged appropriate for apprehending robustness. To qualify a solution as robust, it is possible to use a multicriteria approach based on these k measures. They should, as much as possible, be uncorrelated. It is thus a question of generalizing, for k robustness measures, what can be done with only one measure. Two possibilities can be envisaged. The first consists of bringing into play only the way robustness is measured for qualifying a solution as robust as described in section 1.5.1. The second consists of bringing into play, along with the k criteria of R, one or more other complementary criteria that provide additional information destined to help the decision-maker determine what "robustness" means to him/her, as described in section 1.5.2a).

The interesting attempt of Hites et al. [23] takes the direction of the first possibility. Assuming the set S of scenarios or variable settings to be finite, the authors consider that for each $s \in S$, $v_s(\mathbf{x})$ provides information that will be pertinent for evaluating the relative robustness of solution $\mathbf{x}$. This leads them to state $R = \{v_s(\mathbf{x})/s \in S\}$. They point out the limitations of the traditional multicriteria decision-aiding tools for defining, based on R, the solutions that warrant being qualified as robust. However, they also emphasize that a solution can only be qualified as robust if it is non-dominated by the criteria set R.

I know of only one article that goes in the direction of the second possibility stated above. Jia and Ierapetritou [24] focus on the design phase of a process for synthesizing chemical products in small or intermediate quantities. Interested readers can consult a brief description by Aissi and Roy [3, 4] (section 4.3). They will also find at the end of section 4.3 a general proposal for the case in which S is finite. This proposal is based on using two robustness measures, completed by a performance indicator $w(\mathbf{x})$ of the same type as the one introduced in section 1.5.2a).

1.5.3 Abandoning the First of the Three Characteristic Features

One way to break away from feature #1is not to introduce an initial performance preference in the FR model but rather, from the beginning, to seek to apprehend the preferences in terms of robustness. This assumes that the FR model brings into play a family of several criteria (the case of one single criterion can, it seems to me, be left aside), each of these criteria translating one perspective of robustness. I do not know any work using this approach, other than the one in which I participated (see [32]), which concerned the regulation of dense traffic on a rail network (timetable elaboration and local network improvements). Interested readers can consult section 4.2 of the article by Aissi and Roy [3] for a brief description.

Another probably more natural way to break away from feature #1 leads to focusing on the cases in which the FR preference model is based on a family F of $n(\geq 2)$ criteria. Each criterion v_i models one component of the multidimensional performance of a solution $\mathbf{x}$, evaluated without taking the robustness concern into account. Let $v_{is}(\mathbf{x})$ be this performance when the real-life context is assumed to conform to the scenario or variable setting $s \in S$. Depending on whether one or several robustness measures are used to give meaning to the qualifier "robust", two cases can be distinguished.

a) With a single robustness measure

This single robustness measure must synthesize, for a single dimension characterizing the robustness of a solution **x**, the totality of the performances $v_{is}(\mathbf{x})$, not only $\forall s \in S$ as in section 1.5.1, but also for $i = 1,..,n$. I know of no publication that has proposed such a general measure. It is obviously possible to begin by aggregating the n criteria into a single criterion, which bring us back to the cases mentioned in section 1.5.1. I limit myself below to presenting only a brief description of a different path already presented in sub-section 3.3 of the article by Aissi and Roy [3, 4].

First, a specific robustness measure (see 1.5.1) is associated to each of the n criteria. Second, a way to aggregate the n measures is sought. If these measures are expressed on a common scale, the min or max can be used as a definition of the synthesis measure. If the role played by the different robustness measures needs to be differentiated according to the criterion to which they are associated, such operators as the Ordered Weighed Average (OWA) or Choquet's integral can, with certain precautions, be used.

b) With several robustness measures

The set R of robustness measures that can be brought into play may be conceived in two ways. The first consists of apprehending the robustness of a solution from various points of view, as in sub-section 1.5.2b). The second consists of associating a specific robustness measure to each of the n criteria, as in sub-section 1.5.3a). Like monocriterion preference models, multicriteria preference models do not seem to have been considered in the works that examine the robustness of a solution from a multicriteria perspective. Aissi and Roy [3, 4] (subsection 4.4) have described the only two studies I know of: Bescharati and Azarm [10] and Fernàndez et al. [17].

1.6 Fourth Proposal: Seek to Construct "Robust Conclusions"

Stating a robust conclusion constitutes a form of response to the robustness concern, which generalizes those presented in the previous section. After reviewing several definitions, I present diverse examples of robust conclusions and end this section by evoking some of the approaches that allow this kind of conclusions to be obtained.

1.6.1 Definitions

A ***conclusion*** *is an assertion that take the results $R(P,V)$ into account, either for all pairs $(P,V) = s \in S$, or only for the elements $\widehat{S} \subset S$. These assertions must help the decision-maker to frame, mark out or restrict his/her range of reflection and action.*

A ***robust conclusion*** *is a conclusion that contains in its statement a set of conditions in which its validity has been established.* This means that the robustness of a conclusion is contingent on both the validity range $\widehat{S} \subset S$ to which it belongs and the way that the validity conditions are formulated. Focusing only on the case of $\widehat{S} = S$ would lead to restricting the diversity and the importance of robust conclusion to no useful purpose. It would be the same for requirements that the set $\widehat{S}$ be rigorously defined and that the conditions of validity contain no ambiguities. These considerations led me (see [34, 35]) to distinguish three types of robust conclusions:

A conclusion is qualified as perfectly robust when its validity is rigorously established $\forall s \in \widehat{S}$, $\widehat{S}$ subset of S perfectly identified.

A conclusion is qualified as approximately robust when it has been established without ambiguity for "almost" all $s \in \widehat{S}$, $\widehat{S}$ subset of S perfectly identified. "Almost" means that the exceptions are related to pairs (P,V) that are not necessarily perfectly identified but which can be neglected because they are not very realistic or are almost totally without interest.

A conclusion is qualified as pseudo robust when the conditions that establish its validity contain a degree of ambiguity, and this validity is only established for the elements of a subset of $\widehat{S}$, which is not necessarily perfectly identified but is judged representative of $\widehat{S}$.

Any perfectly robust conclusion is *a fortiori* approximately robust, and any approximately robust conclusion is *a fortiori* pseudo robust.

1.6.2 Examples

The following assertion constitutes the most common example of a robust conclusion: "in S, $\mathbf{x}^*$ is the only solution that optimizes the robustness measure $r(\mathbf{x})$". More generally, since Π defines a set of properties that a solution $\mathbf{x}$ must have to be qualified as robust, the assertion "$\mathbf{x}^*$ is a robust solution because the properties Π are validated without ambiguity for all S" is a robust conclusion.

The concept of robust conclusion would have little interest if it was only applied to assertions that affirm the robustness of a solution on all S in a perfectly well-defined manner. The analyst may be brought to formulate highly interesting robust conclusions for the decision-maker who does not explicitly refer to a definition of a robust solution. In addition, as the following examples show, the analyst may, because he/she lacks time or does not have sufficiently rigorous algorithms available, be led to propose assertions whose validity range is not necessarily all of S and/or which potentially tolerate exceptions.

a) Other examples of perfectly robust conclusions

i) "$\widehat{S}$ being a perfectly identified subset of S, $\forall s \in \widehat{S}$, $\mathbf{x}$ is a solution whose deviation from the optimum according to criterion g never exceeds ε.

ii) "The following solutions... are feasible solutions for all the variable settings of S with the exception of the following..." (the range of validity of this perfectly robust conclusion is S, without the excepted variable settings).
iii) "Solution **x** dominates solution **y**, $\forall s \in S$, except perhaps for the variable settings $s_1, ..., s_q$" (the range of validity of this perfectly robust conclusion is S, without the variable settings $s_1, ..., s_q$).
iv) "The variable settings of $\widehat{S}$ (perfectly identified subset of S) prove the incompatibility of the following objectives ..." (example of objectives: attain a performance at least equal to b_i according to criterion g_i for $i = 1, ..., k$).
v) "$\widehat{S}$ being a perfectly identified subset of S, **x** is a solution that, $\forall s \in \widehat{S}$, has the property P and, $\forall s \notin \widehat{S}$, has the property Q" (for example, with a finite solution set, property P: **x** is always among the α best solutions; property Q: **x** is never among the β worst solutions).

b) Examples of approximately robust conclusions

i) "$\widehat{S}$ being a perfectly identified subset of S, $\forall s \in \widehat{S}$, **x** is a solution whose deviation from the optimum according to criterion g only exceeds ε for certain variable settings that, because they correspond to option combinations that are not very realistic, can be neglected.
ii) "A probability distribution having been defined on S, **x** is a feasible solution with a probability at least equal to $1 - \varepsilon$" (as long as the risk ε of non-feasibility of a solution is judged negligible, this type of conclusion can be considered as approximately robust).
iii) "Solution **x** dominates solution **y**, $\forall s \in \widehat{S}$, $\widehat{S}$ only differing from S by the variable settings that correspond to combinations of procedures and versions that can be judged as not easily compatible."
iv) "$\widehat{S}$ being a perfectly identified subset of S, **x** is a solution that, for *almost* all variable settings $s \in \widehat{S}$, has the property P, where *almost* means that the variable settings for which it could be considered otherwise are judged negligible."
v) "Solution **x** is non dominated, $\forall s \in S$, except for certain variable settings that, because they are *too different* from a reference variable setting, can be neglected."

c) Examples of pseudo robust conclusions

i) "The performance of the solution **x** according to the criterion g can be considered as always being at least equal to W because a simulation using a very large number of random drawings of variable settings of S has shown that $g(\mathbf{x}) \geq W$ was always verified."
ii) "A systematic but non-exhaustive exploration of S showed that the following solutions ... were always feasible solutions."
iii) "Solution **x** dominates (or almost dominates) solution **y**, $\forall s \in \widehat{S}$ (perfectly defined subset of S), where *dominates (or almost dominates)* means that when the

dominance is violated, it is only violated for one or two criteria, with slight deviations."

iv) "$\widehat{S}$ beings a subset of S that was constructed in order to serve as a *representative* sub-set of the variable settings making up S, the assertions validated on $\hat{S}$ can be judged valid on S."

v) "The assignment of $\mathbf{x}_i$ to the category C_j can, $\forall s \in S$, be considered as robust, given the meaning of this term, although it is not completely without ambiguity."

Interested readers will find concrete illustrations of some of the previous examples in works by Pomerol et al. [32], Roy [34, 35], Roy and Bouyssou [41] (chapter 8), Roy and Hugonnard [42, 2], and Roy et al. [43].

1.6.3 How Can Robust Conclusions be Obtained?

The examples in the previous section show that extremely varied forms of robust conclusions warrant consideration. They can encourage thinking that it should be possible to conceive a list of typical statements that are particularly worthy of interest for each of the three types of robust conclusions. However, the statements on such a list would be of interest only for those robust conclusions to which an approach allowing their validation can be associated. This approach would obviously depend on:

- the formal representation (FR): mono or multicriteria models with solutions characterized by a set of variables subjected to constraints or by a set of numbered actions;
- the way that S was conceived: finite or infinite set, nature of the frailty points concerned for $\widehat{V}$ as well as $\widehat{P}$ (see sections 1.3 and 1.4).

Rather than begin by establishing a list of typical conclusions and then wondering about the context (FR, S) in which they could be validated, it is perhaps preferable to proceed in the opposite manner. This is what seems to have been done by the rare authors who sought to respond to the robustness concern by validating more elaborate robust conclusions than those that are limited to affirming than such a solution is robust according to very precise definitions. I will end this section with a brief presentation of the context in which are situated the three approaches of which I am currently aware. All three focus on cases in which:

- the decision-making possibilities are defined by a finite list of what are called *actions* rather than *solutions*;
- the preference model is multicriteria;
- the problematic can be one of choice, of sorting or of ranking.

a) J. Figueira, S. Greco, V. Mousseau and R. Slowinski have focused on the following context (see [18, 19, 21, 22]):

The FR preference model is an additive value function that aggregates several partial monotonic non-decreasing functions. S takes into account a set $\widehat{P}$ of procedures: the procedures of $\widehat{P}$ are all those that allow an additive value function to be built that is compatible with the preference data that the decision-maker provides about a reference set of actions.

There are two types of robust conclusions:

- those that are validated by all the procedures of $\widehat{P}$ that are qualified as **necessary**;
- those that are validated by at least one procedure of $\widehat{P}$ that are qualified as **possible**.

b) T. Tervonen, J. Figueira, R. Lahdelma, P. Salminen and have focused on the following context (see [48]):

- the FR preference model is a relational outranking system of the ELECTRE type, which takes several pseudo criteria into account; and
- the variable setting that define S are related to the possible options in terms of weights, veto thresholds and discrimination thresholds.

The robust conclusions (stated in S) are those that can be validated for a subset $\widehat{S}$ of S built by simulation, which brings into play the probability distributions of all the possible options for each of the frailty points considered. The interested reader can consult Sörensen [46] for another approach to the robustness concern using the Monte Carlo method.

c) H. Aissi and I have focused on the following context (see [3, 4], subsection 5.3):

- the FR preference model is a relational outranking system of the ELECTRE type that uses pseudo criteria; and
- .S, based on two previously defined sets $\widehat{S}$ and $\widehat{V}$, is finite

The robust conclusions are obtained at the end of a three-step procedure. The first step constructs a subset $\widehat{S}$ of S, which must be the result of a compromise between two relatively conflicting requirements, *calculability* and *representativeness*. The second step highlights two categories of frailty points: those that can greatly influence the results, and those whose influence on the results is negligible. On this basis, we proposed replacing $\widehat{S}$ by a set $\widehat{S'}$ that is at least as representative and, as far as possible, much more limited. The third step, based on a careful analysis of the results $R(P,V), \forall (P,V) \in \widehat{S'}$, must allow robust conclusions pertinent to the problem under study to be obtained. This approach generalizes and extends the insufficiently formalized approach that was used to obtain robust conclusions in two concrete cases (see [41], chapter 8; [42, 2].

The types of statements of robust conclusions that each of the three approaches above can validate warrant a more detailed explanation. There are many other contexts (FR, S) than those mentioned above (especially in relation to mathematical programming); it should thus be possible to propose approaches specifically designed to obtain robust conclusions.

1.7 Conclusion

Most publications that propose ways to respond to robustness concern (as defined in section 1.1) remain limited to the two special channels described in section 1.2. This is the two-fold observation that led me to formulate the four proposals presented above. The first two proposals (see sections 1.3 and 1.4) should help the analyst not to overlook frailty points in what I call the formal representation (see section 1.1). If these frailty points are ignored, the analyst could propose solutions or make recommendations that could, when implemented, lead to results much worse than those expected. The last two proposals (see sections 1.5 and 1.6), in conjunction with the first two, present research directions that, even though some have already begun to be explored, should, it seems to me, help to develop new forms of responses to robustness concern.

Using more general definitions of the concept of robust solution or even forms of responses conceived in terms of robust conclusions, it should be possible to take into account a greater variety of decision-makers' expectations, to provide them with a better understanding of the subjectivity of the robustness concern, to help them choose better compromises between the risk of being poorly protected against very bad performances and the abandoning of hope for good, even very good, performances. The research directions that have been outlined above should not only allow a better understanding of the robustness concern in applications, but also stimulate interesting research on more theoretical levels.

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