Trajectory Optimization of Flexible Mobile Manipulators Using Open-Loop Optimal Control Method

M.H. Korayem and H. Rahimi Nohooji

Robotic Lab, Mechanical Department, Iran University of Science and Technology
hkorayem@iust.ac.ir, hamedrahimi.n@gmail.com

Abstract. In this paper Open-loop optimal control method is proposed as an approach for trajectory optimization of flexible mobile manipulator for a given two-end-point task in point-to-point motion Dynamic equations are derived using combined Euler–Lagrange formulation and assumed modes method. To solve the optimal control problem an indirect method via establishing the Hamiltonian function and deriving the optimality condition from Pontryagin's minimum principle is employed. The obtained equations provide a two point boundary value problem which is solved by numerical techniques. The main advantage of this method is obtaining various optimal trajectories with different characteristics by changing the penalty matrices values which able the designer to choose the best trajectory. Finally, a two-link flexible manipulator with mobile base is simulated to illustrate the performance of the method.

Keywords: Mobile Manipulator, Flexible Link, Optimal Trajectory, Optimal Control.

1 Introduction

Mobile manipulators due to their extended workspace offer an efficient application for wide areas. But these systems are usually “power on board” with limited capacity. So because of heavy component such as links, the mobile manipulator needs greater motor, base, and source of energy which leads to consume more energy for the same movement. Hence, using light and small platforms and motor actuators in order to minimize the inertia and gravity effects on actuators will help a mobile manipulator to work in an energy- efficient manner. But the link deflection is unavoidable when the links are light and long, so it leads us to use the elastic manipulators.

Korayem and Ghariblu used Iterative Linear Programming (ILP) method for the maximum allowable dynamic load (MADL) calculation of a rigid mobile manipulator [1]. Korayem and Nikoobin, used the optimal Control Approach to find the maximum load carrying capacity of rigid mobile manipulators for a given two-end-point task [2]. The assumed mode expansion method is used by Sasiadek and Green [3,4,5] to derive the dynamic equation of fixed base flexible manipulator.

In above mentioned works only mobility of base or flexibility of links have been considered, and the synthesis of mobile base with flexible links has not been studied.

In [6,7] a computational algorithm to MADL determination via linearizing the dynamic equation and constraints is presented on the basis of ILP approach for flexible

C. Xiong et al. (Eds.): ICIRA 2008, Part I, LNAI 5314, pp. 54–63, 2008.

mobile manipulators. But, in ILP method, the linearizing procedure and its convergence to the proper answer is a challenging issue, especially when nonlinear terms are large and fluctuating. As a result in none of these papers the link flexibility has been considered either in the dynamic equation or simulation procedure.

The main contribution of this paper is to propose open loop optimal control method for path planning of wheeled mobile manipulator with flexible link. The dynamic equation of flexible link manipulator is derived by using the generalized Euler-Lagrange formulation and assumed modes method. And the extra DOFs arose from base mobility are solved by using additional constraint functions and the augmented Jacobian matrix. Hamiltonian function for a proper objective function is formed, and then necessary conditions for optimality are obtained from the Pontryagin's minimum principle. The obtained equations establish a Two Point Boundary Value Problem (TPBVP) solved by numerical techniques. The general formulation to find the optimal path at point-to-point motion is derived. In comparison with other method the open-loop optimal control method does not require linearizing the equations, differentiating with respect to joint parameters and using of a fixed-order polynomial as the solution form.

The remainder of the paper is the simulation for a two-link mobile manipulator with flexible links in order to investigate the efficiency of the presented method.

2 Modeling of a Manipulator with Multiple Flexible Links

For general n-link flexible robots, the vibration $v_i(\eta_i,t)$ of each link can be obtained through truncated modal expansion, under the planar small deflection assumption of the link.

$$v_i(\eta_i,t)=\sum_{j=1}^{n_i}\phi_{ij}(\eta_i)e_{ij}(t),\ i=1,\ldots,n \tag{1}$$

where n_i is the number of modes used to describe the deflection of link i; $\phi_{ij}(x_i)$ and $e_{ij}(t)$ are the jth mode shape function and jth modal displacement for the ith link, respectively.

In mobile manipulators if manipulator degrees of freedom is denoted by n_m and the base degrees of freedom by n_b , then the overall system degrees of freedom will be $n=n_m+n_b$. Meanwhile, the end effector degrees of freedom in cartesian space is denoted by m. It is well known that in most mobile manipulator systems, we have $n>m$. As a result, the system has kinematic redundancy or extra degrees of freedom on its motion equal to $r=n-m$. There is a well-known method of redundancy resolutions that applies additional suitable kinematic constraint equations to system dynamics and results in simple and on-line coordination of the mobile manipulator during the motion. This method borrows from the extended Jacobian matrix concept.

By using the Lagrangian assumed modes method the dynamic equation of flexible mobile manipulator in compact form could be obtained as follows :

$$M\ddot{q}+H(q,\dot{q})+G(q)=U \tag{2}$$

where M is the mass matrix, H is the vector of Coriolis and centrifugal forces and G describes the gravity effects and $\vec{q} = \left(q_b^T, q_r^T, q_f^T\right)^T$ is generalized coordinate of the system that q_b, q_r, q_f are defining the mobile base motion, rigid body motion of links and flexibility of links respectively.

Consider a n DOFs mobile manipulator with generalized coordinates $q = [q_i]$, $i = 1,2,...,n,$ and a task described by m task coordinates r_j , $j = 1,2,...,m$ with $m < n$. By applying r holonomic constraints and c non-holonomic constraints to the system, $r+c$ redundant DOFs of the system can be directly determined. Therefore m DOFs of the system is remained to accomplish the desired task. As a result, we can decomposed the generalized coordinate vector as $q = [q_r \quad q_{nr}]^T$, where $q_r \in R^{r+c}$ is the redundant generalized coordinate vector determined by applying constraints and $q_r \in R^m$ is the remain generalized coordinate vector.

The system dynamics can also be decomposed into two parts: one is corresponding to redundant set of variables, q_r, and another is corresponding to non-redundant set of them, q_{nr}. That is,

$$\begin{bmatrix} U_r \\ U_{nr} \end{bmatrix} = \begin{bmatrix} M_{r,r} & M_{r,nr} \\ M_{r,nr} & M_{nr,nr} \end{bmatrix} \begin{bmatrix} \ddot{q}_r \\ \ddot{q}_{nr} \end{bmatrix} + \begin{bmatrix} C_r + G_r \\ C_{nr} + G_{nr} \end{bmatrix} \tag{3}$$

by defining the state vector as:

$$X = [X_1 \quad X_2]^T = [q_{nr} \quad \dot{q}_{nr}]^T \tag{4}$$

non-redundant part of Eq.(3) can be rewritten in state space form as:

$$\dot{X} = \left[\dot{X}_1 \quad \dot{X}_2\right]^T = [X_2 \quad N(X) + D(X)U] \tag{5}$$

where $D = M^{-1}$ and $N = -M^{-1}(H(X_1, X_2) + G(X_1))$. Then optimal control problem is to determine the position and velocity variable $X_1(t)$ and $X_2(t)$, and the joint torque $U(t)$ which optimize a well-defined performance measure when the model is given in Eq.(5).

3 Formulation of the Optimal Control Problem

The basic idea to improve the formulation is to find the optimal path for a specified payload, and then maximum payload is obtained via an iterative algorithm. For the sake of this, the following objective function is considered

$$\underset{U(t)}{Minimize}\, J_0 = \int_{t_0}^{t_f} L(X,U)dt \tag{6}$$

where

$$L(X,U) = \frac{1}{2}\|X_1\|_{W_1}^2 + \frac{1}{2}\|X_2\|_{W_2}^2 + \frac{1}{2}\|U\|_R^2 \tag{7}$$

Integrand *L(.)* is a smooth, differentiable function in the arguments, $\| X \|_K^2 = X^T KX$ is the generalized squared norm, W_1 and W_2 are symmetric, positive semi-definite *(m×m)* weighting matrices and *R* is symmetric, positive definite *(m×m)* matrices. The objective function specified by Eqs. (6) and (7) is minimized over the entire duration of the motion. The designer can decide on the relative importance among the angular position, angular velocity and control effort by the numerical choice of W_1, W_2, and *R* which can also be used to convert the dimensions of the terms to consistent units. According to the Pontryagin's minimum principle, the following conditions must be satisfied

$$\dot{X} = \partial H / \partial \psi, \quad \dot{\psi} = -\partial H / \partial X, \quad 0 = \partial H / \partial U \tag{8}$$

where the Hamiltonian function with define the nonzero costate vector $\psi = \begin{bmatrix} \psi_1^T & \psi_2^T \end{bmatrix}^T$ is defined as:

$$\mathrm{H(X, U,}\psi) = 0.5\left(\|X_1\|_{W_1}^2 + \|X_2\|_{W_2}^2 + \|U\|_R^2\right) + \psi_1^T X_2 + \psi_2^T \left[N(X) + D(X)U\right] \tag{9}$$

So, according to Eq. (8), the optimality conditions can be obtained by differentiating the Hamiltonian function with respect to states, costates and control .The control values are limited with upper and lower bounds, so the optimal control is given by

$$U = \begin{cases} U^+ & -R^{-1}D^T\psi_2 > U^+ \\ -R^{-1}D^T\psi_2 & U^- < -R^{-1}D^T\psi_2 < U^+ \\ U^- & -R^{-1}D^T\psi_2 < U^- \end{cases} \tag{10}$$

The actuators which are used for medium and small size manipulators are the permanent magnet D.C. motor. The torque speed characteristic of such D.C. motors may be represented by the following linear equation:

$$U^+ = K_1 - K_2 X_2 \ , U^- = -K_1 - K_2 X_2 \tag{11}$$

where $K_1 = [\tau_{s1} \ \tau_{s2} \cdots \tau_{sn}]^T$, $K_2 = dig\left[\tau_{s1}/\omega_{m1} \cdots \tau_{sn}/\omega_{mn}\right]$, $\dot{\theta} = \left[\dot{\theta}_1 \ \dot{\theta}_2 \cdots \dot{\theta}_n\right]^T$, τ_s is the stall torque and ω_m is the maximum no-load speed of the motor.

In this formulation, for a specified payload value, 4m differential equations is given in order to determine the 4m state and costate variables. Control equations with this set of differential equations and the boundary conditions construct a standard form of TPBVP, which is solvable with available commands in different software such as MATLAB, C++ or FORTRAN.

4 Simulation for a Flexible Planar Wheeled Mobile Manipulator

A two-link planar flexible manipulator is mounted on a differentially driven mobile base at point F on the main axis of the base as shown in Fig.1. A concentrated payload of mass m_p is connected to the second link.

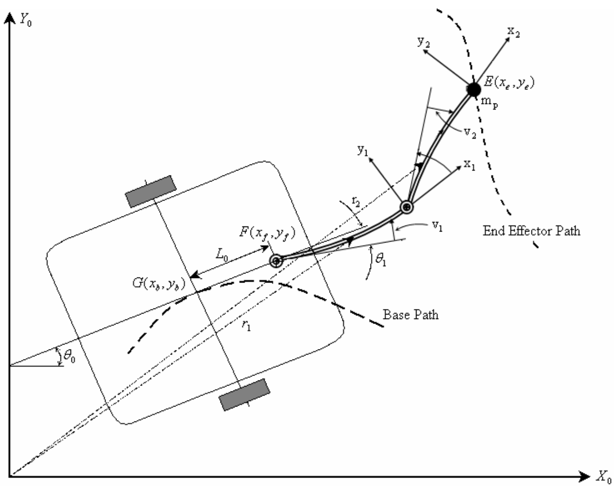


Fig. 1. Two-link mobile manipulator with flexible links

All required parameters of the robot manipulator are given in Table 1.

Table 1. Simulation Parameters

Parameter	Value	Unit
Length of Links	L1=L2=0.5	m
Mass	m1=3, m2=3	Kg
Moment of area (Link)	I1=I2= 2e-10	m4
Module of elasticity (Link)	E1=E2=2e10	Kg.m2
Max. No Load Speed	ωs1=ωs2=3.5	Rad/s
Actuator Stall Torque	τs1=τs2=10	N.m

The load must be carried from an initial point with coordinate (x_e = 0.56m, y_e = 1.25m) to the final point with coordinate (x_e = 2.076m, y_e = 2.362m) during the overall time t_f = 2 s such that optimal trajectory between these two points at a specified time is desired. It should be noted that final load position is not feasible without the base motion. Simultaneously, the mobile base is initially at point (x_0 = 1m, y_0 = 1m, θ_0(1)=0) and moves to final position (x_f = 2m, y_f = 1.5m, θ_0(end)=0.435) and Suppose that L_0 is 40 cm [1].

In order to define the mobile manipulator with flexible link with considering one mode shape for each link the mechanical system generalized coordinates can be chosen as: $q=[q_1 \quad q_2 \quad q_3]=\left[x_f \quad y_f \quad \theta_0 \quad \theta_1 \quad \theta_2 \quad e_{11} \quad e_{21}\right]$, where $q_1=\left[x_f \quad y_f \quad \theta_0\right]$ are generalized coordinates of the base and $q_2=[\theta_1 \quad \theta_2]$ are links angles and $q_3=[e_{11} \quad e_{21}]$ are modal displacement of link. So according to Eq.(1), $v_i(x_i,t)$ can be expressed as follows:

$$v_1=\phi_{11}(x_1)e_{11}(t), \quad v_2=\phi_{21}(x_1)e_{21}(t) \tag{12}$$

where with considering the simply support mode shape [7], ϕ_{ij} can be computed as:

$$\phi_{ij}(x_i)=\sin\left(\frac{j\pi x_i}{L_i}\right), \; i=1\cdots 2 \; and \;\; j=1 \tag{13}$$

Since the motion is in horizontal plan the gravity effects (G_r,G_f) will be zero. And the operational coordinated of the end effector can be specified by $p_{ee}=[x_e \quad y_e]$ and the end effector degrees of freedom in the cartesian coordinate system will be m = 2. The system degree of freedom is equal to n=5, hence the system has redundancy of order R = n-m = 3 and needs three additional kinematical constraints for proper coordination. Meanwhile, the mobile base has one nonholonomic constraint (c=1) i.e. the rolling without slipping condition for the driven wheels: $\dot{x}_f \sin(\theta_0)-\dot{y}_f \cos(\theta_0)+L_0\dot{\theta}_0=0$.

Hence, the number of kinematical constraints which must be applied to system for redundancy resolution is equal to r = R-c = 2. In this case, with the previously specified base trajectory during the motion, the user-specified additional constraints can be considered as the base position coordinates at point $F(x_f, y_f)$, which gives $x_f=X_{1z}$; $y_f=X_{2z}$, where X_{1z} and X_{2z} are functions in terms of time which by differentiating them with can also be obtained. A fifth order polynomial function is considered for the base trajectory along a straight-line path from (1, 1) to (2, 1.5) during the overall time 2 s. Velocity at start and stop time is considered to be zero. From the base motion, $\dot{x}_f$ and $\dot{y}_f$ are known, therefore if the base angle at initial time $\theta_0(t_0)$ be specified, angle and angular velocity of the base $(\theta_0(t),\dot{\theta}_0(t))$, can be determined by solving nonholonomic constraint equations. Here the initial base angle is considered to be zero, $\theta_0(0)=0$, therefore final base angle will be obtained as $\theta_0(end)=0.435$ Rad. By defining the state vectors as follows:

$$X_1=Q^T=\left[x_1 \quad x_3 \quad x_5 \quad x_7\right]^T, \quad X_2=\dot{Q}^T=\left[x_2 \quad x_4 \quad x_6 \quad x_8\right]^T. \tag{14}$$

The sate space form of Eq. (20) can be written as

$$\dot{x}_{2i-1}=x_{2i}, \; \dot{x}_{2i}=F_2(i) \;\; ; i=1,\cdots,4 \tag{15}$$

where $F_2(i)$ can be obtained from Eq.(7). And the boundary condition can be expressed as follows:

$$\begin{aligned} &x_{10}=90^\circ, x_{30}=120^\circ; x_{1f}=30^\circ, x_{30}=60^\circ \\ &x_{2i0}=x_{2if}=x_{(2i-1)0}=x_{(2i-1)f}=0, \quad i=1...4 \end{aligned} \tag{16}$$

In order to derive the equations associated with optimality conditions, penalty matrices can be selected as follows:

$$W_1 = diag\left(w_1, w_3, w_5, w_7\right), \quad W_2 = diag\left(w_2, w_4, w_6, w_8\right); R = diag\left(r_1, r_3\right) \tag{17}$$

Then, by considering the costate vector as $\psi^T = [\varphi_i]^T, i = 1, \cdots, 8$. By differentiating the Hamiltonian function with respect to the states as follows:

$$\dot{\varphi}_i = -\frac{\partial H}{\partial x_i}, \; i = 1, \cdots, 8$$

Control functions are computing by differentiating the Hamiltonian function with respect to control and setting the derivative equal to zero. After using the extrimal bound of control for each motor, by substituting the obtained control equations into (15) and (19), these equations forms 16 nonlinear ordinary differential equations that with 16 boundary conditions given in Eq. (16), constructs a two point boundary value problem. This problem can be solved using the BVP4C command in MATLAB®.

In this case, the payload is considered to be 1 kg and the purpose is to find the optimal path between initial and final point of payload in such a way that the smallest amount control value can be applied and the angular velocity values of motors be bounded in $\pm 0.8\, rad/s$. By considering the penalty matrices as: $W_1 = [0]_{4\times 4}$, $w_2 = w_4 = .1$, $w_6 = w_8 = 0$ and $R = diag\{0.1, 0.1\}$ the optimal path with minimum effort can be obtained, but the angular velocities are greater than $|0.8|$ rad/s. Therefore for decreasing the velocities, w_2 and w_4 must be increased. A range of values of w_2 and w_4 which is used in simulation are given in Table 3. W_1, R, w_6 and w_8 remain without changes.

Table 2. The values of W_2 used in simulation

case	1	2	3	4
w_2 and w_4	Diag(0.1)	Diag(1)	Diag(10)	Diag(100)

The end-effector trajectories in XY plane are shown in Fig. 6 for these cases. Fig. 2 shows the angular position of joints with respect to time. This graph shows that by increasing the w_2 and w_4, the angular position change to approach approximately to a straight line. Fig. 3 shows the angular velocities of the first and second joints. It can be found that by increasing the w_2 and w_4, exterimum values of angular velocity reduce from -1.2 rad/s to -0.8 rad/s. By growing the w_2 and w_4, the angular velocities reduce greatly for the first to second cases whereas at the third case a little reduction has been occurred in spite of great increase in w_2 and w_4.

The computed torque is plotted in Fig. 4. As it can be seen, increasing the w_2 and w_4 causes to raise the torques, so that for the last case the torque curves reach to their bounds at the beginning and end of the path. This result is predictable, because increasing the w_2 and w_4, decreases the proportion of R and the result of this is increasing the control values.

The mode shapes is plotted in Fig. 5 shows the flexible deformation of system in case3.At last Arm motions with related end-effector trajectory in this case are plotted in Fig. 7. (in each figure -, --, -., .., denotes cases1, 2, 3 and 4 respectively)

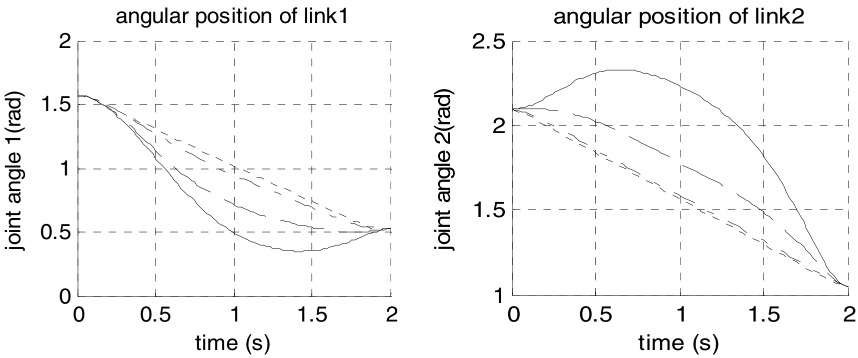


Fig. 2. Angular positions of joint 1 and 2

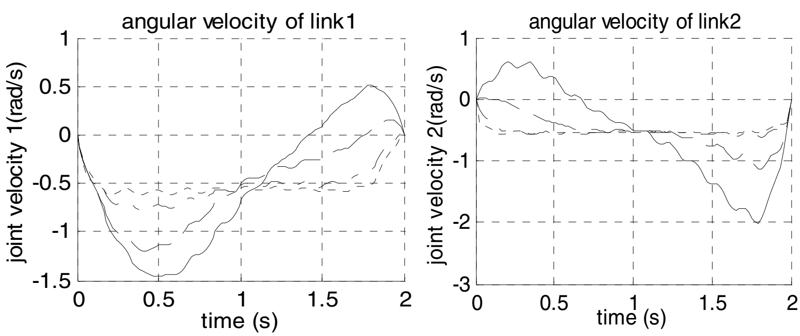


Fig. 3. Angular velocities of joint 1 and 2

Therefore, the first path is the optimal path with the least control values, whereas its angular velocity is the largest magnitude. Finally, the optimal path is the third path which its velocity magnitude is bounded in $\pm 0.62\, rad / s$ interval and the torque values is the lowest. On the basis of the objective contrast principle, there is not the solution that satisfies all the desired objectives simultaneously e.g. the optimal path with minimum effort has maximum velocity and the optimal path with minimum velocity has maximum effort. Consequently, in this method, designer compromises between different objectives by considering the proper penalty matrices.

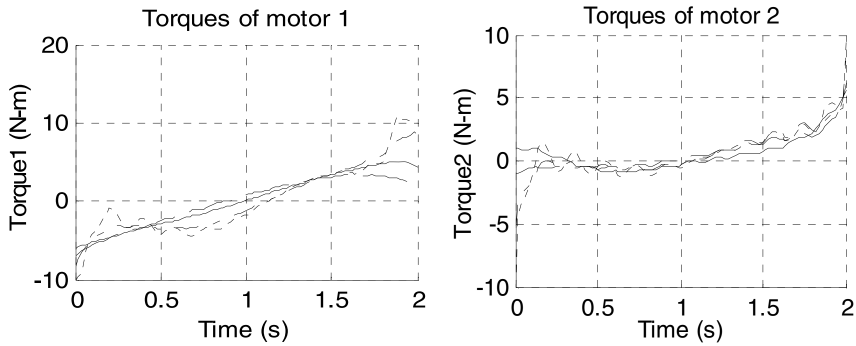
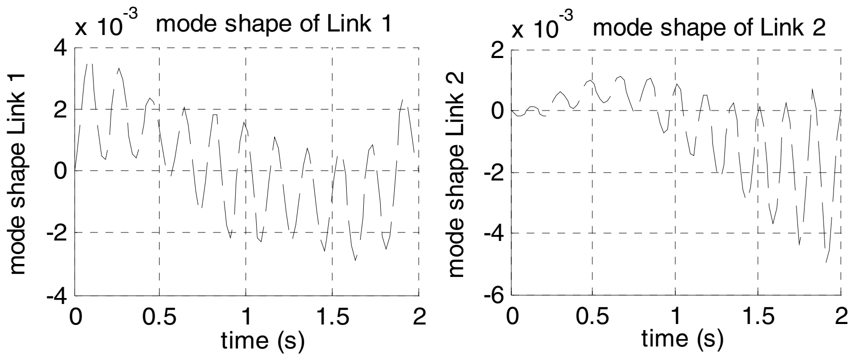


Fig. 4. Torques of motor 1 and



2

Fig. 5. mode shape of link 1 and 2

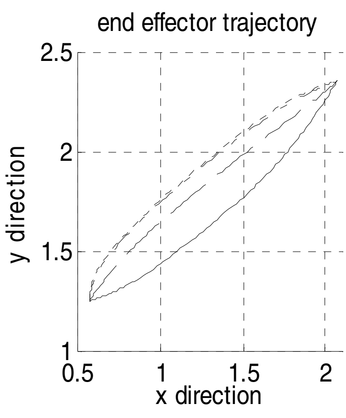


Fig. 6. End effector trajectory in XY plane

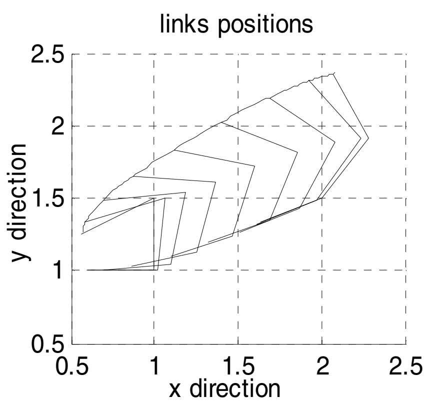


Fig. 7. Arm motion in XY plane

5 Conclusion

In this paper, formulation of the trajectory optimization for mobile flexible links manipulator in point-to-point motion, based on the open-loop optimal control approach is presented. In this method, the complete form of the obtained nonlinear equation is used, and unlike the previous works linearizing the equations or using of a fixed-order polynomial as the solution form is not required. This formulation can be used for path planning of flexible mobile manipulators via defining the proper objective function and changing the penalty matrices to achieve the desired requirements. Therefore, an efficient algorithm on the basis of TPBVP solution is proposed to optimize the path in order to achieve the predefined objective. One of the advantage of this method that designer can compromise between different objectives by considering the proper penalty matrices and is able to choose the proper trajectory among the various paths.

References

1. Korayem, M.H., Ghariblu, H.: Maximum allowable load of mobile manipulator for two given end points of end-effector. Int. J. Adv. Manuf. Technol. 24(9-10), 743–751 (2004)
2. Korayem, M.H., Nikoobin, A., Azimirad, V.: Maximum Load Carring Capacity of Mobile Manipulators using Optimal Control Approach. Robotica Journal (in press)
3. Green, A., Sasiadek, J.Z.: Dynamics and trajectory tracking control of a two-link robot manipulator. Journal of Vibration and Control 10(10), 1415–1440 (2004)
4. Green, A., Sasiadek, J.Z.: Robot Manipulator control for rigid and assumed mode flexible dynamics models. In: AIAA Guidance, Navigation, and Control Conference and Exhibit (2003)
5. Sasiadek, J.Z., Srinivasan, R.: Dynamic modeling and adaptive control of a single link flexible manipulator. AIAA Journal of Guidance, Control and Dynamics 12(6), 838–844 (1989)
6. Korayem, M.H., Gariblu, H.: Analysis of wheeled mobile flexible manipulator dynamic motions with maximum load carrying capacities. Robotics and Autonomous Systems 48(2-3), 63–76 (2004)
7. Gariblu, H., Korayem, M.H.: Trajectory optimization of flexible mobile manipulators. Robotica 24(3), 333–335 (2006)