A Nonlinear Adaptive Variable Structure Trajectory Tracking Algorithm for Mobile Robots

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Abstract. Trajectory tracking control is an important subject for mobile robots, and lots of new algorithms have been presented recently. The tracking algorithm for a scenario of a WMR (wheeled mobile robot) tracking a moving target is discussed in this paper. The tracking guidance model of WMR and target is derived, and a nonlinear variable structure tracking algorithm is designed based on Lyapunov stability theory according to the tracking mathematical model and the uncertainty of target acceleration. Meanwhile, an adaptive algorithm is proposed for the uncertain item of the variable structure control law so as to increase the robustness of the tracking algorithm. A numerical example of maneuvering target tracking verifies the rightness of the tracking model and the effectiveness of the proposed method.

Keywords: Trajectory tracking; Mobile robots; Adaptive variable structure control; Robustness.

1 Introduction

Now the tracking algorithm for wheeled mobile robots has been paid more and more attention by scholars. Ref.1 and Ref.2 had designed the feedback controller based on the method of small perturb linearization for error model, which can only obtain local stability. Adaptive control theory was used to design the trajectory tracking algorithm for mobile robots with unknown parameters in Ref.3, which can accomplish the global tracking engagement. However, this kind of method need complex parameters choice and with bad robustness. Ref.4 and Ref.5 had studied the trajectory tracking algorithm for mobile robot based on the back stepping method and this algorithm can achieve the global trajectory tracking for some model satisfied certain conditions. Based on the back-stepping method, a switch function for variable structure control was designed and a sliding mode controller with globally asymptotic stability was derived in Ref.6. These analysis and computations of the methods mentioned above were all designed to control the velocity of the mobile robot center to accomplish the trajectory tracking, which imported the tracking model and control errors. However, the rotation and velocity of the mobile robot center were implemented by the rotation of the two rear wheels usually. A variable structure control tracking method was designed to implement global tracking, which directly used the velocities of the two rear wheels as the control variables in Ref.7. But this method had not taken the unknown parameters into account.

C. Xiong et al. (Eds.): ICIRA 2008, Part I, LNAI 5314, pp. 892–900, 2008.

The mobile robot trajectory tracking system was characterized by nonlinearity and uncertainty. The method based on variable structure control theory has been widely used recently to solve the nonlinear problem, which had been used in missile autopilot design in Ref.8 and Ref.9, and the simulation result had shown that this kind of control method had the merit of robustness and stability. Ref.10 and Ref.11 had proposed a kind of nonlinear variable structure control algorithm for missile interception, which can satisfy the tracking requirement despite the uncertain perturbation and disturbance. So a nonlinear tracking algorithm was presented based on adaptive sliding mode control theory in this paper, considering the uncertainty of target acceleration, and an adaptive algorithm was proposed to compute the target acceleration at the same time.

2 Kinematics Model

2.1 The Mathematical Model of Two Wheeled Robots

In this paper, a two wheeled mobile robot[12] is discussed. The mechanism of WMR includes a body, two driven rear wheels and a castor wheel. The castor wheel can be neglected in the kinematics, which is used only to support the body. XOY is the Inertial coordinate system, and x, y are the position of the mobile robot center corresponding to XOY coordinate system. θ is the orientation relative to the axis X . v and ω are the motion velocity and the angular velocity of the midpoint of the rear axis, and d is the distance of the two wheels. V_L and V_R are the velocity of the left and the right wheel respectively, which are the tracking control variables. The rotation of the mobile robot was driven by the two rear wheels, so was the motion of the mobile robot center. Then, the aim of the tracking algorithm design is how to design the velocity of the two wheels to satisfy the requirements of catching the moving target.

Under the assumptions that the wheels of the mobile robot do not skid on the ground and the control inputs of the system are translational velocity(v) and angular velocity(ω) of the mobile robot, the motion of the mobile robot can be described by the following equations

$$\frac{dz}{dt} = BU \tag{1}$$

where

$$z = \begin{bmatrix} x & y & \theta \end{bmatrix}^T, \quad U = \begin{bmatrix} v & \omega \end{bmatrix}^T, \quad B = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \tag{2}$$

and

$$v = (V_R + V_L)/2, \ \omega = (V_R - V_L)/d \tag{3}$$

From above we can see that the left wheel velocity and the right wheel velocity controlled the motion of the mobile robot, so we can use the two variables as the control variables, which should be designed via the new nonlinear tracking algorithm.

2.2 The Mathematical Model of Tracking Engagement

We first describe mathematically the tracking situation where a mobile robot guided by the tracking algorithm pursues a maneuvering target. To simplify the dynamic equations of the pursuit situation, we assume that: The target is a point mass, the dynamics of the mobile robot is fast enough to be neglected, and the maneuvering accelerations of the target can only change the speed direction.

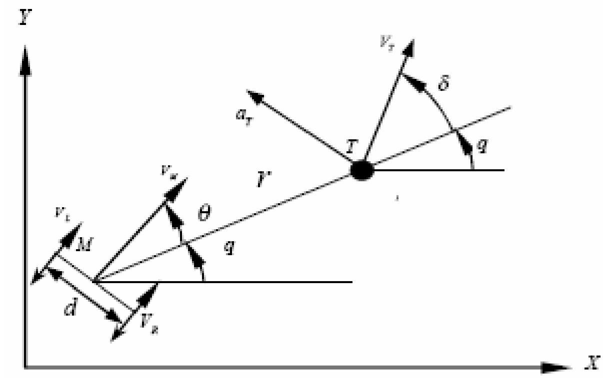

Fig. 1. The geometric diagram of tracking geometry

The target T is located ahead of the mobile robot M . The range between the target and mobile robot is r , and (XOY) is the Inertial coordinate system. The velocity and the maneuvering acceleration of the target are denoted by V_T and a_T respectively, and the velocity of the mobile robot center is denoted by V_M . q is the LOS(Line Of Sight) angel relative to the fixed reference. The angle δ is the target motion direction relative to the LOS, and the angle θ is the instantaneous mobile robot direction of movement relative to the LOS. Then we can derive the following equations from the definitions of the coordinate system and the classical principles of dynamics

$$\begin{aligned} \dot{r} &= V_T \cos\delta - V_M \cos\theta \\ \dot{\theta} &= (V_R - V_L)/d - (V_T \sin\delta - V_M \sin\theta)/r \\ \dot{\delta} &= a_T / V_T - (V_T \sin\delta - V_M \sin\theta)/r \\ \dot{q} &= (V_T \sin\delta - V_M \sin\theta)/r \\ V_M &= (V_R + V_L)/2 \end{aligned} \tag{4}$$

Trajectory tracking algorithm required that the speed of mobile robot is higher than that of the target, so we define the parameter N as the speed ratio:

$$N = V_M / V_T > 1 \tag{5}$$

To catch the target it is not only required that $r = 0$ at the tracking end point, but also required both target and the mobile robot will move in the same direction. So the mobile robot lead angel θ are required to be proportional to the target movement direction relative to the LOS, hence:

$$\theta = n\delta \tag{6}$$

where, n is the guidance constant. So the relation (6) can guarantee that θ vanishes with δ. Implementation of the tracking algorithm in a mobile robot with realistic maneuver dynamics and limits requires more consideration. It is necessary to find the relation between the angular condition of (6) and the speed of the mobile robot using a new kind of control method.

According to (4), the variables are defined as: $x = [x_1, x_2, x_3]^T = [r, \theta, \delta]^T$; $u = V_R - V_L$ is the control variable, and $w = a_T$ is the acceleration of target, which is the system disturbance. Hence we can get the system function:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} V_T \cos x_3 - V_M \cos x_2 \\ -V_T \sin x_3 / r + V_M \sin x_2 / r \\ -V_T \sin x_3 / r + V_M \sin x_2 / r \end{bmatrix} + \begin{bmatrix} 0 \\ 1/d \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 1/V_T \end{bmatrix} w \tag{7}$$

3 Nonlinear Trajectory Tracking Algorithm

3.1 Variable Structure Control Law

Considering the nonlinear Multiple-Input Multiple-Output (MIMO) uncertain system:

$$\dot{X} = f(X,t) + g_1(X,t)U(t) + g_2(X,t)W(t) \tag{8}$$

where, $X \in R^n$ is the state vector. $U(t) \in R^p$ is the control vector, and $f(X,t)$ is the uncertain nonlinear item. $g_1(X,t)$ and $g_2(X,t)$ are vector functions, which have suitable dimensions. $W(t) \in R$ is the acceleration disturbance of target and limited by $0 < \|W(t)\| < b$, where b is a positive constant. The aim of the control method is to design the control signal $U(t)$ such that the system approaches to the equilibrium point $X_s = 0$ asymptotically, i.e.

$$\lim_{t \to t_s} X(t) = 0 \tag{9}$$

According to the variable structure control theory, the sliding surface is defined as:

$$S = CX \tag{10}$$

where, $C \in R^{p \times n}$ is a matrix satisfying $|Cg_1(X,t)| \neq 0$. Then we design the control law as follows:

$$U = U_{eq} + U_s + U_n \tag{11}$$

where, U_{eq} is the equivalent control item for the system without disturbance. U_s is the proportion item. U_n represents the nonlinear feedback control for suppression of the effect of the uncertainty [13,14]. $K \in R^{p\times p}$ is nonnegative definite matrix.

$$U_{eq} = -[Cg_1(X,t)]^{-1}[Cf(X,t)] \tag{12}$$

$$U_s = -[Cg_1(X,t)]^{-1}KS \tag{13}$$

$$U_n = -[Cg_1(X,t)]^{-1}\|Cg_2(X,t)\|bS/\|S\| \tag{14}$$

Lemma 1. Using the control law of (11), the system (8) is asymptotically stable.

Proof: Considering the Lyapunov function $V = S^T S/2$ and differentiating (10), the following equation is achieved:

$$\dot{S} = -KS + Cg_2(X,t)W - \|Cg_2(X,t)\|bS/\|S\| \tag{15}$$

Then:

$$\begin{aligned}\dot{V} &= S^T\dot{S}\\ &= -S^T KS + S^T Cg_2(X,t)W - b\|Cg_2(X,t)\|\|S\| < -S^T KS < 0\end{aligned} \tag{16}$$

We can see that $V > 0 \Rightarrow \dot{V} < 0$. It is easily checked that $V = 0 \Rightarrow X = 0$. Hence, the system is asymptotically stable, and the conditions of variable structure control theory are satisfied, thus Lemma 1 holds.

3.2 Adaptive Variable Structure Control Law

The above variable structure control law need to know the upper limit of b, but it is difficult to get in actual situation usually. So we propose a simple adaptation law for the bound of norm $\|W(t)\|$, and design a variable structure controller using this adaptive upper bound. We suppose that $\hat{b}$ is the estimation of b, then design the adaptive variable structure control item U_n as follow:

$$U_n = -[Cg_1(x)]^{-1}\|Cg_2(X,t)\|\hat{b}S/\|S\|, \dot{\hat{b}} = \gamma\|Cg_2(X,t)\|\|S\| \tag{17}$$

Lemma 2. Suppose γ is a positive constant, using the control law of (11) with the adaptive item U_n of (17), the system (8) is also asymptotically stable.

Proof: Considering the Lyapunov function

$$V = \frac{1}{2}S^T S + \frac{1}{2\gamma}(b-\hat{b})^2 \tag{18}$$

Differentiating (10), the following equation is achieved:

$$\dot{S} = -KS + Cg_2(x)W - \|Cg_2(x)\|\hat{b}\frac{S}{\|S\|} < -KS + \|Cg_2(x)\|b - \|Cg_2(x)\|\hat{b}\frac{S}{\|S\|} \tag{19}$$

Then:

$$\dot{V} = S^T\dot{S} - \frac{1}{\gamma}(b-\hat{b})\dot{\hat{b}} < -S^T KS + \|Cg_2(X,t)\|(b-\hat{b})\|S\| - \frac{1}{\gamma}(b-\hat{b})\dot{\hat{b}} = -S^T KS < 0 \tag{20}$$

We can see that $V > 0 \Rightarrow \dot{V} < 0$ and $V = 0 \Rightarrow X = 0$. Hence, the system is asymptotically stable, and Lemma 2 holds.

3.3 Design of Nonlinear Guidance Law

It follows that to realize the control law only need to know the scope of acceleration of target, and this method can be applied in the course of tracking the unknown acceleration target. No control is taken in the LOS direction usually, as long as other parameters are kept sliding on the sliding surface. When the mobile robot catches the target, the tracking control is finished[15]. The other coupling parameters can be treated as disturbances.

The aim of tracking algorithm design is to bring the system into the sliding surface and keep the dynamical characteristics of the system. According to (7), we design: $c = [0 \ \ 1 \ \ -n]$ and $x = [x_1, x_2, x_3]^T = [r, \theta, \delta]^T$. Hence we can get the system function and the sliding variable as follow:

$$s = cx = x_2 - nx_3 \tag{21}$$

According to (7),we can get:

$$f(X,t) = \begin{bmatrix} V_T \cos x_3 - V_M \cos x_2 \\ -V_T \sin x_3 / r + V_M \sin x_2 / r \\ -V_T \sin x_3 / r + V_M \sin x_2 / r \end{bmatrix} \tag{22}$$

$$g_1(X,t) = [0 \quad 1/d \quad 0]^T \tag{23}$$

$$g_2(X,t) = [0 \quad 0 \quad 1/V_T]^T \tag{24}$$

Obviously, $Cg_1(X,t) = 1/d$. Considering the above mentioned design method of nonlinear adaptive variable structure tracking algorithm, the tracking law can be written as:

$$u_{eq} = (n-1)d(V_M \sin x_2 - V_T \sin x_3)/r = (n-1)d\dot{q} \tag{25}$$

$$u_s = -dks \tag{26}$$

$$u_n = -nd/V_T\hat{b}\,\mathrm{sgn}(s) \tag{27}$$

where, $\hat{b}$ is the estimation of upper limit of the target acceleration, and k is a positive constant. Then the guidance law can be derived as:

$$V_R - V_L = u_{eq} + u_s + u_n = (n-1)d\dot{q} - dks - nd / V_T \hat{b}\,\mathrm{sgn}(s) \tag{28}$$

According to (4) and (27), the speeds of the two wheels can be obtained:

$$V_R = V_M + (n-1)d\dot{q}/2 - dks - nd\hat{b}\,\mathrm{sgn}(s)/(2V_T) \tag{29}$$

$$V_L = V_M - (n-1)d\dot{q}/2 + dks + nd\hat{b}\,\mathrm{sgn}(s)/(2V_T) \tag{30}$$

This tracking algorithm requires to knowing some information including velocity of the mobile robot, velocity of the target and LOS angles. The information can be obtained from standard onboard sensors or by simple computation.

4 Numerical Simulation

According to Ref.16 and Ref.17, we assume $n < 1/N$, and the initial angle satisfy: $|\theta_0| < \sqrt{6(Nn-1)/(Nn^3-1)}$ and $V_M > da_T/(2V_T)$. In this section a scenario with a maneuvering target is studied via tracking simulations. The performance of the tracking algorithm is investigated using the simulation parameters summarized in Table 1. Suppose d=$0.25m$, and the initial position of the target and mobile robot are $(0,0)$ and $(9.85,1.74)$ respectively.

Table 1. Simulation parameters

Kinematics	parameters	Initial conditions
$V_m = 0.5(1+0.1\cos(2\pi t/8))m/s$	$n = 0.2$	$r_0 = 10\ m$
$V_t = 0.2(1+0.1\sin(2\pi t/8))m/s$	$k = 2$	$\theta_0 = 20^o$
$a_T = 0.2m/s^2$	$\gamma = 2$	$\delta_0 = 20^o$

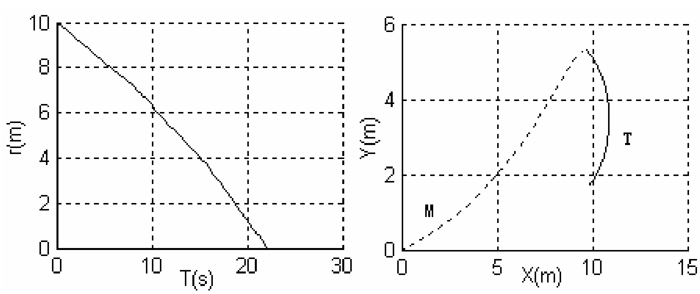

Fig. 2. The range of the mobile robot and the target as well as their trajectories in an Inertial coordinate system. Despite the large initial heading error of 20 degrees, the mobile robot gradually approaches the target and tracks it until catching up the target, and it can be seen that the tracking task was completed in thirty seconds.

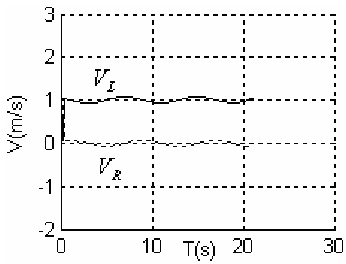

Fig. 3. The simulation of velocities of the mobile robot. V_L and V_R are the variables of the trajectory tracking algorithm design. We can see that the maximal speeds of the two wheels are $1m/s$ about, and it is feasible for mobile robots.

5 Conclusion

In this paper, we have analyzed extensively the performance of the trajectory tracking engagement. The relative kinematics model of the target and the mobile robot is proposed, and a new adaptive sliding mode trajectory tracking algorithm for catching a moving target is designed based on Lyapunov stability theory. A scenario with a mobile robot tracking maneuvering target is studied, and performance is demonstrated via simulation for the tracking of a maneuvering target with large initial heading errors. It can be seen that the trajectory tracking is implemented in thirty seconds, and the guidance algorithm has the robust character for the uncertain maneuvering acceleration and velocity. At the same time the left wheel velocity and the right wheel velocity are designed according to the tracking algorithm. The simulation results indicate the feasibility of the proposed engagement using this tracking algorithm.

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