Dimensional Synthesis and Analysis of the 2-UPS-PU Parallel Manipulator

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Abstract. A new 2-UPS-PU parallel manipulator with one translational and two rotational degrees of freedom (DOF) has been proposed in this paper, which has the advantages of high rotational capability and high load carrying capacity, etc. The mobility of the robot is analyzed. The inverse and forward kinematics problems are solved. The velocity and force Jacobian of the new parallel manipulator are given. An effective method for computing this manipulator's workspace is proposed. The workspace is represented in an ingenious selected cylindrical coordinate system. An optimal dimensional synthesis method is presented to optimize all the parameters of this manipulator for a prescribed workspace with the excellent mechanical performance.

Keywords: Parallel manipulator; Kinematics; static; dimensional synthesis.

1 Introduction

Compare with conventional serial robots, parallel manipulators has the advantages of higher accuracy, higher stiffness, and higher ratio of force-to-weight, so it has been intensively researched and evaluated by industry and intuitions over the last two decades [1].

It is well known that a main drawback of parallel manipulator is their reduced workspace. Furthermore computing this workspace is not an easy task as, at the opposite of classical serial robot, the translational and orientation workspace are coupled [2]. A number of authors have described the workspace of a parallel mechanism by discretizing the Cartesian workspace [3]. In the case of three degree of freedom (3-DOF) manipulators, the workspace is limited to a region of the three-dimensional Cartesian space.

A more challenging problem is designing a parallel manipulator for a given workspace. Merlet [4] propounded an algorithm to determine all the possible geometries of Gough-type 6-DOF parallel manipulators whose workspace must include a desired one. Boudreau and Gosselin [5] proposed an algorithm that allows for the determination of some parameters of the parallel manipulators using a genetic algorithm method in order to obtain a workspace as close as possible to a prescribed one. Kosinska et al. [6] presented a method for the determination of the parameters of a Delta-4 manipulator, where the prescribed workspace has been given in the form of a set of points. Snyman et al. [7] proposed an algorithm for designing the planar 3-RPR manipulator parameters, for a prescribed two-dimensional physically reachable output workspace.

C. Xiong et al. (Eds.): ICIRA 2008, Part I, LNAI 5314, pp. 141–151, 2008.
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Laribi et al. [8] presented an optimal dimensional synthesis method of the DELTA parallel manipulator for a prescribed workspace. This problem was generally solved numerically, and none of the authors mentioned above took driving force into account.

In this paper, the 2-UPS-PU Parallel manipulator is designed to have a specified workspace. The algorithm is proposed to solve the optimization problem, which not only takes into account the leg-length limits, the mechanical limits on the passive joints, and interference between links, but also the driving forces of the three legs.

This paper is organized as follows: Section 2 is devoted to the description of the 2-UPS-PU Parallel manipulator. Section 3 deals with the position analysis of the Parallel manipulator. Section 4 is devoted to the kinematic analysis of the Parallel manipulator. Section 5 deals with the Statics analysis of the Parallel manipulator. In Section 6, we carry out the formulation of the optimization problem. Section 7 contains some conclusions.

2 Displacement Analysis

The 2-UPS-PU parallel manipulator is shown in Fig. 1a. This manipulator consists of three kinematic chains, including two UPS legs with identical topology and one PU leg, connecting the fixed base to a moving platform. In this parallel manipulator, the UPS legs, from base to platform, consist of a fixed Universal joint, an actuated prismatic joint and a spherical joint attached to the platform. The PU leg connecting the base center to the platform consists of a prismatic joint attached to the base, a universal joint attached to the platform. This branch is used to constrain the motion of the platform to the three degrees of freedom.

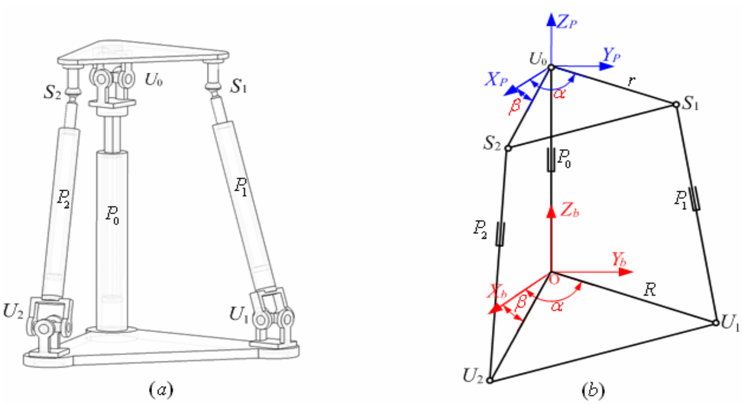

Fig. 1. The 2-UPS-PU parallel manipulator

The reference frame O-$X_bY_bZ_b$ is fixed on the base and mobile frame U_0-$X_pY_pZ_p$ is fixed on the moving platform (see Fig. 1b). At the initial position, the X_p-axis and Y_p-axis of mobile frame are coincidence with the axes of the Universal joints U_0 respectively. The orientation of the first axis of U_0 is fixed. The description of the orientation and position of the mobile frame can be represented by following transform matrix

$$\boldsymbol{T} = Trans_{Z_p}(l_0) R_{X_p}(\theta_1) \cdot R_{Y_p}(\theta_2) = \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 & 0 \\ \sin\theta_1 \sin\theta_2 & \cos\theta_1 & -\sin\theta_1 \cos\theta_2 & 0 \\ -\cos\theta_1 \sin\theta_2 & \sin\theta_1 & \cos\theta_1 \cos\theta_2 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

where l_0 is the distance between the Universal joint U_0 and reference point O, θ_1 and θ_2 are two Euler angles about two axes of U_0, respectively.

The spherical joints (S_1 and S_2) of the UPS legs are arranged on the moving platform and their distances to the Universal joint U_0 on the moving platform is r. The Universal joints (U_1 and U_2) are fixed on the base platform and the distances to the reference point O on the base is R. The coordinate of U_0, S_1 and S_2 in mobile frame and the coordinate of U_1 and U_2 in reference frame are expressed as:

$$\boldsymbol{U}_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \boldsymbol{S}_1 = \begin{pmatrix} r\cos\alpha \\ r\sin\alpha \\ 0 \end{pmatrix} \quad \boldsymbol{S}_2 = \begin{pmatrix} r\cos\beta \\ r\sin\beta \\ 0 \end{pmatrix} \quad \boldsymbol{U}_1 = \begin{pmatrix} R\cos\alpha \\ R\sin\alpha \\ 0 \end{pmatrix} \quad \boldsymbol{U}_2 = \begin{pmatrix} R\cos\beta \\ R\sin\beta \\ 0 \end{pmatrix} \tag{2}$$

2.1 Inverse Kinematics

For a given position and orientation of the mobile platform, we can compute the related link lengths, denoted by l_i, using the following relation:

$$\begin{aligned} l_0 &= l_0 \\ l_1 &= \left\| \boldsymbol{T} \cdot \boldsymbol{S}_1 - \boldsymbol{U}_1 \right\| = \sqrt{a^2 + b^2 + c^2} \\ l_2 &= \left\| \boldsymbol{T} \cdot \boldsymbol{S}_2 - \boldsymbol{U}_2 \right\| = \sqrt{d^2 + e^2 + f^2} \end{aligned} \tag{3}$$

where

$$\begin{aligned} a &= r\cos\alpha\cos\theta_2 - R\cos\alpha \\ b &= r\cos\alpha\sin\theta_1\sin\theta_2 + r\sin\alpha\cos\theta_1 - R\sin\alpha \\ c &= r\sin\alpha\sin\theta_1 - r\cos\alpha\cos\theta_1\sin\theta_2 + l_0 \\ d &= r\cos\beta\cos\theta_2 - R\cos\beta \\ e &= r\cos\beta\sin\theta_1\sin\theta_2 + r\sin\beta\cos\theta_1 - R\sin\beta \\ f &= r\sin\beta\sin\theta_1 - r\cos\beta\cos\theta_1\sin\theta_2 + l_0 \end{aligned}$$

Eq. (3) is the solution of the so-called inverse kinematics problem.

2.2 Direct Kinematics

If set the second axis of Universal joint U_0 pass through either S_1 or S_2 (α=90° or α=0°), we can simply get analytical direct kinematics solution. For example, let α=90°, then cosα=1 and sinα=0. As a result, Eq. (3) for α=90° is simplified as:

$$\begin{aligned} &l_1^2 = r^2 + R^2 + l_0^2 + 2rl_0 \sin\theta_1 - 2rR\cos\theta_1 \\ &l_2^2 = 2rl_0 \sin\beta\sin\theta_1 - 2rR\sin\beta\sin\beta\cos\theta_1 - 2rR\cos\beta\sin\beta\sin\theta_1\sin\theta_2 \\ &\quad - 2rl_0\cos\beta\cos\theta_1\sin\theta_2 - 2rR\cos\beta\cos\beta\cos\theta_2 + r^2 + R^2 + l_0^2 \end{aligned} \tag{4}$$

Then, we can calculate θ_1 and θ_2 by

$$\begin{aligned} \theta_1 &= \sin^{-1}\left(\frac{l_2^2 - r^2 - R^2 - l_0^2}{2r\sqrt{l_0^2 + R^2}}\right) + \tan^{-1}\left(\frac{R}{l_0}\right) \\ \theta_2 &= \sin^{-1}\left(\frac{a+b}{\sqrt{c^2+d^2}}\right) - \tan^{-1}\left(\frac{d}{c}\right) \end{aligned} \tag{5}$$

where

$$\begin{aligned} &a = \left(r^2 + R^2 + l_0^2 - l_1^2\right)/2r & &b = l_0\sin\beta\sin\theta_1 - R\sin^2\beta\cos\theta_1 \\ &c = l_0\cos\beta\cos\theta_1 + R\cos\beta\sin\beta\sin\theta_1 & &d = R\cos^2\beta \end{aligned},$$

It is easy to see that Eq. (5) must satisfies: $\theta_1 \in [-90°, 90°]$, $\theta_2 \in [-90°, 90°]$
Obvious, the same calculation can be drawn when α=0°.

3 Velocity Equation

To differentiae Eq. (3) allows us to obtain the velocities equation as:

$$\begin{pmatrix} \dot{l}_1 \\ \dot{l}_2 \\ \dot{l}_0 \end{pmatrix} = [\boldsymbol{q}] \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{l}_0 \end{pmatrix} = \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{l}_0 \end{pmatrix} \tag{6}$$

where $[\boldsymbol{q}]$ is the kinematic jacobian matrix, and

$$q_{11} = r\left[l_0\left(\cos\alpha\sin\theta_1\sin\theta_2 + \sin\alpha\cos\theta_1\right) + R\sin\alpha\left(\sin\alpha\sin\theta_1 - \cos\alpha\cos\theta_1\sin\theta_2\right)\right]/l_1$$

$$q_{12} = r\cos\alpha\left[R\cos\alpha\sin\theta_2 - R\sin\alpha\sin\theta_1\cos\theta_2 - l_0\cos\theta_1\cos\theta_2\right]/l_1$$

$$q_{13} = l_0/l_1 + r\left(\sin\alpha\sin\theta_1 - \cos\alpha\cos\theta_1\sin\theta_2\right)/l_1$$

$$q_{21} = r\left[l_0\left(\sin\beta\cos\theta_1 + \cos\beta\sin\theta_1\sin\theta_2\right) + R\sin\beta\left(\sin\beta\sin\theta_1 - \cos\beta\cos\theta_1\sin\theta_2\right)\right]/l_2$$

$$q_{22} = r\cos\beta\left[R\cos\beta\sin\theta_2 - l_0\cos\theta_1\cos\theta_2 - R\sin\beta\sin\theta_1\cos\theta_2\right]/l_2$$

$$q_{23} = l_0/l_2 + r\left(\sin\beta\sin\theta_1 - \cos\beta\cos\theta_1\sin\theta_2\right)/l_2$$

4 Statics Analysis

The workloads can be simplified as a wrench $\boldsymbol{F}_{\mathrm{w}}$ applied onto moving platform at U_0. When ignoring the friction in all the joints and the mass of all the parts, the wrench $\boldsymbol{F}_{\mathrm{w}}$ is balanced by three active forces f_{i} (i = 1,2, 3), two constrained forces f_{x} and f_{y}, and a constrained torque m_{z}. Each of f_{i} is applied on and along the axes of three legs;

f_x, f_y and m_z are applied on the moving platform about $\boldsymbol{X}_p$, $\boldsymbol{Y}_p$, $\boldsymbol{Z}_p$, respectively. the formulae for solving active forces and constrained forces are derived as

$$\begin{pmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{pmatrix} = \begin{bmatrix} \boldsymbol{S}_1 & \boldsymbol{S}_2 & \boldsymbol{S}_z & \boldsymbol{S}_x & \boldsymbol{S}_y & \boldsymbol{0} \\ \boldsymbol{S}_{01} & \boldsymbol{S}_{02} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{S}_z \end{bmatrix} \cdot \begin{pmatrix} f_1 \\ f_2 \\ f_0 \\ f_x \\ f_y \\ m_z \end{pmatrix} \tag{7}$$

where

$\boldsymbol{S}_x = (1 \quad 0 \quad 0)^T$, $\boldsymbol{S}_y = (0 \quad 1 \quad 0)^T$, $\boldsymbol{S}_z = (0 \quad 0 \quad 1)^T$,

$\boldsymbol{S}_i = (\boldsymbol{T} \cdot \boldsymbol{S}_i - \boldsymbol{U}_i)/l_i$, $\boldsymbol{S}_{0i} = [\boldsymbol{S}_i \times (\boldsymbol{T} \cdot \boldsymbol{S}_i - \boldsymbol{U}_i)] / [\boldsymbol{S}_i \times (\boldsymbol{T} \cdot \boldsymbol{S}_i - \boldsymbol{U}_i)]$ $i = 1, 2$

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5 Workspace

5.1 Mechanical Constraints

There are four main mechanical constraints that limit the workspace of a parallel manipulator [1]: (i) Workspace singularities, (ii) the actuators stroke, (iii) the range of the passive joints, and (iv) the link interference.

5.1.1 Workspace Singularities

The theoretical workspace of the manipulator is surrounded by singular surface. In the theoretical workspace, the determinant of jacobian matrix ($|\boldsymbol{q}|$) should be always greater or less than zero. The relationship between the determinant of jacobian matrix

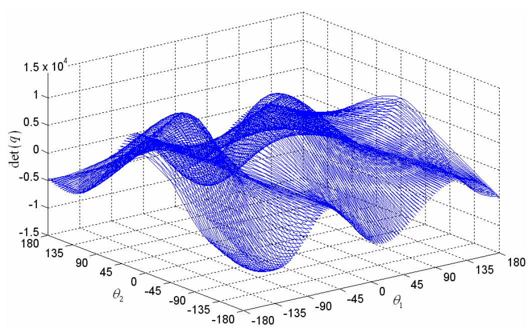

Fig. 2. The relationship between the determinant of jacobian matrix and θ_1, θ_2

and parameters of θ_1、θ_2, for r =150, R =200, α=π/2, β=π/6 and l_0=300, is shown in Fig. 2. If we set θ_1=θ_2=0 as the initial oriention, the determinant of jacobian matrix should be always greater than zero.

$$|q|>0 \tag{8}$$

5.1.2 Actuators' Stroke

The limited stroke of actuator i imposes a length constraint on link i, such that

$$\left(l_{i\max}-l_{i\min}\right)/l_{i\min}\le 0.8 \qquad i=0,1,2 \tag{9}$$

where $l_{i\min}$ and $l_{i\max}$ are, respectively, the minimum and maximum lengths of leg i.

5.1.3 Range of the Passive Joints

Each passive joint has a limited range of motion. The limits on each Universal joint impose a constraint, such that

$$\theta_U \le \theta_{U\max} \tag{10}$$

Similarly, the limits on each spherical joint impose a constraint, such that

$$\theta_S \le \theta_{S\max} \tag{11}$$

5.1.4 Link Interference

Let us assume that the links can be approximated by cylinders of diameter D. This imposes a constraint on the relative position of all pairs of links, such that

$$distance\left(l_i \quad l_j\right)\ge D \quad i,j=0,1,2 \quad i\ne jl \tag{12}$$

or the minimum distance between every two line segments corresponding to the links of the parallel manipulator should be greater than or equal to D. The minimum distance between two line segments is not given by a simple formula but can be obtained through the application of a multi-step algorithm. Due to space limitations, we will not present that algorithm here but refer the reader to the well-detailed one given in [9].

5.2 Workspace Representation

According to the characteristics of the 2-UPS-PU parallel manipulator, we present a cylindrical coordinate system (see Fig. 3), where φ and γ are exactly the polar coordinates representing the orientation of the moving platform and l_0 is the z-coordinate.

The unit vector of Z_p-axis in reference frame is expressed as:

$$\boldsymbol{Z}_p=\begin{pmatrix}\sin\gamma\cos\varphi\\ \sin\gamma\sin\varphi\\ \cos\gamma\end{pmatrix}=\begin{pmatrix}\sin\theta_2\\ -\sin\theta_1\cos\theta_2\\ \cos\theta_1\cos\theta_2\end{pmatrix} \tag{13}$$

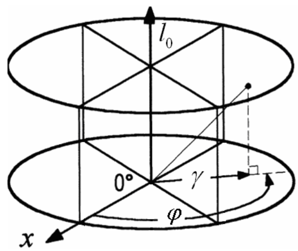

Fig. 3. Cylindrical coordinate system

Then, we can calculate θ_1 and θ_2 by

$$\begin{aligned} \theta_2 &= \sin^{-1}\left(\sin\gamma\cos\varphi\right) \\ \theta_1 &= -\sin^{-1}\left(\sin\gamma\sin\varphi/\cos\theta_2\right) \end{aligned} \tag{14}$$

5.3 Algorithm for the Workspace

Step 1: Initialize double arrays A, B and Z, with dimensions (n+2)×m, where n is the number of equally spaced planes l_0 between $l_{0\min}$ and $l_{0\max}$ at which the workspace will be computed, and m is the number of points to be computed at each plane l_0=const. These arrays will store, respectively, the values of φ, γ and l_0 for the points defining the workspace boundary.
Step 2: Set r, R, h, α, β and D.
Step 3: Set θ_{umax}, θ_{smax}, $l_{i\min}$ and $l_{i\max}$, Where i =0,1 ,2.
Step 4: Set $l_0=l_{0\min}$.
Step 5: For the current l_0, construct a polar coordinate system at (φ,γ). Starting at m equally spaced angles, increment the polar ray, solve the inverse kinematics, and apply the constraint checks defined by Eqs. (8)-(12) until a constraint is violated. The values for φ, γ and l_0 at the point of constraint violation are written into the three double arrays.
Step 6: Set $l_0 = l_0 + \Delta l_0$, where $\Delta l_0 = (l_{0\max} - l_{0\min})/n$
Step 7: Repeat steps 5-6 until l_0 becomes greater than $l_{0\max}$.
Step 8: Transfer A and B into X and Y, so that X[i,j]=B[i,j]cos(A[i,j]), Y[i,j]=B[i,j]sin(A[i,j]), where i=1..n+1 and j=1..m.

The proposed algorithm was implemented in MATLAB for the 2-UPS-PU parallel manipulator whose data are r=100, R=120,α=90°, β=25°, D=5, $l_{\min}$=320, $l_{\max}$=578, $l_{0\min}$=320, $l_{0\max}$=570,θ_{Umax}=90° and θ_{Smax}=60°. The workspace of the manipulator is presented in Fig. 4. and the approximated projected workspaces for l_0=350, 450 and 500 shown in Fig. 5.

6 Optimal Design

The aim of this section is to develop and to solve the multidimensional optimization problem of selecting the geometric design variables for the 2-UPS-PU parallel manipulator having a prescribed workspace with better driving capability.

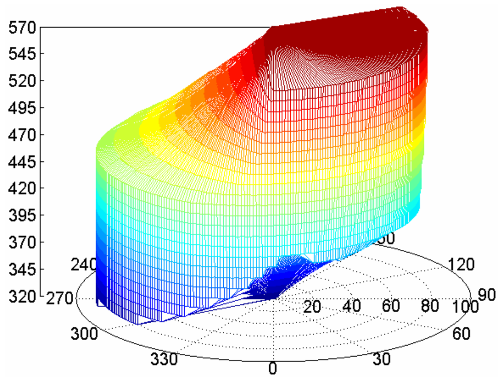

Fig. 4. Isometric view of the Parallel manipulator's workspace

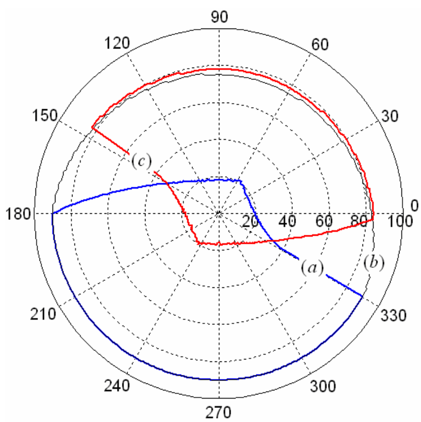

Fig. 5. The projected workspace of the Parallel manipulator for the positions (a) l0=350, (b) l0=450 and (c) l0=500

The prescribed workspace of the parallel manipulator is defined as a cylinder with radius γ and height h and the actual workspace is convex (see Fig.4). The prescribed workspace is inside the workspace of the parallel manipulator if all the points in the boundary curves are inside the actual workspace.

6.1 Objective Function

The parameters to be optimized are the minimum lengths of three legs ($l_{0\min}$, $l_{1\min}$ and $l_{2\min}$), the dimension of the base and the platform (R, r), and the relative angular position of the two UPS chains (α, β). Without losing generality, let r be normalized by h, and R, $l_{0\min}$ be normalized by r such that

$$kr = r / h, \quad kR = R / r, \quad kl_0 = l_{0\min} / r \tag{15}$$

The objective function of the multi-parameter optimization problem, can be stated as

$$\begin{aligned} &\min \quad k = \pi R^2 l_{0\min} F \\ &\text{Subject to } f(\boldsymbol{I}, p) \le 0 \text{ for all the points } p \text{ inside the specified workspace.} \end{aligned} \tag{16}$$

where $\boldsymbol{I}$ is the unknown vector of parameters, and F is the maximum of the driving force of the three legs.

6.2 Algorithm for Dimensional Synthesis

Step 1: Initialize double arrays A, B, C and D, with dimensions 361×1. Set the parameters γ_u and h representing the prescribed workspace.

Step 2: Set α=90°, k=0, and the allowable parameter ranges for β, kr, kR, and $kl_{0\min}$.

Step 3: Set the cycle number n.

Step 4: Random select the parameters β, kr, kR, and $kl_{0\min}$ by Monte Carlo method in ranges.

Step 5: Set l_0= $kl_{0\min}$ and γ=γ_u. Starting at 360 equally spaced angles, increment the angle φ, solve the inverse kinematics, write the length of the UPS legs into array A and B. set $l_{1\min}$=$l_{2\min}$=min(min(A),min(B)), and $l_{1\max}$=$l_{2\max}$=1.8×$l_{1\min}$.

Step 6: Set flag=0. For the current l_0, construct a polar coordinate system at (φ,γ). Starting at 360 equally spaced angles, increment the polar ray, solve the inverse kinematics, and apply the constraint checks defined by Eqs. (8)-(12) until γ>γ_u. Set flag=1 at the point of constraint violation and return to Step 12.

Step 7: If flag=0, then set l_0= $kl_{0\min}$+h. For the current l_0, construct a polar coordinate system at (φ,γ). Starting at 360 equally spaced angles, increment the polar ray, solve the inverse kinematics, and apply the constraint checks defined by Eqs. (8)-(12) until γ>γ_u. Set flag=1 at the point of constraint violation and return to Step 12.

Step 8: If flag=0, then set l_0= $kl_{0\min}$ and γ=γ_u. Starting at 360 equally spaced angles, increment the angle φ, solve the Statics, and write the maximum driving forces of the three legs into array C.

Step 9: If flag=0, then set l_0= $kl_{0\min}$+h and γ=γ_u. Starting at 360 equally spaced angles, increment the angle φ, solve the Statics, and write the maximum driving forces of the three legs into array D.

Step 10: Set F=max(max(C), max(D)), and k_d= $FRl_{0\min}$.

Step 11: If k=0 or k_d <k, set Or= kr, OR= kR, O l_0= k l_0, O$l_{\min}$= $l_{1\min}$ and O$l_{\max}$= $l_{1\max}$.

Step 12: Repeat steps 4-11 until the cycle time is n.

The proposed algorithm was again implemented in MATLAB. The optimal solution obtained for this example is presented in Table 1 for γ_u=60° and h=100

Fig. 6 shows the workspace of the 2-UPS-PU parallel manipulator.

Fig. 7 shows the relationship between the minimum of γ and l_0. One can notice the actual workspace include the prescribed workspace.

Table 1. The optimal dimensions of 2-UPS-PU parallel manipulator

β	R	r	$l_{1,2min}$	$l_{1,2max}$	l_{0min}	l_{0max}
27°	186	124	396	712	450	650

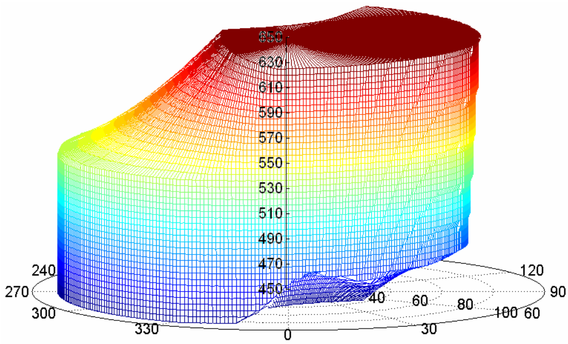

Fig. 6. Isometric view of the Parallel manipulator's workspace

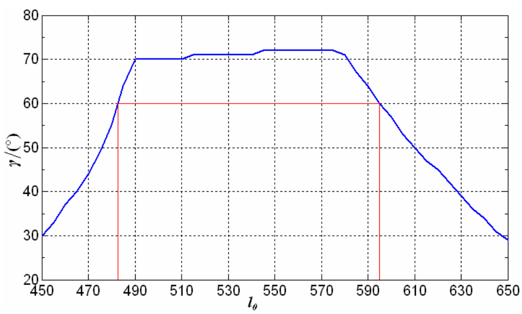

Fig. 7. The relationship between the minimum of γ and l_0

7 Conclusion

This paper proposed a novel 2-UPS-PU parallel manipulator with simple structure, high rotational capability, high load carrying capacity.

Based on the cylindrical coordinate system, an algorithm for computing three-dimensional workspace of the manipulator has been proposed in this paper. The boundary of the workspace on the specific plan is found out quickly by step-searching along the selected ray line.

An optimal dimensional synthesis method was presented to optimize this manipulator for a prescribed workspace. The driving force parameters were introduced into the object function. All the dimensional parameters, including the length of the legs, were included into the optimizing algorithm.

The methods proposed in this parper can be adopted universally. The results of this paper provide solid theoretical basis for further theoretical studies and practical application of this manipulator.

References

1. Huang, Z., Kong, L.F., Fang, Y.F.: Theory on Parallel Robotics and Control. Machinery Industry Press, Beijing (1997) (in Chinese)
2. Merlet, J.-P.: Still a long way to go on the road for parallel mechanisms. In: R. ASME DETC Conference, Montréal (2002)
3. Huang, T., Wang, J.S., Whitehouse, D.J.: Closed form solution to the position workspace of Stewart parallel manipulators. J. Science in China, Series E: Technological Sciences 41, 394–403 (1998)
4. Merlet, J.-P.: Designing a parallel manipulator for a specific workspace. J. International Journal of Robotics Research 16, 545–556 (1997)
5. Boudreau, R., Gosselin, C.M.: The synthesis of planar parallel manipulators with a genetic algorithm. J. ASME J. Mech. 121, 533–537 (1999)
6. Kosinska, A., Galicki, M., Kedzior, K.: Design and optimization of parameters of Delta-4 parallel manipulator for a given workspace. J. Robot. Syst. 20, 539–548 (2003)
7. Snyman, J.A., Hay, A.M.: Optimal synthesis for a continuous prescribed dexterity interval of 3-DOF parallel planar manipulator for different prescribed output workspaces. In: Proceeding of CK 2005, 12th International Workshop on Computational Kinematics Cassino (2005)
8. Laribi, M.A., Romdhane, L., Zeghloul, S.: Analysis and dimensional synthesis of the DELTA robot for a prescribed workspace. J. Mechanism and Machine Theory 42, 859–870 (2007)
9. Masory, O., Wang, J.: Workspace evaluation of Stewart platforms. J. ASME, Design Engineering Division 45, 337–345 (1992)