Applying ELECTRE and Maximizing Deviation Method for Stock Portfolio Selection under Fuzzy Environment

Chen-Tung Chen and Wei-Zhan Hung

Abstract. The purpose of stock portfolio selection is how to allocate the capital to a large number of stocks in order to bring a most profitable return for investors. In most of past literature, expert considered portfolio problem only based on past data. It is very important for experts to use their experience and knowledge to predict the performance of each stock. In this paper, 2-tuple linguistic variables are used to express the opinions of experts to predict the performance of each stock with respect to each criterion. According to experts' linguistic evaluations, we use maximizing deviation method to derive the weight of each criterion. And then, the linguistic ELECTRE method is used to derive the credibility matrix and calculate the net credibility degree of each stock. Based on the outranking index and selection threshold, we can easily obtain portfolio set and decide the investment ratio of each stock. An example is implemented to demonstrate the practicability of proposed method.

Keywords: stock portfolio selection, 2-tuple linguistic variable, ELECTRE, maximizing deviation method.

1 Introduction

The purpose of stock portfolio selection is how to allocate the capital to a large number of stocks in order to bring a most profitable return for investors [8]. Markowitz proposed the mean–variance method for the portfolio selection problem in 1952 [10]. The capital asset pricing model (CAPM), Black model and two-factor model are derived from the mean–variance method. In 1980, Saaty proposed Analytic Hierarchy Process (AHP) to deal with portfolio selection problem [12]. Edirisinghe and Zhang selected the securities in the context of data envelopment analysis (DEA) [2]. In the aforementioned portfolio selection models, experts decide investment portfolio only based on past numerical data except AHP. However, AHP is a subjective method and has the consistent problem of judgment by experts. In real situation, expert can use his experience and knowledge to

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predict the performance of each stock; it is risky to select stock to invest only based on past numerical data in more and more competitive environment. The Elimination et choice in Translating to Reality (ELECTRE) method is a highly developed multi-criteria analysis model which takes into account the uncertainty and vagueness in the decision process [11]. It is based on the axiom of partial comparability; it can simplify the evaluation procedure of stock selection. Due to imprecise and subjective information that often appears in stock selection process, crisp values are inadequate for solving the problems. A more realistic approach may be to use linguistic assessments instead of numerical values [*3*].

In fact, experts can apply 2-tuple linguistic variables to express their opinions and obtain the final evaluation result with appropriate linguistic variable. The 2-tuple linguistic representation model is based on the concept of symbolic translation [3, 16]. It is an effective method to reduce the mistakes of information translation and avoid information loss through computing with words [6].

The maximizing deviation method is proposed by Wang [14] to compute the weight of each criterion in multiple attribute decision making (MADM) problems with numerical information. If some criterion makes the performance values among all the stocks have obvious differences, such a criterion plays a more important role in choosing the best stock. The distinguish ability and objectivity of the maximizing deviation method is better than AHP which is based on expert's subjective opinion.

This paper is organized as follows. In section 2, we present the context of the 2-tuple linguistic variable. In section 3, we discuss the concept and formula of the maximizing deviation method. In section 4, we describe the detail of the proposed method, and then an example is implemented to demonstrate the procedure for the proposed method and compare with the method of Tiryaki and Ahlatcioglu [13]. Finally, the conclusion is discussed at the end of this paper.

2 The 2-Tuple Linguistic Representation

Let $S=\{s_0, s_1, s_2, ..., s_g\}$ be a finite and totally ordered linguistic term set. A 2-tuple linguistic variable can be expressed as (s_i, α_i), where s_i is the central value of i-th linguistic term in S and α_i is a numerical value representing the difference between calculated linguistic term and the closest index label in the initial linguistic term set. The symbolic translation function Δ is presented in [4] to translate a crisp value into a 2-tuple linguistic variable. The generalized translation function can be represented as [1] $\Delta:[0,1]\rightarrow S\times[-(1/2g),(1/2g))$, $\Delta(\beta)=(s_i,\alpha_i)$ where $i=round(\beta\times g)$, $\alpha_i=\beta-(i/g)$, $\alpha_i\in[-(1/2g),(1/2g))$ and $\beta\in[0,1]$. A reverse function Δ^{-1} is defined to return an equivalent numerical value β ($\beta\in$ [0,1]) from 2-tuple linguistic variable (s_i, α_i). According to the symbolic translation, an equivalent numerical value β is obtained as $\Delta^{-1}(s_i,\alpha_i)=(i/g)+\alpha_i=\beta$ [1]. Let $x = \{(r_1, \alpha_1),\ldots,(r_n, \alpha_n)\}$ be a 2-tuple linguistic variable set. The arithmetic mean $\bar{x}$ is computed as $\bar{X}=\Delta\left(\frac{1}{n}\sum_{i=1}^{n}\Delta^{-1}(r_i,\alpha_i)\right)=(s_m,\alpha_m)$ [7]. In general, decision makers would use the different

Table 1 Different types of linguistic variables

Linguistic variable	Figure
Very Poor (s_0^5) , Poor (s_1^5) , Fair (s_2^5) , Good (s_3^5) , Very Good (s_4^5)	Fig. 1(type 1)
Very Poor (s_0^7) , Poor (s_1^7) , Medium Poor (s_2^7) , Fair (s_3^7) , Medium Good (s_4^7) , Good (s_5^7) , Very Good (s_6^7)	Fig. 2(type 2)

2-tuple linguistic variables based on their knowledge or experiences to express their opinions [5]. For example, the different types of linguistic variables show as Table 1. Each 2-tuple linguistic variable can be represented as a triangle fuzzy number [3].

3 The Maximum Deviation Method

If the performance values among all the alternatives are little differences with respect to criterion, it shows that the criterion plays a less important role in the decision-making procedure. Contrariwise, if one criterion makes the performance values among all the alternatives have obvious differences, such a criterion plays a more important role in choosing the best alternative. According to the concept, the maximizing deviation method [15] is applied to calculate the weight of each criterion.

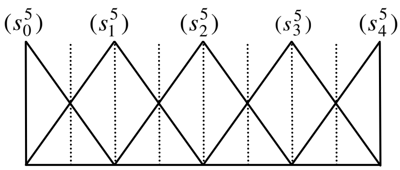

Fig. 1 Membership functions of linguistic variables at type 1 (t=1)

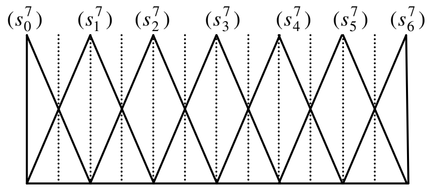

Fig. 2 Membership functions of linguistic variables at type 2 (t=2)

Assume that an expert group has K experts, and the fuzzy rating of alternative A_i respect to criterion C_j of each expert E_k (k = 1,2,...,K) can be represented as a 2-tuple linguistic variable $\tilde{x}_{ij}^k = \left(s_{ij}^k, \alpha_{ij}^k\right)$. The deviation method is used to compute the differences of the performance values of each alternative with respect to all criteria. For the expert E_k and the criterion C_j , the deviation of alternative A_i to all the other alternatives can be defined as $H_{ij}^k(w) = \sum_{l=1}^{n} \left(\Delta^{-1}\left(\tilde{x}_{ij}^k\right) - \Delta^{-1}\left(\tilde{x}_{lj}^k\right)\right)^2 w_j$ and

$$H_j^k(w) = \sum_{i=1}^{n} \sum_{l=1}^{n} \left(\Delta^{-1}\left(\tilde{x}_{ij}^k\right) - \Delta^{-1}\left(\tilde{x}_{lj}^k\right)\right)^2 w_j .$$

The $H_j^k(w)$ represents the deviation value of all alternatives to other alternatives with respect to the criterion C_j by the expert E_k . Based on the maximum deviation method, a non-linear programming model can be constructed as [15]

$$\max \quad H(w)=\sum_{k=1}^{K}\lambda_k \sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{l=1}^{n}\left(\Delta^{-1}\left(\tilde{x}_{ij}^{k}\right)-\Delta^{-1}\left(\tilde{x}_{lj}^{k}\right)\right)^2 w_j \quad s.t. \quad w_j \geq 0, \quad \sum_{j=1}^{m} w_j^2 = 1 \tag{1}$$

where λ_k the represents the weight of expert E_k .The weight (w_j) of criterion C_j can be calculated as [15]

$$w_j^* = \frac{\sum_{k=1}^{K}\lambda_k \sum_{i=1}^{n}\sum_{l=1}^{n}\left(\Delta^{-1}\left(\tilde{x}_{ij}^{k}\right)-\Delta^{-1}\left(\tilde{x}_{lj}^{k}\right)\right)^2}{\sum_{j=1}^{m}\sum_{k=1}^{K}\lambda_k \sum_{i=1}^{n}\sum_{l=1}^{n}\left(\Delta^{-1}\left(\tilde{x}_{ij}^{k}\right)-\Delta^{-1}\left(\tilde{x}_{lj}^{k}\right)\right)^2} \tag{2}$$

4 Proposed Method

In general, stock selection problem may be described as a multiple criteria decision making (MCDM) problem with multiple experts. The fuzzy rating of each expert E_k (k = 1,2,...,K) can be represented as a 2-tuple linguistic variable $\tilde{x}_{ij}^{k}=\left(s_{ij}^{k},\alpha_{ij}^{k}\right)$. The aggregated linguistic ratings ($\tilde{x}_{ij}$) of stocks with respect to each criterion can be calculated as $\tilde{x}_{ij} = \Delta\ (\frac{1}{n}\sum_{k=1}^{K}\Delta^{-1}(S_{ij}^{k},\alpha_{ij}^{k})) = (S_{ij},\alpha_{ij})$. A linguistic decision matrix can be concisely expressed as $\tilde{D}=[\tilde{x}_{ij}]_{m\times n}$ with $\tilde{x}_{ij}=(S_{ij},\alpha_{ij})$. According to the ELECTRE method, the concordance index $C_j(S_i,S_l)$ is calculated for each pair of stocks (S_i , S_l) with respect to each criterion as

$$C_j(S_i,S_l)=\begin{cases} 1 & ,\Delta^{-1}(\tilde{x}_{ij})\geq\Delta^{-1}(\tilde{x}_{lj})-q_j \\ \dfrac{\Delta^{-1}(\tilde{x}_{ij})-\Delta^{-1}(\tilde{x}_{lj})+p_j}{p_j-q_j} & ,\Delta^{-1}(\tilde{x}_{lj})-q_j\geq\Delta^{-1}(\tilde{x}_{ij})\geq\Delta^{-1}(\tilde{x}_{lj})-p_j \\ 0 & ,\Delta^{-1}(\tilde{x}_{ij})\leq\Delta^{-1}(\tilde{x}_{lj})-p_j \end{cases} \tag{3}$$

where q_j and p_j are indifference and preference threshold values for criterion C_j, $p_j > q_j$ .The discordance index $D_j(S_i,S_l)$ is calculated for each pair of stocks with respect to each criterion as

$$D_j(S_i,S_l)=\begin{cases} 1 & ,\Delta^{-1}(\tilde{x}_{ij})\leq\Delta^{-1}(\tilde{x}_{lj})-v_j \\ \dfrac{\Delta^{-1}(\tilde{x}_{lj})-p_j-\Delta^{-1}(\tilde{x}_{ij})}{v_j-p_j} & ,\Delta^{-1}(\tilde{x}_{lj})-p_j\geq\Delta^{-1}(\tilde{x}_{ij})\geq\Delta^{-1}(\tilde{x}_{lj})-v_j \\ 0 & ,\Delta^{-1}(\tilde{x}_{ij})\geq\Delta^{-1}(\tilde{x}_{lj})-p_j \end{cases} \tag{4}$$

where v_j is the veto threshold for criterion C_j, $v_j > p_j$.

Calculate the overall concordance index $C(S_i,S_l)$ as $C(S_i,S_l)=\sum_{j=1}^{n}W_j^* C_j(S_i,S_l)$. The credibility matrix $S(S_i,S_l)$ of each pair of the stocks is calculated as

$$S(S_i,S_l)=\begin{cases} C(S_i,S_l), \ \mathit{if} \ \ D_j(S_i,S_l)\le C(S_i,S_l) \ \ \forall j \\ C(S_i,S_l) \prod\limits_{j\in J(S_i,S_l)} \dfrac{1-D_j(S_i,S_l)}{1-C(S_i,S_l)}, \ \mathit{otherwise} \end{cases} \tag{5}$$

where $J(S_i,S_l)$ is the set of criteria for which $D_j(S_i,S_l)>C(S_i,S_l)$.

The concordance credibility and discordance credibility degrees are defined as $\phi^+(S_i)=\sum\limits_{S_l\in S} S(S_i,S_l)$ and $\phi^-(S_i)=\sum\limits_{S_l\in S} S(S_l,S_s)$ [9].

Then, the net credibility degree is defined as $\phi(S_i)=\phi^+(S_i)-\phi^-(S_i)$. If the net credibility degree is higher, then represents a higher attractiveness of stock. In order to determine the ranking order and the investment ratio, the outranking index of stock S_i can be defined as $OTI(S_i)=((\phi(S_i)/(m-1))+1)/2$. A portfolio set for investment can be determined based on threshold value β as $\Omega=\{S_i \mid OTI(S_i)\ge\beta\}$. Finally, investment ratio of stocks can be calculated as $P(S_i)=\left(OTI(S_i)/\left(\sum\limits_{S_i\in\Omega} OTI(S_i)\right)\right), S_i\in\Omega$

and $P(S_i)=0 \ ,S_i\notin\Omega$.

5 Numerical Example

In this paper, the data of Tiryaki and Ahlatcioglu [13] are used to implement in order to demonstrate the practicability of the proposed method. In their paper [13], three experts make the portfolio selection decision. They consider six criteria and 22 stocks. All of the experts use linguistic variables with 7 scale of linguistic term set to express their opinions (see Table 1). According to the proposed method, the computational procedures of the problem are summarized as follows.

Step 1. Each expert expresses his opinion about the performance of each stock refer to the data in [13].

Step 2. Assume that the importance of each expert is equal. We use maximizing deviation method to compute the weight of each criterion as 0.145, 0.230, 0.168, 0.098, 0.117, and 0.242.

Step 3. Calculate the aggregated linguistic ratings of each stock are shown in Table 2.

Step 4. The indifference threshold, preference threshold, and veto threshold values of each criterion can be determined in accordance with linguistic term set as $q_j=1/6,\ p_j=2/6,\ v_j=3/6,\ j=1,2,...6.$

Step 5. Calculate the concordance credibility degree, the discordance credibility degree, the net credibility degree, and the outranking index as Table 3.

Step 6. The investment ratio of each stock in accordance with different thresholds is shown as Table 4. For example, the portfolio set is $\{S_1,S_2,S_7,S_8,S_9,S_{10}\}$ in accordance with $\beta=0.7$. Compared with the method [13], the advantage of our method is that it provides a more flexible and reasonable tool to select the stock portfolio and investment ratio of each stock.

Table 2 The aggregated linguistic ratings of each stock

Stock	C_1	C_2	C_3	C_4	C_5	C_6	Stock	C_1	C_2	C_3	C_4	C_5	C_6
s_1	0.833	0.778	0.889	0.667	0.500	0.778	s_{12}	0.722	0.167	0.389	0.500	0.778	0.444
s_2	0.611	0.611	0.722	0.667	0.444	0.778	s_{13}	0.389	0.500	0.111	0.278	0.333	0.389
s_3	0.556	0.778	0.444	0.389	0.667	0.444	s_{14}	0.389	0.222	0.389	0.556	0.500	0.444
s_4	0.556	0.056	0.500	0.500	0.444	0.056	s_{15}	0.278	0.222	0.222	0.556	0.222	0.389
s_5	0.556	0.222	0.611	0.444	0.500	0.278	s_{16}	0.333	0.111	0.056	0.222	0.222	0.389
s_6	0.444	0.611	0.722	0.667	0.389	0.500	s_{17}	0.500	0.222	0.500	0.556	0.444	0.389
s_7	0.778	0.611	0.611	0.667	0.611	0.778	s_{18}	0.556	0.778	0.222	0.556	0.278	0.722
s_8	0.944	0.611	0.778	0.778	0.444	0.833	s_{19}	0.389	0.333	0.667	0.611	0.333	0.500
s_9	0.611	0.611	0.722	0.500	0.444	0.556	s_{20}	0.500	0.722	0.333	0.611	0.333	0.333
s_{10}	0.722	0.667	0.611	0.500	0.722	0.722	s_{21}	0.722	0.444	0.444	0.556	0.722	0.556
s_{11}	0.778	0.611	0.389	0.444	0.611	0.389	s_{22}	0.500	0.167	0.389	0.389	0.333	0.333

Table 3 The concordance credibility degree, the discordance credibility degree, the net credibility degree and the outranking index

Stock	$\phi^+(s_i)$	$\phi^-(s_i)$	$\phi\ (s_i)$	OTI	Stock	$\phi^+(s_i)$	$\phi^-(s_i)$	$\phi\ (s_i)$	OTI
s_1	21.845	6.422	15.423	0.867	s_{12}	11.420	14.059	-2.639	0.437
s_2	21.495	9.854	11.641	0.777	s_{13}	8.538	19.615	-11.077	0.236
s_3	18.688	11.790	6.898	0.664	s_{14}	11.717	20.868	-9.150	0.282
s_4	6.341	20.400	-14.059	0.165	s_{15}	6.989	21.595	-14.605	0.152
s_5	11.790	18.550	-6.761	0.339	s_{16}	4.362	21.758	-17.396	0.086
s_6	18.629	12.606	6.023	0.643	s_{17}	12.431	20.189	-7.758	0.315
s_7	21.888	10.012	11.876	0.783	s_{18}	11.809	11.559	0.250	0.506
s_8	21.689	6.224	15.465	0.868	s_{19}	14.034	16.882	-2.849	0.432
s_9	21.026	12.532	8.495	0.702	s_{20}	14.272	13.097	1.176	0.528
s_{10}	21.774	9.866	11.909	0.784	s_{21}	19.519	14.628	4.891	0.616
s_{11}	17.850	13.949	3.901	0.593	s_{22}	9.858	21.510	-11.653	0.223

Table 4 Investment ratio with different threshold and Comparison with Tiryaki's result

Rank	1	2	3	4	5	6
F.Tiryaki's result	S_1 ,0.216	S_8 ,0.196	S_2 ,0.157	S_7 ,0.157	S_{10} ,0.157	S_9 , 0.117
$\beta = 0.6$	(S_8 ,0.129),(S_1 ,0.129),(S_{10} ,0.117),(S_7 ,0.117),(S_2 ,0.116),(S_9 ,0.105) (S_3 ,0.099),(S_6 ,0.096),(S_{21} ,0.092)					
$\beta = 0.7$	S_8 ,0.182	S_1 ,0.181	S_{10} ,0.164	S_7 ,0.164	S_2 ,0.163	S_9 ,0.147
$\beta = 0.8$	S_8 ,0.500	S_1 ,0.500				

6 Conclusions

In general, the problem of stock selection and evaluation adhere to uncertain and imprecise data, and fuzzy set theory is adequate to deal with it. In this proposed model, 2-tuple linguistic variables are applied to express the subjective judgment of each expert. Expert can easily express his opinion by different types of 2-tuple linguistic variables. According to experts' opinions, the weight of each criterion can be determined by maximizing deviation method. The linguistic ELECTRE method is used to derive the credibility matrix and calculate the net credibility degree of each stock. Based on the outranking index and selection threshold, we can easily obtain portfolio set and decide the investment ratio of each stock. In the future, a decision support system will be developed based on the proposed method for dealing with the stock selection problems.

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