Construction of Minimum Connected Dominating Set in 3-Dimensional Wireless Network

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Abstract. Connected Dominating Set (CDS) has been a well known approach for constructing a virtual backbone to alleviate the broadcasting storm in wireless networks. Previous literature modeled the wireless network in a 2-dimensional plane and looked for the approximated Minimum CDS (MCDS) distributed or centralized to construct the virtual backbone of the wireless network. However, in some real situations, the wireless network should be modeled as a 3-dimensional space instead of 2-dimensional plane. We propose our approximation algorithm for MCDS construction in 3-dimensional wireless network in this paper. It achieves better upper bound $(13 + \ln 10)opt + 1$ than the only known result $22opt$. This algorithm helps bringing the research for MCDS construction in 3-dimensional wireless network to a new stage.

1 Introduction

Due to the lack of pre-defined infrastructure, most routing protocols in wireless network involve flooding, which usually cause serious broadcasting storm[8]. Connected Dominating Set (CDS) has been a well known approach for constructing a virtual backbone to alleviate this broadcasting storm in wireless networks. With the help of the CDS, average message burden of the network could be reduced so that routing becomes much easier and can adapt quickly to network topology changes[3]. Furthermore, using a CDS as forwarding nodes can efficiently reduce the energy consumption, which is also a critical concern in wireless networks.

Researchers have proved that it is a NP-hard problem to find the MCDS for a given graph in early 70s[6]. With some known mathematical conclusions in 2-dimensional graphs like unit disk graphs, most previous literatures modeled the wireless network in a 2-dimensional plane and looked for the approximated

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Minimum CDS(MCDS) distributed or centralized to construct the virtual backbone of the wireless network[5,9,10,11,12]. They have successfully achieved constant approximation ratios for special graphs like unit disk graphs. However, in some real situations, the wireless network should be modeled as a 3-dimensional space instead of 2-dimensional plane. For example, in an three-dimensional under water-Acoustic sensor networks for ocean column monitoring like figure 1, networks of sensors whose depth can be controlled are included. Sensor nodes float at different depths in order to observe a given phenomenon. This kind of network is used for surveillance applications or monitoring of ocean phenomena (eg. ocean bio-geo-chemical processes, water streams, pollution, etc). It is similar for the temperature sensing system in ocean. Obviously, 2-dimensional modeling is far from enough for real applications like these.

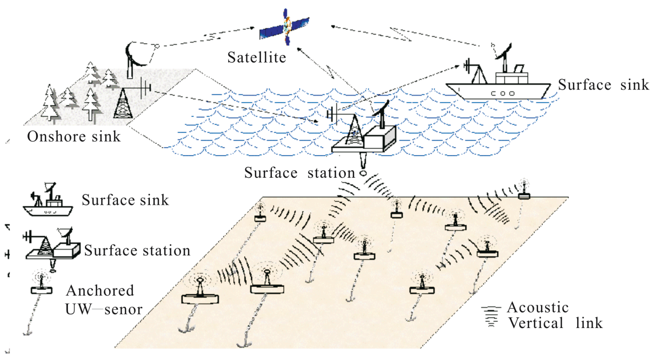

Fig. 1. 3-dimensional Under Water Acoustic Sensor Networks

In this paper, we study the problem of constructing MCDS in 3-dimensional wireless network. We model the 3-dimensional wireless network using unit ball graphs. A graph is a called unit ball graph if its vertices can be represented as points in 3-dimensional Euclidean space and two vertices are adjacent if and only if the distance between the two corresponding points is less than 1. Unit ball graphs have been quite popular in modeling wireless network (eg. ad hoc wireless network) nowadays. It provides a more reasonable representation of a real-life wireless network. As unit disk graphs could be viewed as a subclass of the unit ball graphs and MCDS construction problem in it has been proved to be NP-hard problem[6], it is a NP-hard problem for MCDS in unit ball graphs as well. The only known result for MCDS in 3-dimensional wireless network is given by Butenko et al.[1], which gives a 22-approximation algorithm. We propose our approximation algorithm for MCDS construction in 3-dimensional wireless network in this paper. This algorithm could achieve better approximation ratio of $13 + \ln 10 \approx 15.303$ than Butenko's.

The remainder of this paper is organized as follows. Section 2 discusses the related work of CDS problem and some existing work of MCDS in unit ball graphs. The detailed approximation algorithm and proof are given in section 3. Section 4 concludes the whole work.

2 Related Work

The research of Connected Dominating Set has started from early 80s. It was first introduced as a virtual backbone for routing in Mobile radio network by Ephremides et al.[4]. They thought that it should consist of relatively small number of nodes and remain connected. Meanwhile nodes within it should be able to communicate to points within and outside the central network. Guha and Khuller[5] proposed the first two approximation algorithms for CDSs construction in 1998. In their first algorithm, the CDS is built up at one node first, then they restricted the searching space of the next dominator(s) to the current dominatees. The CDS in this algorithm expands until there is no white nodes. In their second algorithm, all the possible dominators are determined in the first phase, then they are connected through some intermediate nodes in the second phase. Lots of improvements have been done based on it afterwards[5,9,10,12]. Some of them improved the approximation ratio(PR)or computation complexity. For example, Ruan et al.[9] designed a 1-phase greedy algorithm with PR of $2 + \ln \Delta$ where the Δ is the maximum degree in the graph. Some of them implemented distributed versions. For example, Wu and Li[12] proposed a distributed algorithm, which was proved to have a PR of $O(n)$ later.

Recently, a new kind of methodology for constructing CDSs was proposed, which constructed a Maximum Independent Set(MIS) first and then interconnect it into a CDS. Wan et al.[11] proposed two 2-phase distributed algorithms. In these two algorithms, a spanning tree is first constructed. Every node in the spanning tree is then labelled as either a dominator or a dominatee based on a ranking scheme. The algorithms are employed upon Unit Disk Graphs (UDG) to obtain a constant performance ratio of 8. Some other examples are[2,7].

All of these existing research work studied the CDS within the 2-dimensional Euclidean plane R^2 only. They model the network using 2-dimensional graphs, for example, general disk graphs and unit disk graphs. While, in fact, the real network is 3-Dimensional instead of 2-Dimensional. A popular modeling of the 3-Dimensional network is ball graphs, a special case of which is the unit ball graphs(similar to unit disk graphs). Thus CDS construction under ball graphs, especially under unit ball graphs is of great interests nowadays. Few work has been done on this topic as far as we know. The only research result ever known up till now is given by Butenko et al.[1]. They proposed an algorithm in their paper and proved it achieved 22 PR.

3 A 15.303-Approximation Algorithm

We proposed an approximation algorithm for the minimum connected dominating set problem in unit ball graphs in this section, which could achieve a $(13 + \ln 10)$ approximation ratio, better than all other existing algorithms ever known[1].

3.1 Preliminaries

We model the 3 dimensional network using a unit ball graph $G = (V, E)$, in which a vertex represents a point of the network in 3-dimensional Euclidean space $\Re^3$. Two vertices u and v in the graph G are connected by an edge e in E if and only if the balls of radius 1 centered in points u and v have a nonempty intersection. We denote a maximal independent set of the graph G as $MIS(G)$. Meanwhile let *opt* be the size of the minimum connected dominating set for the unit ball graph G.

3.2 Detailed Algorithm

In our algorithm, we first construct a maximal independent set MIS for the unit ball graph. The detailed process is illustrated in Algorithm 1. For better explanation, we make use of the coloring scheme in this algorithm. Initially, color all nodoes white. Then select a root and color it black and all its neighbors grey. Each round we pick the vertex in the tree that is still white in the graph to add into MIS. By coloring its adjacent nodes grey, we could have a new coloring graph. This algorithm ends when all the vertex in the graph is either in the MIS or grey(adjacent to one of the node in MIS). The size of the MIS constructed by this algorithm, as proved in [1], is bounded by $11opt+1$. Also as a centralized version of MIS constructions, it maintains the property that any pair of complementary subsets of the MIS have a distance of exactly two hops [2,11].

In the second step, we consider the $MIS(G)$ as the terminal set of the graph G, constructing the minimum set of steiner nodes to interconnect them. We employ a greedy approximation algorithm as well, which select the vertex that could connect the maximum number of connected components together each time. We will show that a combination of this greedy approximation and the Algorithm 1 gives the upper bound as $(13 + \ln 10)opt + 1$, which is approximately $15.303opt + 1$.

Algorithm 1. MIS_UBG($G(V, E)$)

1: Color all nodes in V white
2: Randomly select a root node $r \in V$ and color it black and its neighbors grey
3: **while** there is a white node $x \in V$ **do**
4: Color x in black and all of its white neighbor grey
5: **end while**
6: $M = \emptyset$
7: Put all black nodes in M and return M.

The pseudo-code of the detailed algorithm for the second step is presented in Algorithm 2. With input $MIS(G)$ and $G = (V, E)$, we color vertex in $MIS(G)$ black first and let all other vertices in G remain grey. In the algorithm, we will change some nodes from grey to blue to interconnect the $MIS(G)$. A *black-blue component* is a connected component of the subgraph induced only by black and blue nodes *without considering connections between blue nodes.*

So, the union of the set $MIS(G)$ and S(the result of the Algorithm 2) is the connected dominating set for unit ball graph G.

Algorithm 2. CDS_UBG(MIS(G),G=(V,E))

Initialize the set of blue nodes S as Φ
while there are more than one connected black-blue component exist **do**
  choose the vertex v that connects the maximum number of
  black-blue components, change it's color from grey to blue
  and set S=$S \bigcup v$;
end while
return blue nodes set S.

Theorem 1. *Let opt be the size of the minimum connected dominating set in the unit ball graph G. Then the size of the connected dominating set $MIS(G) \bigcup S$ is up-bounded by $(13 + \ln 10)opt + 1$.*

Proof. *Suppose $v_1, v_2, \ldots, v_k$ are selected in turn by the algorithm 2. Let $y_1, y_2, \ldots, y_{opt}$ be a minimum connected dominating set and for any i, $y_1, y_2, \ldots, y_i$ induces a connected subgraph. Denote $C_i = MIS(G) \bigcup \{v_1, v_2, \ldots, v_i\}$ and $C_j^* = \{y_1, y_2, \ldots, y_j\}$.*

Let $f(C)$ be the number of connected components of the subgraph induced by vertex set C and $\Delta_y f(C) = f(C \bigcup y) - f(C)$. Since the number of black nodes that y_i could dominate in $C_i \bigcup C_{j-1}^*$ is at most one more than the number of black nodes that it could dominate in C_i. Then we have

$$\begin{aligned}
&-\Delta_{y_j} f(C_i \bigcup C_{j-1}^*) + \Delta_{y_j} f(C_i) \\
&= f(C_i \bigcup C_{j-1}^*) - f(C_i \bigcup C_j^*) + \Delta_{y_j} f(C_i) \\
&= f(C_i \bigcup C_{j-1}^*) - f(C_i \bigcup C_j^*) + f(C_i \bigcup y_j) - f(C_i) \\
&\leq 1.
\end{aligned}$$

As v_{i+1} is the vertex that could connect the maximum number of connected components, $-\Delta_{v_{i+1}} f(C_i) \geq -\Delta_{y_j} f(C_i)$ for all $1 \leq j \leq opt$. Thus,

$$\begin{aligned}
&-\Delta_{v_{i+1}} f(C_i) \\
&\geq (-\sum_{1 \leq j \leq opt} \Delta_{y_j} f(C_i))/opt \\
&\geq (-\sum_{2 \leq j \leq opt} \Delta_{y_j} f(C_i) - \Delta_{y_1} f(C_i))/opt \\
&= (-opt + 1 - \sum_{2 \leq j \leq opt} \Delta_{y_j} f(C_i \bigcup C_{j-1}^*) \\
&\quad -\Delta_{y_1} f(C_i))/opt \\
&= (-opt + 1 - \sum_{1 \leq j \leq opt} \Delta_{y_j} f(C_i \bigcup C_{j-1}^*))/opt \\
&= (-opt + 1 - f(C_i \bigcup C_{opt}^*) + f(C_i))/opt \\
&\geq (-opt - 1 + f(C_i))/opt.
\end{aligned}$$

So $-f(C_{i+1}) \geq -f(C_i) + \frac{-opt-1+f(C_i)}{opt}$.

Denote $a_i = -opt - 1 + f(C_i)$. *Then we have* $a_{i+1} \leq a_i(1 - \frac{1}{opt})$, *which implies*

$$a_i \leq a_0(1 - \frac{1}{opt})_i \leq a_0 e^{-i/opt}.$$

If $a_0 < opt$, *then* $-opt - 1 + |MIS| < opt$, *which means* $|MIS| < 2opt + 1 \leq 2opt$. *Since for any vertices* v_i, *it connects at least two black-blue components, the size of* S *is at most* $|MIS| - 1$. *Hence, the size of the connected dominating set* $MIS(G) \bigcup S$ *is up-bounded by* $4opt - 1$ *now.*

Else, choose i *to be the largest one satisfying* $opt \leq a_i$. *So we have* $opt \leq a_0 e^{-i/opt}$, *which indicates* $i \leq opt \ln(a_0/opt)$. *Meanwhile,*

$$a_k \leq a_{k-1} - 1 \leq a_{k-2} - 2 \leq \ldots \leq a_{i+1} - (k - i - 1)$$

implies that $-opt \leq opt - k + i$ *as* $a_{i+1} \leq opt - 1$. *Furthermore, since* $|MIS| \leq 11opt + 1$, $a_0/opt = (-opt - 1 + |MIS|)/opt \leq 10$. *We have*

$$\begin{aligned} k &\leq 2opt + i \\ &\leq opt(2 + \ln(a_0/opt)). \\ &\leq opt(2 + \ln 10) \\ &= (2 + \ln 10)opt. \end{aligned}$$

Hence, $|S| \leq (2 + \ln 10)opt$ *and the size of the connected dominating set*

$$\begin{aligned} |MIS(G) \bigcup S| &\leq 11opt + 1 + (2 + \ln 10)opt \\ &= (13 + \ln 10)opt + 1 \end{aligned}$$

4 Conclusion

Since not much work has been done for MCDS construction in 3-dimensional wireless network, we propose a new approximation algorithm for it in this paper. Compared with the only existing algorithm proposed by Butenko et al., we could achieve a better result of $(13 + \ln 10)opt + 1$ than theirs. This algorithm helps bringing the research for MCDS construction in 3-dimensional wireless network to a new stage.

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