Approximate Numerical Solution of Hydrodynamic Gas Journal Bearings*

Haijun Zhang[1,2], Changsheng Zhu[1], and Qin Yang[2]

[1] College of Electrical Engineering, Zhejiang University, Hangzhou, Zhejiang Province, China
[2] College of Mechanical-Electrical Engineering, Jiaxing University, Jiaxing, Zhejiang Province, China
zhjzszs_537@hotmail.com

Abstract. Considering small pressure change in the gas film of hydrodynamic gas-lubricated journal bearings, the corresponding nonlinear Reynolds equation is linearized through appropriate approximation and approximate Reynolds equation is derived and solved by means of finite difference method (FDM). The gas film pressure distribution of hydrodynamic gas-lubricated journal bearing is attained and load capacity is calculated. The approximate numerical solution shows better agreement with experimental data than direct numerical solution and demands less computer time. It is of interest to note the eccentricity ratio ε at which approximate numerical solution is better agreement with experimental data is different when bearing number is changing. The approximate numerical solution is slightly larger when the eccentricity ratio is smaller, and becomes slightly smaller when the eccentricity ratio is larger.

Keywords: hydrodynamic gas-lubricated journal bearings; Reynolds equation; nonlinearity; finite difference method (FDM).

1 Introduction

Gas bearings operate with the pressure generated for lubricating film. Because gas bearings have such characteristics as low fiction, high precision and low pollution, they have been successfully used in many commercial applications, such as navigation systems, computer disk drives, high-precision instruments and sensors, dental drills, machine tools, and turbo-compressors [1]. There had been a research boom of gas bearings in the 60-70 of 20th century. In recent years study on gas bearings has been paid much attention to with the advent of MEMS research [2, 3].

Reynolds equation is the fundamental equation with which the performance of hydrodynamic gas-lubricated bearing is investigated. Duo to gaseous compressibility, Reynolds equation is nonlinear and its analytical solution usually is difficult to obtain. Therefore numerical method is an effective means to study gas-lubricated journal bearings. All numerical methods used will be treated in two classes: analytical-numerical and direct-numerical [4]. In the former class either some features of the particular bearing

* This work is supported by National Science Foundation of China (10472101).

C. Xiong et al. (Eds.): ICIRA 2008, Part II, LNAI 5315, pp. 260–268, 2008.

under study are taken advantage of or some approximations are made in order to partially solve the problem by analytical means. Harrison first presented solutions for infinitely long gas-lubricated slider and journal bearings [5]. His work on the infinite journal bearing was improved upon by Katta and Soda [6] who gave an approximate analytical solution. Ausman [7] published the linearized ph solution for small motion of journal bearing of finite length. Methods of the latter class approach the problem immediately with numerical approximations, the accuracy of which is often easier to control. Elrod and Burgdorfer [8] presented solutions for the infinite journal bearings, i.e., length-to-diameter ratio is infinite ($L/D=\infty$). Ramondi [9] obtained solutions for full journal bearings with L/D ratio of 2, 1, and 0.5. Qi [10] transformed Reynolds equation into standard elliptic partial differential equation form and gained solutions using Matlab's PDE solver.

Because increased amount of time is spent on solving the non-linear Reynolds equation with direct numerical approaches, it is essential to find a simple and convenient method for the problem. Considering small pressure change in the gas film of hydrodynamic gas-lubricated journal bearing, the corresponding nonlinear Reynolds equation is linearized through appropriate approximation. Approximate Reynolds equation is derived and solved by means of finite difference method (FDM). The gas film pressure distribution of hydrodynamic gas-lubricated journal bearing is attained and load capacity is calculated. Finally the approximate numerical solution shows better agreement with experimental data than direct numerical solution, which proves its validity.

2 Reynolds Equation for Hydrodynamic Gas Journal Bearings

Reynolds equation, which determines pressure distribution of the bearings, can be derived from the motion equation, continuity equation and energy equation for gas. The gas-lubricated journal bearing model, composed of two plates, is shown in Fig.1, in which the upper plate is stationary and the lower one is moving at the velocity u_0 .

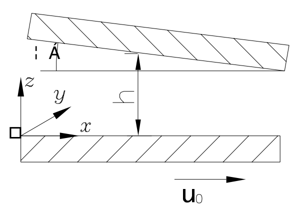

Fig. 1. Gas-lubricated journal bearing model

By assuming steady conditions and neglecting the inertial terms, body forces, x-direction and y-direction components of viscosity force induced by flow gradient, as shown in Fig.1, Reynolds equation is derived

$$\frac{\partial}{\partial x}\left(\frac{\rho \mathrm{h}^3}{\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\rho \mathrm{h}^3}{\mu}\frac{\partial p}{\partial y}\right)=6u_0\frac{\partial(\rho \mathrm{h})}{\partial x}+12\frac{\partial(\rho \mathrm{h})}{\partial t} \tag{1}$$

where ρ is gas density, μ gas absolute viscosity, h is gas film thickness, p is gas film pressure, u_0 is x-direction component of gas velocity, t is time variable.

Reynolds equation relating pressure, density, surface velocity, and film thickness is a 2-dimension elliptic differential one, which is fundamental in gas-lubricated journal bearing field. The solution to the Reynolds equation provides the pressure distribution throughout the film, given the geometry, property, and state parameters. Once the pressure distribution is known, the other properties of the gas film are readily determined.

Gas-lubricated film obeys a polytropic relation [1],

$$p\rho^{-n}=\text{constant} \tag{2}$$

where n is the polytropic gas-expansion exponent, whose value lies between 1 and γ. When flow is adiabatic, i.e., there is no transferred heat and the change in internal energy equals the compression work, (2) with $n=\gamma=c_p/c_v$ follows directly from the equation of state and the energy equation. When flow is isothermal, (2) with $n=1$ derives from the equation of state. c_p is the specific heat per unit weight for constant pressure and c_v is the specific heat per unit weight for constant volume.

The viscosities of common gases increase with temperature but are comparatively insensitive to moderate temperature and pressure changes. Within a gas lubricating film, temperature and pressure changes are normally small, and viscosity is considered to be constant.

Therefore, by assuming constant temperature, (1) becomes

$$\frac{\partial}{\partial x}\left(p\mathrm{h}^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(p\mathrm{h}^3\frac{\partial p}{\partial y}\right)=6\mu u_0\frac{\partial(p\mathrm{h})}{\partial x} \tag{3}$$

For hydrodynamic gas-lubricated journal bearings shown in Fig. 2, (3) becomes

$$\frac{\partial}{\partial\theta}\left(ph^3\frac{\partial p}{\partial\theta}\right)+r^2\frac{\partial}{\partial y}\left(ph^3\frac{\partial p}{\partial y}\right)=6\mu u_0 r\frac{\partial(ph)}{\partial\theta} \tag{4}$$

The non-dimensional form for (4) is

$$\frac{\partial}{\partial\theta}\left(PH^3\frac{\partial P}{\partial\theta}\right)+\frac{\partial}{\partial\zeta}\left(PH^3\frac{\partial P}{\partial\zeta}\right)=\Lambda\frac{\partial}{\partial\theta}(PH) \tag{5}$$

where

$$x=r\theta, y=r\zeta, u_0=r\omega, P=p/p_{\mathrm{a}}, \Lambda=\frac{6\mu\omega}{p_{\mathrm{a}}}\left(\frac{r}{c_0}\right)^2, h=c_0\cdot(1+\varepsilon\cos\theta)=c_0H$$

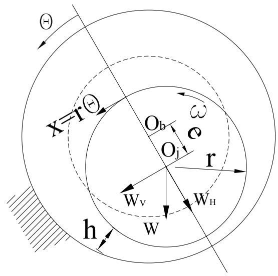

Fig. 2. Schematic diagram of hydrodynamic gas-lubricated journal bearing

where θ, ζ is the non-dimensional circumferential coordinate and axial coordinate respectively, H, P is the non-dimensional gas film thickness and gas film pressure, r is journal radius, ω is journal angular velocity, p_a is ambient pressure, ε is eccentricity ratio and c_0 is radial clearance.

The boundary conditions for (5) are

$$\begin{aligned} &P\left(\theta, \pm\frac{L}{2r}\right) = 1 \\ &P(0,\zeta) = P(2\pi,\zeta) \\ &P(\theta,\zeta) = P(\theta,-\zeta) \end{aligned} \tag{6}$$

where $\theta \in [0, 2\pi]$, $\zeta \in \left[-\frac{L}{2r}, \frac{L}{2r}\right]$

Equation (5), a non-linear partial differential equation with respect to non-dimensional pressure P , can be solved directly by the finite difference method (FDM). However, as mentioned above, the disadvantages of direct-numerical method are: increased amount of computer time, the need for an efficient computer system and instability in the convergence of iteration process.

3 Reynolds Equation Approximation

Because Reynolds equation is a non-linear partial differential one, the calculation method of direct-numerical solution is more complex and occupies much more computer time. In order to simplify calculation, Reynolds equation is linearized as follows. First (5) becomes

$$\begin{aligned} &H^3\left(\frac{\partial^2 P}{\partial \theta^2} + \frac{\partial^2 P}{\partial \zeta^2}\right) + 3H^2\frac{\partial H}{\partial \theta}\frac{\partial P}{\partial \theta} - \Lambda\frac{dH}{d\theta} = \\ &-\frac{H^3}{P}\left[\left(\frac{\partial P}{\partial \theta}\right)^2 + \left(\frac{\partial P}{\partial \zeta}\right)^2\right] + \Lambda\frac{H}{P}\frac{\partial P}{\partial \theta} \end{aligned} \tag{7}$$

Considering the compressibility, small viscosity and small pressure of gas-lubricated journal bearings, $\left(\frac{\partial P}{\partial \theta}\right)^2, \left(\frac{\partial P}{\partial \zeta}\right)^2$ are considered to be small. The pressure gradient of gas film, $\frac{\partial P}{\partial \theta}, \frac{\partial P}{\partial \zeta}$, have the same trend as eccentricity ratio ε, i.e., $\frac{\partial P}{\partial \theta}, \frac{\partial P}{\partial \zeta}$ increase with ε increasing. Therefore it is appropriate to make the following approximation,

$$\left(\frac{\partial P}{\partial \theta}\right)^2 + \left(\frac{\partial P}{\partial \zeta}\right)^2 \approx \varepsilon \frac{\partial P}{\partial \theta} + \varepsilon \frac{\partial P}{\partial \zeta} \tag{8}$$

Harrison [5], Katto and Soda [6] substituted $p_a c$ for ph in deriving linearized ph solution of Reynolds equation, and the same approximation is adopted here. Therefore, the non-dimensional pressure P in the denominator of right hand in (7) is replaced by $\frac{P_0}{1+\varepsilon \cos\theta} = \frac{1}{H}$, (7) becomes

$$\begin{aligned} & H^3\left(\frac{\partial^2 P}{\partial \theta^2} + \frac{\partial^2 P}{\partial \zeta^2}\right) + 3H^2 \frac{\partial H}{\partial \theta}\frac{\partial P}{\partial \theta} - \Lambda \frac{dH}{d\theta} + \\ & \mathrm{H}^2\left(\varepsilon \frac{\partial \mathrm{P}}{\partial \theta} + \varepsilon \frac{\partial \mathrm{P}}{\partial \zeta}\right) - \Lambda \frac{\partial P}{\partial \theta} = 0 \end{aligned} \tag{9}$$

As soon as (9) is solved by the means of FDM, the characteristics of gas bearings, such as pressure distribution, load capacity, etc., are obtained.

Load capacity of hydrodynamic gas-lubricated journal bearings is constituted by two parts: one is W_H in the direction of center-line, and the other is W_V normal to the direction of center-line, as shown in Fig.2.

$$\begin{cases} W = \sqrt{W_H^2 + W_V^2} \\ W_H = 2\int_0^{L/2}\int_0^{2\pi} p \cos\theta r d\theta dy \\ W_V = 2\int_0^{L/2}\int_0^{2\pi} p \sin\theta r d\theta dy \end{cases} \tag{10}$$

After discretization of solution region, (10) becomes

$$\begin{cases} W_H = p_a \cdot 2\pi r L \sum_{i=1}^{n-1}\sum_{j=1}^{m-1} P_{i,j} \cos\theta_i \Delta\theta\Delta\zeta \\ W_V = p_a \cdot 2\pi r L \sum_{i=1}^{n-1}\sum_{j=1}^{m-1} P_{i,j} \sin\theta_i \Delta\theta\Delta\zeta \end{cases} \tag{11}$$

where n, m are the circumferential and axial nodes of gas-lubricated journal bearings.

4 Load Capacity Calculation and Comparison

To demonstrate the validity of approximation numerical solution, numerical results are compared with experimental data in the literature. The parameters for gas-lubricated journal bearings have the same value as that in the literature [11], as shown in Table 1.

Table 1. Gas-lubricated journal bearings parameters

Parameters	Value
Journal radius r (mm)	25.4
Radial clearance c_0 (mm)	12.7×10^{-3}
Bearing width L (mm)	76.2
lubricant	Argon
Absolute viscosity μ ($N\cdot s/m^2$)	2.29×10^{-5}
Ambient pressure p_a (N/m^2)	1.033×10^5

Fig.3 demonstrates the pressure distribution of hydrodynamic gas-lubricated journal bearings. It can be seen that pressure distribution for gas bearings is different from that for oil bearings because of gas compressibility. (1) There is large pressure peak value in the convergent region while small pressure peak value in the divergent region, i.e., the magnitude of pressure in the convergent region is larger than that in the divergent region. (2) The pressure in the convergent region ascends smoothly while that in the divergent region descends quickly. (3) Contrary to oil bearings, the magnitude of pressure change is small, which explains smaller load capacity of gas bearings.

Comparing with direct numerical solution, approximate numerical solution demands less amount of computer time. When the geometry parameters of journal bearings and computation nodes are the same and the bearing number is different, the computation efficiency of the two methods in solving bearings' pressure distribution can be compared from CPU time data of Table2.

Table 2. CPU times for the approximate numerical solution and direct numerical solution

Bearing number Λ	Direct numerical solution (s)	Approximate numerical solution (s)
3	124.5156	105.0313
5	128.2500	81.7813
10	110.4531	46.9219
20	85.9531	17.9844

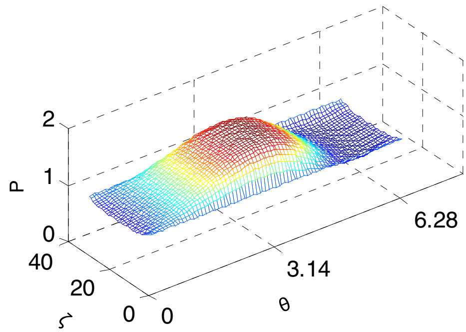

Fig. 3. The non-dimensional pressure distribution of hydrodynamic gas-lubricated journal bearings

It is of interest to compare the calculated values with experimental data. The comparisons of load capacity versus eccentricity ratio ε among direct-numerical solution, approximation numerical solution and experimental data for different bearing number Λ are shown in Fig.4, 5, 6.

It can be seen that load capacity for hydrodynamic gas-lubricated journal bearings increases with increasing eccentricity ratio ε. For the same bearing number Λ, both the value of direct numerical solution and that of approximation numerical solution are larger than experimental data. The difference between approximation numerical solution and experimental data is smaller than that between direct numerical solution and experimental data. Especially when the bearing number Λ increases, the consistency between approximation numerical solution and experimental data is better.

In addition, the eccentricity ratio ε at which approximating numerical solution is better agreement with experimental data is different when bearing number is changing. The approximating numerical solution is slightly larger when the eccentricity ratio is smaller, and becomes slightly smaller when the eccentricity ratio is larger.

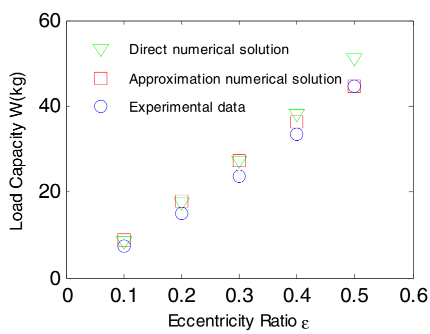

Fig. 4. Comparison when the bearing number Λ is 1.63

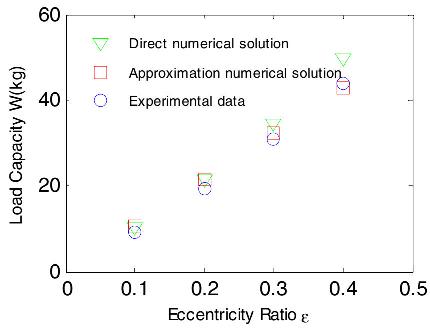

Fig. 5. Comparison when the bearing number Λ is 3.26

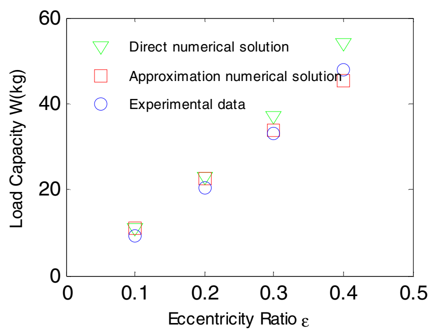

Fig. 6. Comparison when the bearing number Λ is 4.89

5 Conclusion

As a non-linear partial differential equation, Reynolds equation for hydrodynamic gas-lubricated journal bearing is difficult to obtain exact analytical solution. It is simple and convenient to transform Reynolds equation into linear partial differential one through proper approximation and attain numerical solution by the means of FDM. The study demonstrates that the approximating numerical solution shows better agreement with experimental data than direct numerical solution and demands less computer time, from which we think it superior to the direct numerical solution. It is of interest to note the eccentricity ratio ε at which approximating numerical solution is better agreement with experimental data is different when bearing number Λ is changing. The approximating numerical solution is slightly larger when the eccentricity ratio ε is smaller, and becomes slightly smaller when the eccentricity ratio ε is larger.

References

1. Gross, W.A., et al.: Fluid Film Lubrication. John Wiley & Sons, New York (1980)
2. Epstein, A.H., Senturia, S.D.: Microengineering: macro power from micro machinery. Science 276(5316), 1211 (1997)

3. Gad-el-Hak, M.: The Fluid Mechanics of Microdevices—The Freeman Scholar Lecture. Journal of Fluid Engineering 121(1), 5–33 (1999)
4. Castelli, V., Pirvics, J.: Review of numerical methods in gas bearing film analysis. Journal of Lubrication Technology 90(4), 777–792 (1968)
5. Harrison, W.J.: The dydrodynamical theory of lubrication with special reference to air as a lubricant. Trans. Cambridge Philosophical Society 22, 34–54 (1913)
6. Katto, Y., Soda, N.: Theory of lubrication by compressible fluid with special reference to air bearing. In: 2nd Japan National Congress for Applied Mechanics Tokyo, pp. 267–270 (1952)
7. Ausman, J.S.: An improved analytical solution for self-acting, gas-lubricated journal bearings of finite length. Journal of Basic Engineering 83, 188–194 (1961)
8. Elrod, H.G., Burgdorfer, A.: Refinement of the theory of gas-lubricated journal bearings of infinite length. In: 1st International Symposium on Gas Lubricated bearings, Washington, D.C (1959)
9. Raimondi, A.A.: A numerical solution for the gas lubricated full journal bearing of finite length. Trans. ASLE 4, 131–155 (1961)
10. Qi, S.M., Geng, H.P., Yu, L.: New method for the calculation of the characteristics of aerodynamic bearings. Journal of Mechanical Strength 28(3), 369–373 (2006) (in Chinese)
11. Whitley, S., Betts, C.: Study of Gas-Lubricated, Hydrodynamic, Full Journal Bearing. British Journal of Applied Physics 10, 455–463 (1959)