Toward Global Minimum through Combined Local Minima

Ho Yub Jung, Kyoung Mu Lee, and Sang Uk Lee

Department of EECS, ASRI, Seoul National University, 151-742, Seoul, Korea
hoyub@diehard.snu.ac.kr, kyoungmu@snu.ac.kr, sanguk@ipl.snu.ac.kr

Abstract. There are many local and greedy algorithms for energy minimization over Markov Random Field (MRF) such as iterated condition mode (ICM) and various gradient descent methods. Local minima solutions can be obtained with simple implementations and usually require smaller computational time than global algorithms. Also, methods such as ICM can be readily implemented in a various difficult problems that may involve larger than pairwise clique MRFs. However, their short comings are evident in comparison to newer methods such as graph cut and belief propagation. The local minimum depends largely on the initial state, which is the fundamental problem of its kind. In this paper, disadvantages of local minima techniques are addressed by proposing ways to combine multiple local solutions. First, multiple ICM solutions are obtained using different initial states. The solutions are combined with random partitioning based greedy algorithm called Combined Local Minima (CLM). There are numerous MRF problems that cannot be efficiently implemented with graph cut and belief propagation, and so by introducing ways to effectively combine local solutions, we present a method to dramatically improve many of the pre-existing local minima algorithms. The proposed approach is shown to be effective on pairwise stereo MRF compared with graph cut and sequential tree re-weighted belief propagation (TRW-S). Additionally, we tested our algorithm against belief propagation (BP) over randomly generated 30×30 MRF with 2×2 clique potentials, and we experimentally illustrate CLM's advantage over message passing algorithms in computation complexity and performance.

1 Introduction

Recently, there are great interests in energy minimization methods over MRF. The pairwise MRF is currently the most prominent MRF which became most frequent subject of study in computer vision. Also, in the forefront, there is a movement toward 2×2 and higher clique potentials for de-noising and segmentation problems [1,2,3,4,5,6]. They claim better performance through larger clique potentials that can give more specified constraints.

However, the conventional belief propagation which has been so effective in the pairwise MRF, is shown to have severe computational burden over large cliques. In a factor graph belief propagation, the computational load increases exponentially as the size of clique increases, although for the linear constraint

D. Forsyth, P. Torr, and A. Zisserman (Eds.): ECCV 2008, Part IV, LNCS 5305, pp. 298–311, 2008.

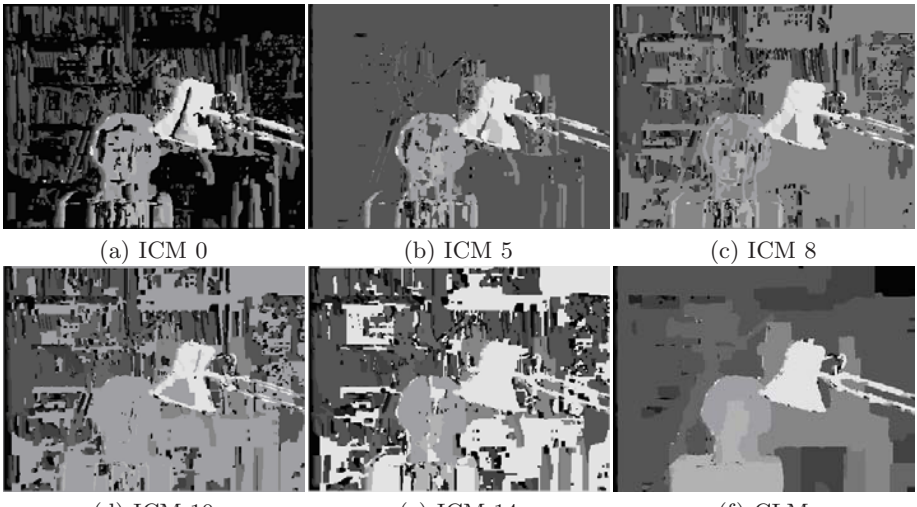
MRFs, the calculation can be reduced to time linear [6,3]. Graph cut based methods are also introduced for energy functions with global constraints and larger clique potentials with pair-wise elements [5,4]. However, these methods are targeted toward a specific category of energy functions and the applicability limitations are high.

A practical and proven method for minimizing even the higher order MRFs is simulated annealing. Gibbs sampler, generalized Gibbs sampler, data-driven Markov chain Monte Carlo and Swendsen-Wang cut were respectively applied to de-noising, texture synthesizing and segmentation problems that involved large clique potentials [7,8,9,10]. However, simulated annealing is considered impractically slow compared to belief propagation and graph cut even in pairwise MRFs [10,11]. More recently, simulated annealing has been modified by localized temperature scheduling and additional window scheduling to increase its effectiveness [12,13].

Another approach that is often being ignored is the greedy local minimum algorithms. With the introductions of theoretically sound graph cut and belief propagation over pairwise MRF, older methods such as ICM [14] and various gradient descent methods are often disregarded as an under-performing alternatives [11]. However, methods like ICM and other local minimum algorithms do not have any constraints over the size of cliques in MRF. Gradient descent method was readily implemented over 5×5 and 3×2 clique potential in the de-noising problem [2,1]. Texture synthesis and segmentation problems were model by high order MRF and the energy was minimized using ICM [15]. Thus, when considering both the computational time and performance, local greedy methods that depend largely on the initial states are still viable in many of the high order MRFs.

In this paper we propose a new algorithm to effectively combine these local minima to obtain a solution that is closer to the global minimum state. First, local solutions are calculated from various initial states. Then, they are combined by random partitioning process such that the energy is minimized. The proposed Combined Local Minima (CLM) approach is very simple but it can effectively find lower energy state than graph cut and belief propagation. CLM is tested on the pairwise stereo MRFs provided by [16,17,18,19,20], and it is shown that the performance can be better than graph cut [21] and sequential tree reweighted belief propagation (TRWS) [22]. We also performed tests over randomly generated 2×2 clique MRFs, and showed that the proposed method converges not only faster but finds lower energy state than belief propagation. However, the biggest advantage of the proposed algorithm is that it can bring further improvement over various local minima algorithms that are applicable to general energy functions.

Section 2 will review ICM algorithm. Section 3 presents proposed CLM. In the experiment section, CLM is shown to be competative over pairwise MRF and superior over 2×2 clique MRF. The paper will close with conclusion and possible future work.



(a) ICM 0 (b) ICM 5 (c) ICM 8

(d) ICM 10 (e) ICM 14 (f) CLM

Fig. 1. (a) to (e) show ICM solutions from different initial states. Homogeneous states of disparity 0, 5, 8, 10, and 14 are respectively used as the initial states of (a), (b), (c), (d), and (e). Combined local minima algorithm effectively combines these ICM solutions into an lower energy state (f).

2 Iterated Conditional Mode (ICM)

For obtaining the local minima states, there are various different methods to choose from. However, in this section, iterated conditional mode will be reviewed for discrete MRF.

MRF consists of a set of nodes $V = \{v_1, v_2, ...v_N\}$. For each nodes $v \in V$, a label l can be assigned from a set L, producing a state x. The number of nodes in V is denoted as N, and the number of labels in L is Q. In a discrete labelling problem, the number of possible states will be Q^N. The energy function $\varphi(x)$ is a function of N dimension vector $x = (x_1, x_2, x_3, ..., x_N)$.

ICM is a simple algorithm that determines the minimum energy at each vertex $v \in V$. For a non-convex energies (such as pairwise energy function), ICM produces a local minima solutions that depends upon the starting state. Following pseudo code minimizes the energy function $\varphi(x)$ in a labelling problem with nodes $v_i \in \{v_1, v_2, ...v_N\}$ and labels $l_j \in L = \{l_1, l_2, ...l_Q\}$.

Iterated Conditional Modes: ICM

1. Determine the initial state x
2. Repeat until no further energy minimization.
3. For $i = 1$ to $i = N$
4. For $j = 1$ to $j = Q$
5. Assign l_j to v_i
   if $\varphi(x_1, ..., x_i = l_j, .., x_N) < \varphi(x_1, ..., x_N)$.

The problem of choosing the right initial state is the big disadvantage of ICM. Figure 1 (a) to (e) show the ICM solutions for Tsukuba stereo MRF. The solutions in Figure 1 are obtained with different initial homogeneous states. Even though the energy minimization cannot be low as graph cut or belief propagation, the computational time is very small because the comparative inequality of step 5 can be evaluated in $O(1)$ for most of the energy functions, including pairwise functions. ICM guarantees to converge but the performance is very poor as shown in figure 1. Also, because of its simplicity, ICM can be applied to high order MRF with larger cliques where graph cut and BP are having problems with.

3 Combined Local Minima

The simplest way to overcome the initial state dilemma of greedy algorithm is to take multiple initial states. Among the multiple local minima obtained from ICM, the lowest energy state can be chosen as the final solution. However, this approach is problematic for MRF with very large dimensions, and obtaining comparable solutions to graph cut and belief propagation is near impossible. Thus, greedy algorithms are not often used for the MRF problems. In this section, however, we assume that each local minima solution has a subset that is a match to a subset of global minima state. We believe that a random partition combination of local minima solution can be used to obtain energy level closer to global minima.

3.1 Combined Space

In a typical labelling problems such as segmentation and stereo, the nodes are presented by the pixel positions. The number of all possible states for such set up will be Q^N. However, the combination of local minima will produce a smaller space. In this section, the general notations will be defined for the proposed algorithm.

The solution space for N number of nodes and set of labels $L = \{l_1, l_2, ..., l_Q\}$ is $\Omega = \{L \times L \times, ..., \times L\}$, where Ω is N dimension space. However, we are proposing to minimize energy over reduced solution space that is obtained from the combinations of local solutions. First, k number of local minima set $\{s_1, s_2, ..., s_k\}$ are found using ICM such that each s_i is N dimension vector having following labels.

$$s_i = \left(l^1_{s_i}, l^2_{s_i}, l^3_{s_i}, ..., l^N_{s_i}\right). \tag{1}$$

$l^j_{s_i}$ is the label value for $v_j \in V$ node of s_i local minima state. $\Omega_S \subseteq \Omega$ is the new solution space composed of the new sets of labels $L_j \subseteq L$.

$$\Omega_S = (L_1 \times L_2 \times L_3 \times, ..., \times L_N). \tag{2}$$

L_j is obtained from the set of the local solutions such that $L_j = \{l^j_{s_1}, l^j_{s_2}, l^j_{s_3}, ..., l^j_{s_k}\}$.

The search for the minimum energy state will be over Ω_S, although there is no guarantee that the global minima is in the reduced space. Choosing the right combinations of local minima for CLM will admittedly be heuristic for

each problem. More on the choices of local minima will be discussed in the later sections. However, when the sufficient number and variety of local minima are present in the proposed CLM, the solution space will be the original Ω.

3.2 Combined Local Minima

The proposed combinatorial algorithm for local minima is very simple and intuitive, however, it is shown to be very effective over traditional pairwise MRF and randomly generated 2×2 clique MRF. For the pairwise MRF, QPBO algorithm can effectively combine two minima solutions together [23]. However, QPBO algorithm is viable only for pairwise MRF, thus we rely on random partitioning technique which is simpler and can be applicable to higher order MRF.

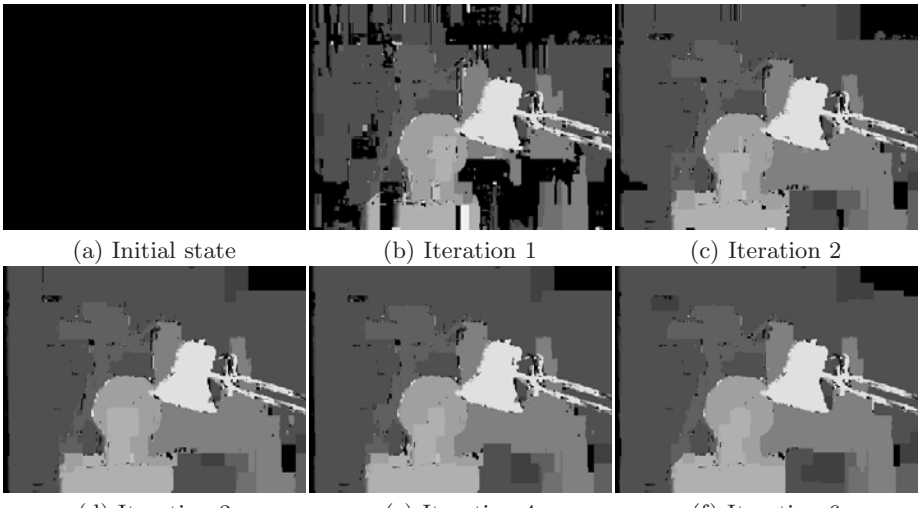
We propose following algorithm to minimize energy from a set of local minima. CLM partitions both current state and local minima states and replaces a part of current state to one of the local minima states' such that energy is reduced for current state. It is a basic greedy algorithm over partitioned states.

Combined Local Minima: CLM

1. Given k number of local minima states from k different initial states, $s_1 = \left(l^1_{s_1}, l^2_{s_1}, l^3_{s_1}, ..., l^N_{s_1}\right)$, $s_2 = \left(l^1_{s_2}, l^2_{s_2}, l^3_{s_2}, ..., l^N_{s_2}\right)$, ...,$s_k = \left(l^1_{s_k}, l^2_{s_k}, l^3_{s_k}, ..., l^N_{s_k}\right)$. and the current state $x = \left(l^1_x, l^2_x, l^3_x, ..., l^N_x\right)$, repeat for specified number of iterations.
2. Randomly partition both the current state x and local minima states $s_1, s_2, ..., s_k$ into same m number of partitions such that $x = \left(V^1_x, V^2_x, V^3_x, ..., V^m_x\right)$, $s_1 = \left(V^1_{s_1}, V^2_{s_1}, V^3_{s_1}, ..., V^m_{s_1}\right)$, $s_2 = \left(V^1_{s_2}, V^2_{s_2}, V^3_{s_2}, ..., V^m_{s_2}\right)$,...,$s_k = \left(V^1_{s_k}, V^2_{s_k}, V^3_{s_k}, ..., V^m_{s_k}\right)$.
3. Repeat for $i = 1$ to $i = m$.
4. Make $k+1$ proposal states $\{x_0, x_1, x_2, ..., x_k\}$ in combinations of current state x and $s_1, ..., s_k$ such that V^i_x vector partition of x is replaced by the V^i_s of local minima states. See below. $x_0 = x = \left(V^1_x, V^2_x, V^3_x, ..., V^m_x\right)$, $x_1 = \left(V^1_x, V^2_x, ..., V^i_{s_1}, ..., V^m_x\right)$, $x_2 = \left(V^1_x, V^2_x, ..., V^i_{s_2}, ..., V^m_x\right)$,..., $x_k = \left(V^1_x, V^2_x, ..., V^i_{s_k}, ..., V^m_x\right)$. Among set $S = \{x_0, x_1, ..., x_k\}$, take the lowest energy state as the current state.

The computational complexity of CLM depends largely on the complexity of evaluating $\varphi(x_i)$. If $\varphi(x_i)$ is needed to be calculated in $O(N)$, ICM's complexity will be $O(kmN)$. If m is randomly chosen, the worst case would be for $m = N$, and the time complexity will be $O(kN^2)$ per iteration. However, if the maximum clique size is small compared to MRF size, both the worst and best complexity will be $O(kN)$ because only V^i and areas around V^i are needed to be evaluated to find the lowest energy among $S = \{x_0, x_1, ..., x_k\}$. Also, the complexity can still be lowered using various computation techniques such as integral image method [24].

The proposed algorithm is greedy and guarantees that the energy does not increase for each iteration. Figure 2 shows the iterative results of the proposed



(a) Initial state (b) Iteration 1 (c) Iteration 2

(d) Iteration 3 (e) Iteration 4 (f) Iteration 6

Fig. 2. (a) shows the initial state of CLM. (b), (c), (d), (e), and (f) show respectively the first, second, third, fourth, and sixth iterations of combined local minima algorithm.

CLM over Tsukuba stereo pair MRF. $k = 16$ number of local minima were used. Few of local minima are shown in Figure 1. With only a small number of iterations, CLM can output energy minimization result far superior to ICM method, and with enough iterations it can be effective as the message passing and graph cut algorithms.

However, there are two heuristics that must be resolved for CLM. First, it is unclear how current state x and $\{s_1, s_2, ..., s_k\}$ should be randomly partitioned in step 2 of the algorithm. Second, the choice of local minima and the value of k are subject to question. These two issues are important to the performance of the proposed algorithm and the basic guidelines are provided in next subsections.

3.3 Obtaining k Local Minima

It is intuitive to assert that if large number of local minima is used for CLM, the obtained energy will be lower. However, for the price of lower energy, more computational time and memory are required. The right tradeoff between computation resources and desired energy level is essential to CLM. This is both advantage and disadvantage of proposed method because by using CLM, you can control the level of performance and computing resources.

Another factor that contributes to the performance of CLM is the variety of local minima. For example, if all the local minima solutions are same, the energy will not be lowered no matter how many times they are combined. Usually, variety of initial states for ICM result in the variety of minima solutions. However, some heuristics may be needed for obtaining different local minima. We have empirically developed few precept for both of these issues.

Thus, in order to have different local minima states, ICM with different homogeneous initial states were used. See experimental section and Figure 4 and 5. In both of the comparison tests, the number of local minima are set to Q, the number of labels. $\{s_1, ..., s_Q\}$ are obtained from ICM with homogeneous initial state, respectively having labels $l_1, l_2, ..., l_Q$. In both stereo MRF and randomly generated MRF, such initial states resulted in the energy minimization comparable to message passing algorithms. Thus, the rule of thumb is to use Q number of local minima derived from the respective homogeneous initial states.

However, by increasing the number of local minima as shown in Figure 5, much lower energy can be achieved with incremental addition to computation time. In Figure 5, CLM200 minimizes energy using total of 200 local minima composed of Q homogeneous initial states and $200 - Q$ number of ICM solutions obtained from random initial states. CLM200 achieves much lower energy than belief propagation. Although, random initial states are used here, more adaptive initial states can also be applied for different problems.

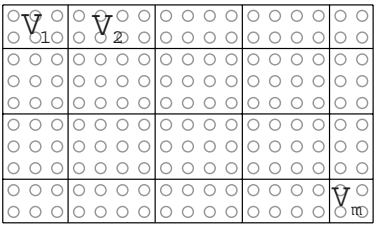
3.4 Random Partition

In this paper, we use rectangular partition method for step 2 of CLM algorithm, much like window annealing of [13]. See Figure 3 (a). 4 integers are randomly chosen, and MRF can be partitioned accordingly. Such method is used because of simplicity of computation and the fact that it can commodate the square lattice structure of digital images. Furthermore, by having rectangular partitions, the energy value of state can be obtained very fast using integral image technique [24], which was used for the stereo pair experiment. However, the integral image technique is not essential to the CLM. In Figure 5, integral image technique is not used during the operations of CLM, and it has superior performance over belief propagation.

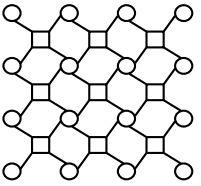
For MRFs with random structure, rectangular partition can not be applied. A possible random partitioning algorithm that can be used is the one that was applied in Swendsen-Wang cut algorithm [10]. In Swendsen-Wang cut, the edges between the nodes are probabilistically cut, and the connected nodes after the random cut would make a single cluster. This method was not used in the experimentation section because of needless complexity over square lattice MRF. Again, the partition method can be specified to each problem at hand. However, V^i should be no larger than N obviously, and there should be a positive probability that size of V^i could be 1, so that the optimization can be occur over single nodes.

4 Experiments

In order to show the effectiveness of the proposed CLM, we compared it's performance with graph cut and TRW-S over pairwise stereo MRF. Additionally, window annealing (WA) [13] results are included in the test. Pairwise stereo MRF is known to be effectively optimized by alpha expansion graph cut (GC) and TRW-S [21,22], but very ill posed for greedy algorithms such as ICM. The experiments were performed over stereo pairs provided by [16,17,18,20,19].



(a) Rectangular Partition



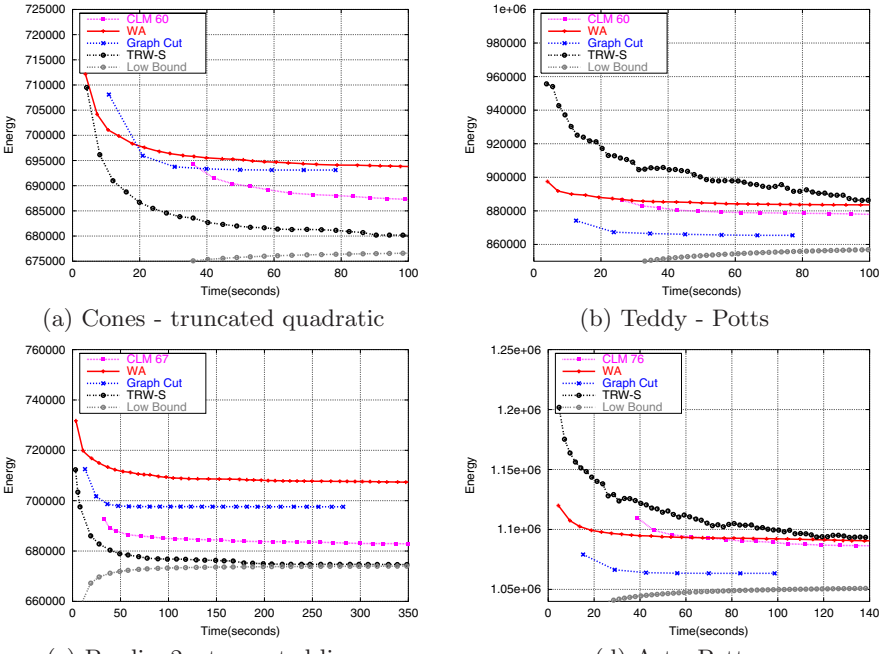
(b) 2×2 clique MRF

Fig. 3. (a) shows an example of rectangular partitioning of a square lattice MRF. A state can be partitioned into rectangular clusters $V = \{V^1, V^2, ..., V^m\}$. Such partition method allows simple calculation of energy function by integral image technique which was used for the pairwise MRF test. In this Figure (b), 4×4 MRF with 2×2 clique potentials is depicted. The circle nodes are $v \in V$. The square factor nodes define the cliques of MRF by connected the neighbors, N_g. MRF is built by assigning random clique potentials from an uniform distribution. In the randomly generated 2×2 clique MRF, integral image technique is not used for computational speed up.

Also recently, larger than pairwise clique models are often proposed for vision problems. Gradient descent and belief propagation are used over 2×2 and larger clique MRF to attack such problems as de-noising, and shape from shading [6,1,2,3]. Thus, we tested our algorithm over randomly generated MRF with 2×2 clique potentials, see Figure 5 (a). Alpha expansion algorithms cannot deal with randomly generated larger than pairwise MRF, and it was excluded from the test. CLM reaches a lower energy faster than belief propagation (BP) and WA methods. The computational complexity of proposed method is $O(kN)$, allowing CLM to be a practical minimization scheme over large clique MRFs. All computations are done over a 3.4GHz desktop.

4.1 Pair-Wise Stereo MRF

Pairwise and sumodular MRF is most common MRF used in computer vision. Also, it has been the subject to many comparative tests. Particularly, the stereo MRF has been an frequent in comparison tests of energy minimization methods [11,25,22]. However, the performance differences between two state of art methods, graph cut and message passing algorithms, are still not clear when the computational time is an issue. Although, TRW-S may eventually find lower energy than graph cut, it can take many more iterations to do so. In some cases, TRW-S is faster and finds lower energy than graph cut. In this test, we tried to present energy functions that are fair to both graph cut and TRW-S. As shown in Figure 4, for Cones and Bowling2 MRF, TRW-S clearly outperforms the graph cut. Otherhand, for Teddy and Art MRF, the graph cut finds lower energy much faster. The performance of each methods seem to depend largely upon the strength of discontinuity costs. Simulated annealing, otherhand, depends large on the temperature scheduling. Awhile WA is competitive with previous methods in speed, usually it could not find lower energy. Although it is possible to tweak



(a) Cones - truncated quadratic (b) Teddy - Potts

(c) Bowling2 - truncated linear (d) Art - Potts

Fig. 4. (a)Cones uses truncated quadratic discontinuity cost. (c) Bowling2 is the result for truncated linear discontinuity cost. (b) Teddy and (d) Art use Potts discontinuity cost. CLM 60 means 60 local minima are used in CLM algorithm. The CLM's performance is shown to be in-between TRW-S and GC. The performance difference to state-of-art methods are very small, however, CLM performance does not seem to strongly vary according to the discontinuity model apposed to TRW-S and graph cut.

the annealing scheduling for lower minimization, we kept the same temperature and window scheduling of [13].

For the energy function, gray image Birchfield and Tomasi matching costs [26] and Potts, truncated linear and truncated quadratic discontinuity cost are used.

$$\varphi(x) = \sum_{p \in V} D(p) + \sum_{(p,q) \in \mathrm{N_g}} V(p,q). \tag{3}$$

$D(p)$ is a pixel-wise matching cost between left and right image. $V(p,q)$ is pair-wise discontinuity costs. The implementations of graph cut and TRW-S by [11,21,27,28,29,22] are used in this experiment.

For the implementation of CLM, Q number of local minima ICM solutions are obtained from following set of initial states $\{(0,0,...,0),(1,1,...,1),...,(Q-1,Q-1,...,Q-1)\}$. As mentioned before, a rule of thumb seems to be Q number of local minima with homogeneous initial states, especially if the MRF is known to have smoothness constraint. For the state partition technique of step 2 of CLM, a simple rectangular partitioning method is used, see Figure 3.

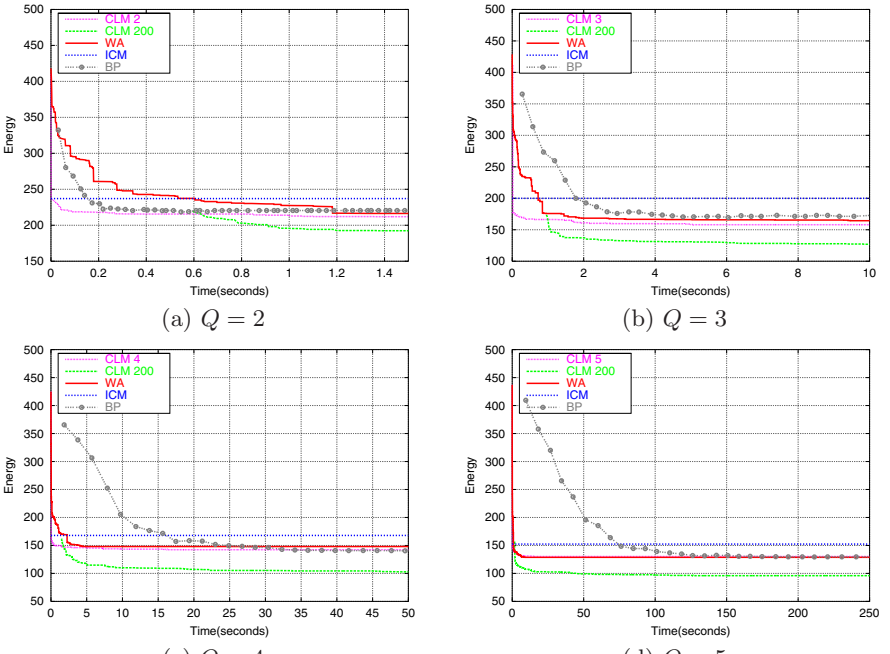


Fig. 5. Energy versus time results of max product BP, ICM, WA, and CLM over 30×30 randomly generated MRF. Figure (a), (b), (c), and (d) respectively have label size $Q = 2$, $Q = 3$, $Q = 4$, and $Q = 5$. CLM using $k = Q$ and $k = 200$ number of local minima are performed for each random MRF. The increase in the local minima allows lower energy state to be achieved in exchange for computation time and memory. However, such price is very small compared to the computation time of BP.

Figure 4 shows energy vs time graph results using Potts, truncated linear, truncated quadratic discontinuity model. Qualitatively, there is a very small difference between TRW-S, graph cut, WA, and CLM, see Figure 6. However, the energy versus time graphs show more edifying comparison. The first iteration of the CLM takes much longer time than the other iterations because all the local solutions are needed to be computed. Overall performance of the proposed CLM stands in the middle of graph cut and TRW-S. However, compared with window annealing, CLM outperforms it everywhere except for the initial calculations.

4.2 Randomly Generated 2×2 Clique MRF

However, the biggest advantage of proposed CLM is that the computational complexity does not increase exponentially. Belief propagation based methods, however, the time complexity of message calculation goes up exponentially as the number of clique size increases [6]. In this section, the proposed CLM is tested over randomly generated 2×2 clique MRF. Below equation describes the energy function as sum of clique potentials.

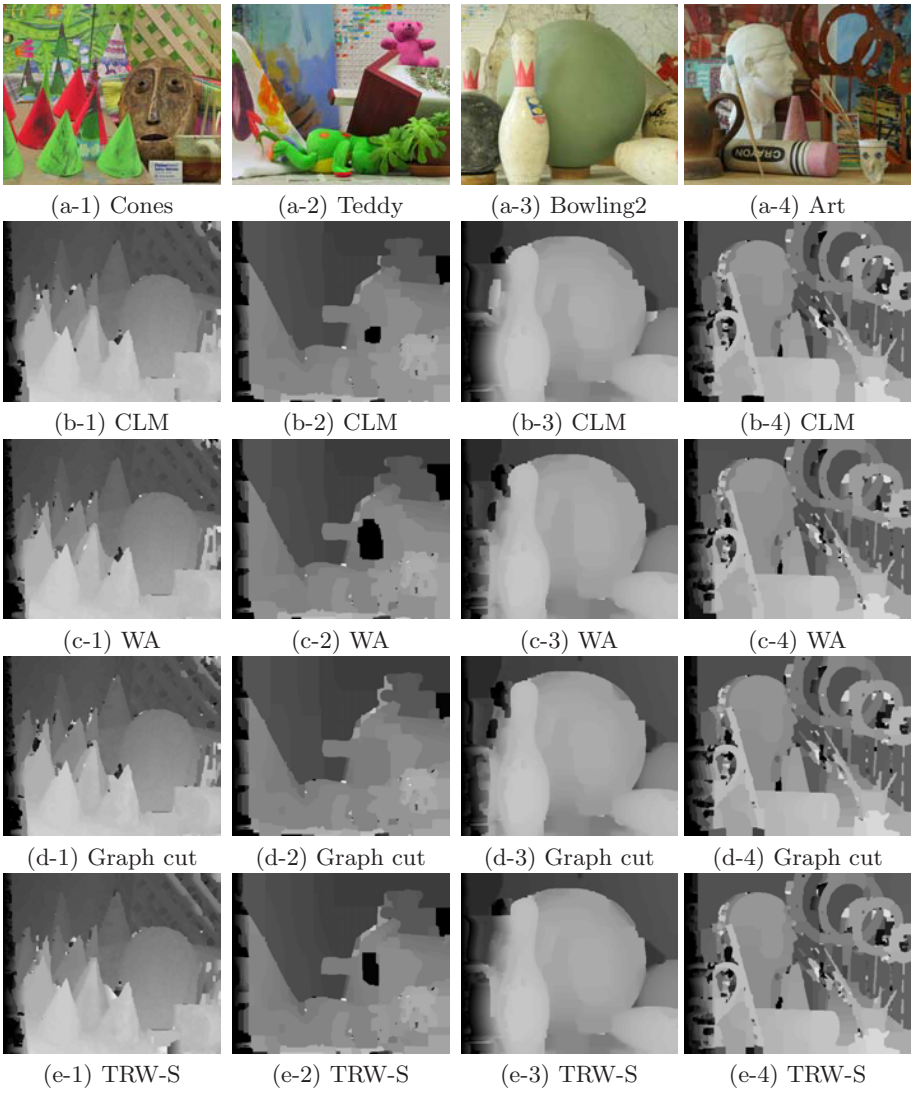


Fig. 6. This figure shows the qualitative stereo energy minimization results at roughly at same computation time. (a-1) to (a-4) are left reference stereo images. (b-1) to (b-4) are the results of proposed CLM. (c), (d), and (e) respectively show the results of window annealing, graph cut and TRW-S. For each stereo pair, the same energy function is used. The qualitative differences between 4 methods are very small, except for Teddy image where graph cut's lower energy makes a difference over the roof area of image. Otherwise, the energy difference between 4 methods are small enough to make no visible differences.

$$\varphi(x) = \sum_{(p,q,r,s)\in N_g} V(p,q,r,s) \tag{4}$$

The clique potential $V(p,q,r,s)$ is a function of 4 dimensioned vector. The value of each $V(p,q,r,s)$ is randomly assigned from an uniform distribution $[0,1]$. In Figure 3 (b), the square factor nodes are connected to 4 variable nodes p,q,r,s. 30×30 variable nodes with 2×2 clique potentials are generated for the comparison test. The energy minimization results of CLM, ICM, and BP are shown in Figure 5; (a) to (d) are the results obtained for MRFs with label size $Q = 2$ to $Q = 5$, respectively.

For the implementation of belief propagation, the factor nodes are transformed into variable nodes with Q^4 number of labels having corresponding $V(p,q,r,s)$ as the unary costs. The pairwise potentials are assigned either 0 or ∞ based on the consistency requirement. CLM is implemented using $k = Q$ local minima and also for $k = 200$ local minima. For $k = Q$, local minima are found by ICM over Q homogeneous states like the stereo problem, even though the smoothness assumption is no longer viable in this problem. For CLM200, additional $200 - Q$ ICM minima obtained from random initial states are used. Same rectangular partitioning is used but the integral image technique is not used.

In these 4 tests, it is clear that the proposed CLM converges faster than BP and WA. The difference from BP is more evident for Figure 5 (c) and (d), because even though BP is fast as CLM200 for $Q = 2$ Figure 5 (a), but as the label size increases, BP could not keep up with speed of CLM. Thus, as the number of labels and clique size become larger, message passing algorithms will become practically ineffective awhile proposed CLM can maintain reasonable computational time. Furthermore, with larger number of local minima, the CLM can reach much lower energy than BP and WA with comparably insignificant addition to computation resources.

5 Conclusion and Future Work

In this paper, we propose a new a method to combine local minima solutions toward more global minimum by random partition method. CLM's performance is compared with state-of-art energy minimization methods over most well known pairwise stereo MRF. Combined local minima is shown to be effective as graph cut and TRW-S. Furthermore, tests over randomly generated 2×2 clique MRFs show that the computation complexity of CLM is much smaller than traditional message passing algorithms as the clique and label size become larger.

Additionally, we included window annealing method in the experiment. However, due to heuristics of simulated annealing and the proposed method, it is hard to say which method is better. Nevertheless, both algorithms show clear advantages over the high ordered MRF compared to existing methods awhile maintaining competitiveness in the pairwise MRFs. We hope that such conclusion will encourage other computer vision researchers to explore more complex MRFs involving larger clique potentials. In the future, MRF with random structure (non square lattice) will be studied using Swendsen Wang cut like partition method.

Acknowledgement

This research was supported in part by the Defense Acquisition Program Administration and Agency for Defense Development, Korea, through the Image Information Research Center under the contract UD070007AD, and in part by the MKE (Ministry of Knowledge Economy), Korea under the ITRC (Information Technolgy Research Center) Support program supervised by the IITA (Institute of Information Technology Advancement) (IITA-2008-C1090-0801-0018).

References

1. Roth, S., Black, M.J.: Steerable random fields. In: ICCV (2007)
2. Roth, S., Black, M.J.: Field of experts: A framework for learning image priors. In: CVPR (2005)
3. Potetz, B.: Efficient belief propagation for vision using linear constraint nodes. In: CVPR (2007)
4. Kohli, P., Mudigonda, P., Torr, P.: p^3 and beyond: Solving energies with higher order cliques. In: CVPR (2007)
5. Rother, C., Kolmogorov, V., Minka, T., Blake, A.: Cosegmenation of image pairs by histogram matching- incorporating a global constraint into mrfs. In: CVPR (2006)
6. Lan, X., Roth, S., Huttenlocher, D., Black, M.J.: Efficient belief propagation with learned higher-order markov random fields. In: Leonardis, A., Bischof, H., Pinz, A. (eds.) ECCV 2006. LNCS, vol. 3952, pp. 269–282. Springer, Heidelberg (2006)
7. Geman, S., Geman, D.: Stochastic relaxation, gibbs distributions, and the bayesian restoration of images. PAMI 6 (1984)
8. Zhu, S.C., Liu, X.W., Wu, Y.N.: Exploring texture ensembles by efficent markov chain monte carlo: Toward a trichromacy theory of texture. PAMI 22(6) (2000)
9. Tu, Z., Zhu, S.C.: Image segmentation by data-driven markov chain monte carlo. PAMI 24 (2002)
10. Barbu, A., Zhu, S.C.: Generalizing swendsen-wang cut to sampling arbitrary posterior probabilities. PAMI 27 (2005)
11. Szeliski, R., Zabih, R., Scharstein, D., Veksler, O., Kolmogorov, V., Agarwala, A., Tappen, M., Rother, C.: A comparative study of energy minimization methods for markov random fields. In: Leonardis, A., Bischof, H., Pinz, A. (eds.) ECCV 2006. LNCS, vol. 3952, pp. 16–29. Springer, Heidelberg (2006)
12. Woodford, O.J., Reid, I.D., Torr, P.H.S., Fitzgibbon, A.W.: Field of experts for image-based rendering. BMVC (2006)
13. Jung, H.Y., Lee, K.M., Lee, S.U.: Window annealing over square lattice markov random field. ECCV (2008)
14. Besag, J.: On the statistical analysis of dirty pictures (with discussion). Journal of the Royal Statistical Society Series B 48 (1986)
15. Mignotte, M.: Nonparametric multiscale energy-based model and its application in some imagery problems. PAMI 26 (2004)
16. `http://vision.middlebury.edu/stereo/`
17. Scharstein, D., Szeliski, R.: A taxonomy and evaluation of dense two-frame stereo correspondence algorithms. In: IJCV (2002)
18. Scharstein, D., Szeliski, R.: High-accuracy stereo depth maps using structured light. In: CVPR (2003)

19. Hirshmuller, H., Szeliski, R.: Evaluation of cost functions for stereo matching. In: CVPR (2007)
20. Scharstein, D., Pal, C.: Learning conditional random fields for stereo. In: CVPR (2007)
21. Boykov, Y., Veksler, O., Zabih, R.: Fast approximate energy minimization via graph cuts. PAMI 23 (2001)
22. Kolmogorov, V.: Convergent tree-reweighted message passing for energy minimization. PAMI 28 (2006)
23. Lempitsky, V., Rother, C., Blake, A.: Logcut - efficient graph cut optimization for markov random fields. In: ICCV (2007)
24. Crow, F.: Summed-area tables for texture mapping. SIGGRAPH (1984)
25. Tappen, M.F., Freeman, W.T.: Comparison of graph cuts with belief propagation for stereo, using identical mrf parameters. In: ICCV (2003)
26. Birchfield, S., Tomasi, C.: A pixel dissimilarity measure that is insensiitive to image samplin. PAMI 20 (1998)
27. Kolmogorov, V., Zabih, R.: What energy functions can be minimized via graph cuts? PAMI 26 (2004)
28. Boykov, Y., Kolmogorov, V.: An experimental comparison of min-cut/max-flow algorithms for energy minimization in vision. PAMI 26 (2004)
29. Wainwright, M.J., Jaakkola, T.S., Willsky, A.S.: Map estimation via agreement on trees: Message-passing and linear-programming approaches. IEEE Trans. Information Theory 51(11) (2005)