Decision Support for Combinatorial Reverse Auction with Multiple Buyers and Sellers

Fu-Shiung Hsieh and Cheng Chung Hua

Abstract. We consider a multiple buyers/sellers combinatorial reverse auction problem in which multiple buyers want to acquire items from a set of sellers to process the task on hand. Each seller owns a set of items to bid for the required items requested by the buyers. The problem is to determine the winners to minimize the total cost to perform acquire the required items. The main results include: (1) a problem formulation for the winner determination problem; (2) a solution methodology based on Lagrangian relaxation; (3) analysis of numerical results obtained by our algorithms.

1 Introduction

Auctions are popular, distributed and autonomy preserving ways of allocating items or tasks among multiple agents to maximize revenue or minimize cost. Combinatorial auctions [1, 3] are beneficial, when complementarities exist between the items to be auctioned. Allowing bids for bundles of items is the foundation of combinatorial auctions. However, combinatorial auctions have been notoriously difficult to solve from a computational point of view [4, 8, 10, 11]. Many algorithms have been developed for combinatorial auction [2, 8, 4, 7, 12, 13]. However, in real world, there are usually multiple buyers and sellers involved in auction. Motivated by the deficiency of the existing methods, we consider a multiple buyers/sellers combinatorial reverse auction problem (MBSCRA) in which multiple buyers want to acquire items from a set of sellers. Each buyer requests a minimum bundle of items that can be provided by a set of bidders. The problem is to determine the winners to minimize the total cost. The remainder of this paper is organized as follows. In Section 2, we present the problem formulation. In Section 3, we propose the solution algorithms. We analyze the performance of our algorithm in Section 4. We conclude in Section 5.

Fu-Shiung Hsieh and Cheng Chung Hua
Department of Computer Science and Information Engineering,
Chaoyang University of Technology, 168 Jifong E. Rd.,Wufong Township,
Taichung County, 41349 Taiwan
e-mail: {fshsieh, s9627622}@cyut.edu.tw

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2 Multiple Buyers/Sellers Combinatorial Reverse Auction

Fig. 1 illustrates a scenario in which buyers request to purchase three different bundles of items from the sellers. Suppose there are two buyers and five sellers. Buyer 1 requests to purchase 2A, 1B and 2C while Buyer 2 requests to purchase 1A and 3B. There are five sellers, Seller 1, Seller 2, Seller 3, Seller 4 and Seller 5, who place bids. Suppose Seller 1 places the bid: (1A, 1B, 1C, p11, p21), Seller 2 places the bid: (2A, 1B, 0C, p12, p22), Seller 3 places the bid: (0A, 1B, 2C, p13, p23), Seller 4 places the bid: (0A, 3B, 0C, p14, p24) and Seller 5 places the bid: (1A, 2B, 0C, p15, p25), where p_{in} denotes the prices of the bid placed by Seller n to Buyer i . We assume that all the bids entered the auction are recorded.

Let's formulate the multiple buyers/sellers combinatorial reverse auction (MBSCRA) problem. Let I denote the number of buyers in MBSCRA. Each $i \in \{1,2,3,....,I\}$ represents a buyer. Let N denote the number of sellers that place bids in MBSCRA. Each $n \in \{1,2,3,....,N\}$ represents a seller. Let K denote the number of items requested. Let d_{ik} denote the desired units of the $k-th$ items requested by Buyer $i \in \{1,2,3,....,I\}$, where $k \in \{1,2,3,....,K\}$. In MBSCRA, we use a vector $b_n = (q_{in1}, q_{in2}, q_{in3},..., q_{inK}, p_{in},)$ to represent the bid submitted by bidder n , where q_{ink} denotes the quantity of the $k-th$ items and p_{in} denotes the price of the bundle. Bid b_n is an offer to deliver q_{ink} units of the $k-th$ items a total price of p_{in} . We use the variable x_{in} to indicate the bid placed by bidder n is active (x_{in} =1) or inactive (x_{in} =0). We formulate the problem as follows.

Multiple Buyers/Sellers Combinatorial Reverse Auction Problem

$$\min \sum_{i=1}^{I} \sum_{n=1}^{N} x_{in} p_{in}$$

$$s.t. \sum_{n=1}^{N} x_{in} q_{ink} \geq d_{ik} \ \ for\ all\ k \in \{1,2,...K\}, i \in \{1,2,...I\}$$

$$\sum_{i=1}^{I} x_{in} \leq 1 \quad for\ all\ n \in \{1,2,..., N\}, x_{in} \in \{0,1\}$$

We form a Lagrangian function by applying Lagrangian relaxation.

$$L(\lambda) = \min \sum_{i=1}^{I} \sum_{n=1}^{N} x_{in} p_{in} + \sum_{i=1}^{I} \sum_{k=1}^{K} \lambda_{ik} (d_{ik} - (\sum_{n=1}^{N} x_{in} q_{ink}))$$

$$\text{st.} \sum_{i=1}^{I} x_{in} \leq 1\ for\ all\ n, x_{in} \in \{0,1\}$$

$$L(\lambda) = \sum_{i=1}^{I} \sum_{k=1}^{K} \lambda_{ik} d_{ik} + \sum_{n=1}^{N} L_n(\lambda),$$

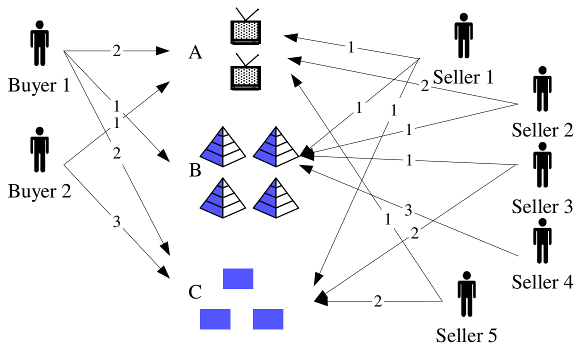

Fig. 1 Multiple Buyers/Sellers Combinatorial Auction

$$where\ L_n(\lambda) = \min \sum_{i=1}^{I} x_{in}(p_{in} - \sum_{k=1}^{K} \lambda_{ik} q_{ink})$$

$$s.t. \sum_{i=1}^{I} x_{in} \leq 1 \quad for\ all\ n \in \{1,2,...,N\},\ x_{ij} \in \{0,1\}$$

Lagrangian relaxation of constraints decomposes the original problem into a number of subproblems that can be solved independently. Lanrange multipliers are determined by solving the dual problem: $\max_{\lambda \geq 0} L(\lambda)$.

3 Solution Algorithms

Our algorithms developed based on Lagrangian relaxation consists of three parts: (1) an algorithm for solving subproblems; (2) a subgradient method for solving the dual problem; (3) a heuristic algorithm for finding a near-optimal feasible solution.

(1) An algorithm for solving subproblems: Given Lagrange multiplier λ, the optimal solution to SS subproblem $L_n(\lambda)$ can be solved as follows. Let $i^* = \arg\min \sum_{i=1}^{I} x_{in}(p_{in} - \sum_{k=1}^{K} \lambda_{ik} q_{ink})$. The optimal solution to $L_i(\lambda)$ is as follows. $x_{in} = \begin{cases} 0\ \forall i \in \{1,2,...,I\} \setminus \{i^*\} \\ 1\ if\ i = i^*\ and\ p_{i^*n} - \sum_{k=1}^{K} \lambda_{i^*k} q_{i^*nk} < 0 \\ 0\ if\ p_{i^*n} - \sum_{k=1}^{K} \lambda_{i^*k} q_{i^*nk} \geq 0 \end{cases}$

(2) A subgradient method for solving the dual problem $\max_{\lambda\geq 0} L(\lambda)$: Let x^l be the optimal solution to the subproblems for given Lagrange multipliers λ^l of iteration l . We define the subgradient of $L(\lambda)$ as $g_{ik}^l = \frac{\partial L(\lambda)}{\partial \lambda_{ik}}\Big|\lambda_{ik}^l = d_{ik} - \sum_{n=1}^{N} x_{in} q_{ink}$, where , $i \in \{1,2,...I\}$ and $k \in \{1,2,..., K\}$. The subgradient method proposed by Polak [9] is adopted to update λ by $\lambda_{ik}^{l+1} = \begin{cases} \lambda_{ik}^l + \alpha^l g_{ik}^l & if\ \lambda_{ik}^l + \alpha^l \lambda_{ik}^l \geq 0; \\ 0 & otherwise. \end{cases}$ where $\alpha^l = c\frac{\overline{L} - L(\lambda)}{\sum_k (g_k^l)^2}$, $0 \leq c \leq 2$ and $\overline{L}$ is an estimate of the optimal dual cost. The iteration step terminates if α^l is smaller than a threshold. Polyak proved that this method has a linear convergence rate and iterative application converges to an optimal dual solution (x^* , λ^*).

(3) A heuristic algorithm for finding a near-optimal, feasible solution based on the solution of the relaxed problem: The solution (x^* , λ^*) may result in one type of constraint violation due to relaxation: assignment of the quantity of items less than the demand of the items. Our heuristic scheme first find $K^0 = \{k \mid k \in \{1,2,3,...., K\}, \sum_{n=1}^{N} x_{in} q_{ink} < d_{ik}\}$ to identify the demand constraints that have not been satisfied. Let $N^0 = \{n \mid n \in \{1,2,3,...., N\}, x_{in}^* = 0\}$ be the set of bidders that is not a winner in solution x^* . To make the set of constraints K^0 satisfied, we first pick $k \in K^0$ with $k = \arg\min_{k\in K^0} d_{ik} - \sum_{n=1}^{N} x_{in}^*\ q_{ink}$. Select $n \in N^0$ and $j \in \{1,2,..., n_i\}$ with $n = arg \min_{n\in N^0, q_{ink}>0} p_{in}$ and set $x_{in}^* = 1$. Then we set $N^0 \leftarrow N^0 \setminus \{n\}$. If the violation of the k -th constraint cannot be completely resolved, the same procedure repeats. If all the constraints are satisfied after applying the aforementioned procedure, a solution is obtained.

4 Numerical Results

The effectiveness of the solution algorithms can be evaluated based on the duality gap, which is the difference between primal and dual objective values. That is, duality gap is defined by $f(x^*) - L(\lambda^*)$. Based on the proposed algorithms for combinatorial reverse auction, we conduct several examples to illustrate the validity of our method.

Table 1 Buyers' Requirements

	Item1	Item2	Item3
Buyer 1	2	1	2
Buyer 2	1	3	0

Table 2 Sellers' Bids

	Item1	Item2	Item3
Seller 1	1	1	1
Seller 2	2	1	0
Seller 3	0	1	2
Seller 4	0	3	0
Seller 5	1	2	0

Example 1: Consider two buyers who will purchase a set of items as specified in Table 1. Five potential sellers' bids as shown in Table 2. For this example, we have $I = 2, \quad N = 5, \quad K = 3, d_{11} = 2, d_{12} = 1, d_{13} = 2, d_{21} = 1,$ $d_{22} = 3, d_{23} = 0.$ According to Table 2, we have:

$q_{111} = q_{211} = 1, q_{112} = q_{212} = 1, q_{113} = q_{213} = 1, q_{121} = q_{221} = 2, q_{122} = q_{222} = 1,$
$q_{123} = q_{223} = 0, q_{131} = q_{231} = 0, q_{132} = q_{232} = 1, q_{133} = q_{233} = 2, q_{141} = q_{241} = 0,$
$q_{142} = q_{242} = 3, q_{143} = q_{243} = 0, q_{151} = q_{251} = 1, q_{152} = q_{252} = 2, q_{153} = q_{253} = 0,$

Suppose the prices of the bids are: $p_{11} = 38, p_{12} = 30, p_{13} = 30, p_{14} = 25, p_{15} = 25,$ $p_{21} = 35, p_{22} = 22, p_{23} = 21, p_{24} = 24, \ p_{25} = 25.$

Suppose we initialize the Lagrange multipliers as follows.
$\lambda(1) = 10.0, \lambda(2) = 10.0, \lambda(3) = 10.0, \lambda(4) = 10.0, \lambda(5) = 10.0$.
Our algorithm the subgradient algorithm converges to the following solution: $x_{13}^* = 1$, $x_{24}^* = 1$ and $x_{in}^* = 0$ for all the other (i, n). As the above solution is a feasible one, the heuristic algorithm needs not be applied. Therefore, $\bar{x}_{13} = 1$, $\bar{x}_{12} = 1$, $\bar{x}_{24} = 1$, $\bar{x}_{22} = 1$. The solution x^* is also an optimal solution. The duality gap of the solution is 3.75%. The duality gap is within 5%. This means the solution methodology generates near optimal solution.

5 Conclusion

We study multiple buyers/sellers combinatorial reverse auction problem. By applying Lagrangian relaxation technique, the original optimization can be decomposed into a number of sellers' subproblems. Numerical results indicate that our proposed algorithms yield near optimal solutions for small problems. Our future research directions are to study the optimality of the near optimal solutions obtained from our algorithms for large problems and compare our algorithms with existing methods.

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