Orientation-Singularity and Orientation-Workspace Analyses of the Stewart Platform Using Unit Quaternion

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Abstract. The orientation-singularity expression of the Stewart platform is deduced by using unit quaternion which can avoid singularity when using Euler angles to represent the orientation of rigid body. The concept and algorithm for nonsingular orientation-workspace at a certain position is proposed. The practical orientation-workspace that is the minimum inscribed sphere of the nonsingular orientation-workspace's boundary surface is put forward. The radius of the sphere namely practical orientation-capability is used to evaluate the practical orientation-workspace. Examples of a Stewart platform are given to illustrate the results including the graphical nonsingular orientation- workspace at a certain position and the practical orientation-capability versus positions within position-workspace of "initial orientation". The research results can be some useful to the manipulator's application.

Keywords: Stewart Platform; Orientation-singularity; Orientation-workspace Unit quaternion.

1 Introduction

During the past more than two decades, parallel manipulators have attracted many scholars' attention. The most important reasons is that parallel manipulators have many specific advantages over serial manipulators. Stewart platform is a well-known six-degree freedom manipulator which was introduced as an aircraft simulator by Stewart [1] in 1965.

The singular configuration is one of important problems in parallel manipulator. It is an intrinsic property of the parallel manipulator and has plenty of effects on it. Many scholars have paid attentions to this topic [2-10]. It is easily can be find that most lectures are concerning constant orientation singularity or position-singularity, which describes that an arbitrary point C in the mobile platform when the mobile one is kept at constant orientation.

The workspace is another important problem and a performance index being worthwhile attended to for parallel manipulators. It is very difficult to represent in 6-dimensional (6-D) space but easy to do it in 3-D space. Therefore a lot of works about algorithms for workspace in 3-D space of parallel manipulators have been reported

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[11-15]. Similarly to the problem of singular configuration, special attention has been paid to constant orientation workspace or position-workspace, which describes the set of all positions can be reached by an arbitrary point C in the mobile platform when the mobile platform is kept at a constant orientation.

From these reported papers above, we can easily find that there are few works exist on the topics of orientation- singularity and orientation-workspace when the point C is kept as fixed. The most relevant investigations respectively have been made in [16] using Euler angles to represent the orientation of rigid body. As well known that, Euler angles may present singularity when they represent the orientation of rigid body (It means that there is no smooth globally solution on inverse problem of rotation matrix).

The Stewart platform is shown in Fig.1, whose mobile platform and fixed one are two similar semiregular hexagons in shape but opposite in direction in the initial pose (position and orientation). B_i and C_i (i =1,2,…,6) are the vertexes of the two hexagons respectively. A_i (i =1,3,5) is the intersection points of the hexagon sides of fixed platform. Each branch is consisted of one prismatic joint and two sphere joints locating at mobile and fixed platform namely B_i and C_i respectively. The mobile platform can arrive at some pose via changing the length of each branch so that the manipulator is six degrees of freedom.

In this paper, the orientation-singularity and orientation-workspace are addressed using unit quaternion, which can avoid singularity when using Euler angles to represent the orientation of rigid body.

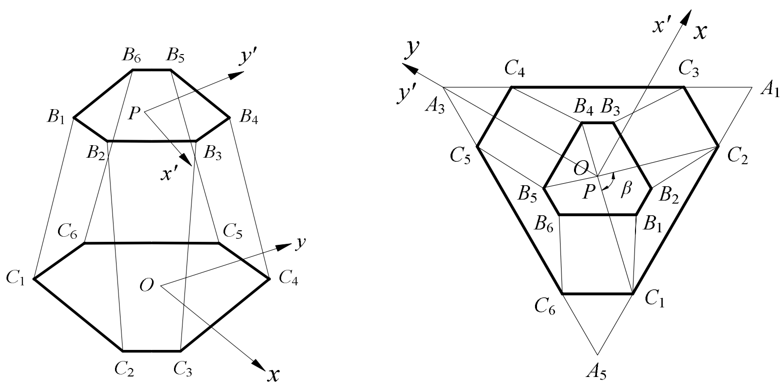


Fig. 1. Schematic map of the Stewart Platform

2 Orientation-Singularity Analyses

A moving reference frame *P-XYZ* and a fixed one *O-XYZ* are respectively attached to the mobile platform and the fixed one of the manipulator, as shown in Fig. 1, where origins P and O are corresponding geometric center of the mobile platform and the fixed one. The Cartesian coordinates of the mobile platform are given by the position of point P with respect to the fixed frame, designated by (X, Y, Z), and the orientation of the mobile platform is represented by unit quaternion. Now let us define a "initial orientation" of the manipulator, which should suffice the following conditions.

(1) The moving reference frame has the same orientation as the fixed one on the fixed platform.

(2) Origin *P* of the moving reference frame is directly on the top of origin *O* of the fixed one.

We can use unit quaternion to represent the orientation of mobile platform. Unit quaternion is defined as $Q = x_0 + x_1\boldsymbol{i} + x_2\boldsymbol{j} + x_3\boldsymbol{k}$ ($x_0^2 + x_1^2 + x_2^2 + x_3^2 = 1$). It means that the mobile platform is rotated by an angle of $2\arccos x_0$ around a line which passes the point *P* and whose direction is $x_1\boldsymbol{i} + x_2\boldsymbol{j} + x_3\boldsymbol{k}$ with respect to the fixed reference. It is defined that the x_0 value is positive namely $x_0 \geq 0$ in this paper. As a result, one unit quaternion defines one certain orientation of the manipulator, vice versa. The rotation matrix here is as follows:

$$\begin{bmatrix} 2(x_0^2 + x_1^2) - 1 & 2(x_1x_2 - x_0x_3) & 2(x_1x_3 + x_0x_2) \\ 2(x_1x_2 + x_0x_3) & 2(x_0^2 + x_2^2) - 1 & 2(x_2x_3 - x_0x_1) \\ 2(x_1x_3 - x_0x_2) & 2(x_2x_3 + x_0x_1) & 2(x_0^2 + x_3^2) - 1 \end{bmatrix} \quad (1)$$

The group operation of unit quaternion and the one of rotation matrix are direct correspondence. Therefore, it can not avoid singularity when using Euler angles to represent the rotary motion of rigid body, but can avoid it using unit quaternion [18].

Substituting the equation (1) and $x_0 = \sqrt{1 - x_1^2 - x_2^2 - x_3^3}$ to the Jacobian matrix of the manipulator [17], and then let the determinant of the matrix to be equal to zero. By expanding and simplifying it, an expression with respect to $x_1x_2x_3$ can be deduced as follows after some rearrangements.

$$\begin{aligned} & f_1x_1^6 + f_2x_1^5x_2 + f_3x_1^5x_3 + f_4x_1^5\sqrt{1 - x_1^2 - x_2^2 - x_3^2} + \cdots + \\ & f_{69}x_2^2 + f_{70}x_2x_3 + f_{71}x_2\sqrt{1 - x_1^2 - x_2^2 - x_3^2} + f_{72}x_3^2 + f_{73} = 0 \end{aligned} \quad (2)$$

Eq.(2) represents the orientation-singularity expression of the manipulator in the 3-D rotation space at a specified position, *P*(*X*, *Y*, *Z*), of the mobile platform. Coefficients of Eq. (2), f_i (i=1,2,…,6), are all functions of geometric parameters, R_a, R_b, β, and orientation parameters, (x_1, x_2, x_3), of the manipulator. R_a and R_b are the circum-radii of the fixed platform and the moving platform respectively. Further inspection shows that there are many terms about $\sqrt{1 - x_1^2 - x_2^2 - x_3^2}$ in the expression, and the highest degree of $x_1x_2x_3$ is 6. It should be found that the orientation-singularity expression with respect to (x_1, x_2, x_3) is very complex.

Graphical representation of the orientation-singularity locus of the manipulator is given to illustrate the result, as shown in figure 2. The geometric parameters used in this example are given as $R_a = 20$ unit, $R_b = 15$ unit, $\beta = 90°$.

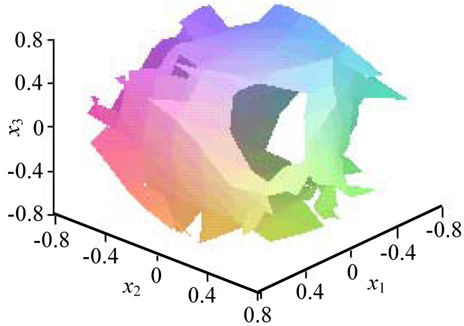


Fig. 2. Orientation-singularity locus at position (0,0,40) unit

Pernkopf and Husty [19] have pointed out that for the Stewart platform with planar base and platform, it must exist a nonsingular void in the three-dimensional rotation space, i.e., (x_1, x_2, x_3), around the orientation origin (0,0,0) for each position (*X*,*Y*,*Z*) in the position-workspace when the mobile platform is kept at the "original orientation", i.e., $(x_1, x_2, x_3) = (0,0,0)$.

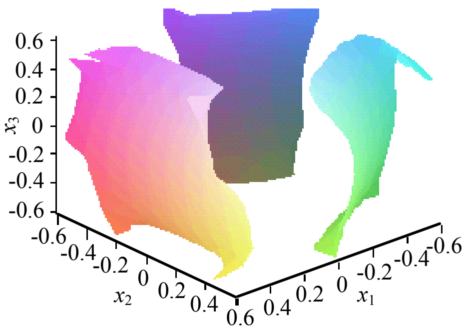


Fig. 3. Nonsingular orientation void at (0,0,40) unit

Fig.3 illustrates that there is certainly an nonsingular orientation-void around the orientation origin (0,0,0) for a given position as presented in [19].

It should be noted that 1) according to the conclusion presented in [19], for each position (*X*, *Y*, *Z*) in the position-workspace, then it is always possible to find a inscribed sphere *C* which is completely inside the nonsingular orientation void and the sphere *C* encloses a region in the rotation space guaranteeing the manipulator can not be singular at that position (*X*, *Y*, *Z*), however, it is of great possibility that the aforementioned inscribed sphere *C* may contain orientations that the mobile platform of the manipulator can't be attained when the mobile platform being fixed to that position (*X*, *Y*, *Z*) in the fixed frame *O-XYZ*, that is, the inscribed orientation sphere *C* may contain orientations which is out of the boundary of the orientation-workspace taking the limitations of active and passive joints and the link interference all into account when the mobile platform being fixed to that position (*X*, *Y*, *Z*); 2) moreover, the orientation-capability of this present manipulator are only analyzed at some chosen discrete positions in the position-workspace, this analysis, however, can be extended to the whole position-workspace of the manipulator.

For the two above-mentioned reasons, orientation-capability of the manipulator can be further analyzed over the whole position-workspace of this present manipulator where singularities, the limitations of active and passive joints and link interference are all taken into consideration can be very constructive in practice, and thus a new concept of "practical orientation-capability" can be introduced as another performance index and design criterion for the orientation-capability of this special class of the Stewart Platform at a certain position (*X*, *Y*, *Z*) in the position-workspace. Similarly, the radius *R* of the inscribed sphere *C* can also be a nice performance index for the measurement of the practical orientation-capability of this present manipulator at a certain position (*X*, *Y*, *Z*) in the position-workspace.

3 Orientation-Workspace Analyses

3.1 Inverse Kinematics

Starting point of the workspace analysis of a mechanism is the solution of the inverse kinematics. Given a pose of the manipulator, we may compute the necessary link-vectors, denoted by l_i , using the following formula.

$$\boldsymbol{l}_i = \boldsymbol{B}_i - \boldsymbol{C}_i \quad (i = 1,2,\cdots,6) \tag{3}$$

$\boldsymbol{B}_i$ and $\mathbf{C}_i$ represent the vector of the i th sphere joint B_i and C_i with respect to the fixed reference *O-XYZ*. l_i represent the length of the i th prismatic joint can be obtained as follows.

$$l_i = |\boldsymbol{l}_i| = |\boldsymbol{B}_i - \boldsymbol{C}_i| \; (i = 1,2,\cdots,6) \tag{4}$$

3.2 Kinematic Constraints

In determining the workspace of the manipulator, there are three main kinematic constraints that limit the workspace of the Stewart platform, which should be considered. They are the actuators' stroke, the range of sphere joints, and the movement interference of actuators.

Actuators' stroke. The limited stroke of the i th actuator imposes a link-length constraint on the i th prismatic joint, that is

$$L_{\min i} \le l_i \le L_{\max i} \quad (i = 1,2,\cdots,6) \tag{5}$$

where $l_{\min i}$ and $l_{\max i}$ are, respectively, the minimum and maximum lengths of the i th actuator.

Range of sphere joints. Let $\theta_{B\max i}$ and $\theta_{C\max i}$ are the range of the i th sphere joint B_i and C_i respectively. We can obtain

$$\theta_{Bi} = \arccos \frac{\boldsymbol{l}_i \cdot (\boldsymbol{G}\boldsymbol{l}_{ni})}{|\boldsymbol{l}_i| \cdot |\boldsymbol{l}_{ni}|} \tag{6}$$

$$\theta_{Ci} = \arccos \frac{\boldsymbol{l}_i \cdot \boldsymbol{l}_{ni}}{|\boldsymbol{l}_i| \cdot |\boldsymbol{l}_{ni}|} \tag{7}$$

where l_{ni} is the i th acuator's vector with respect to the fixed reference when each actuator's length is $0.5(L_{\min i} + L_{\max i})$ and the mobile platform parallels to the fixed one, for this assembly method can effectively broaden the range of each sphere joint. $\boldsymbol{G}$ is the rotation matrix.

Movement interference of actuators. Supposing that each actuator can be approximated by a cylinder of diameter D_i, and $D_{i,i+1}$ $(i, i+1 = 1,2,\cdots,6)$ is the short distance between two adjacent actuators. This imposes a constraint on the relative position of all pairs of actuators, such that

$$D \le D_{i,i+1} \quad (i, i+1 = 1,2,\cdots,6) \tag{8}$$

3.3 Algorithm for Orientation-Workspace

Considering the characteristics of unit quaternion, we use the Spherical Coordinate to compute the orientation-workspace. Above all, (x_1, x_2, x_3) is converted to the formula as follows.

$$\begin{aligned} x_1 &= R \sin\varphi \cos\alpha \\ x_2 &= R \sin\varphi \sin\alpha \quad \varphi \in [0,\pi], \alpha \in [0,2\pi], R \in [0,1] \\ x_3 &= R \cos\varphi \end{aligned} \tag{9}$$

The substitution of expression (9) into (2) lead to the expression with respect to R, φ, α as follows.

$$f(R, \varphi, \alpha) = 0 \tag{10}$$

For any given set of manipulator dimensions, the following steps are taken.

(1) Computation of the position-workspace of the manipulator for the "initial orientation", $(x_1, x_2, x_3) = (0,0,0)$, where limitations of active and passive joints and the movement interference of actuators are all considered by employing the algorithm proposed in [17]. For any given position (X,Y,Z) in the position-workspace, it must exist a nonsingular void in the rotation space, which is inside the orientation-singularity locus. Now we can compute the orientation-workspace at the given position.

(2) R, φ and α, whose variation ranges belong to Eq.(9), should be divided via equal step of value ΔR, $\Delta\varphi$ and $\Delta\alpha$ respectively.

(3) In one subspace of φ, the steps should be carried out as follows.

(a) Increase the value R from zero of step size ΔR with one certain value α and substitute them, R, φ and α, into Eq.(10) and Eq.(11). Note the value R

when one of any Eq.(10-11) is tenable. Otherwise no one is tenable, denote R as value 1.

$$\begin{aligned}
&l_i = L_{\min i};\; l_i = L_{\max i} \\
&\theta_{Bi} = \theta_{B\max i};\; \theta_{Ci} = \theta_{C\max i} \quad (i, i+1 = 1,2,\cdots,6) \\
&D_{i,i+1} = D
\end{aligned} \tag{11}$$

(b) Don't repeat the step (a) but with different value α, until the all of the value α are accomplished.

(4) Repeat the step (3) in every other subspace of φ.

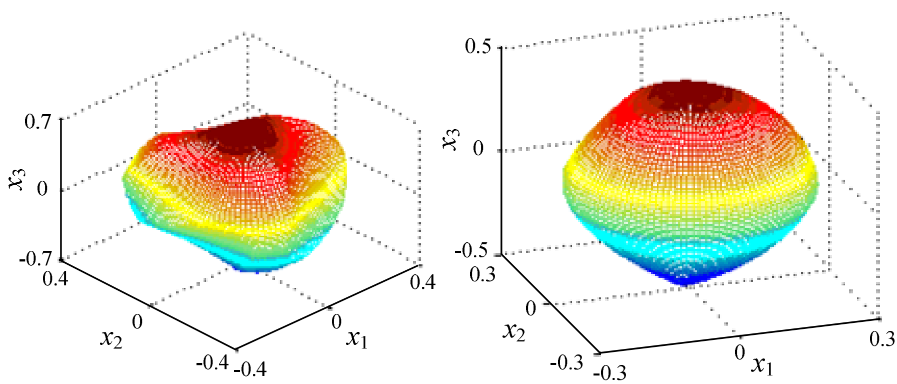
(5) Compute the value, x_1, x_2 and x_3, according to Eq.(9). Then the nonsingular orientation-workspace can be graphically represented using computer.

The practical orientation-capability can be obtained by taking minimal valve of the R computed above at the given position.

4 Numerical Examples

The orientation-workspace, nonsingular orientation-workspace, and practical orientation-workspace of a Stewart platform, which is under investigation at our laboratory, will be numerically studied to demonstrate the aforementioned theoretical results. The design parameters of the manipulator are given as $R_a = 20$ unit, $R_b = 15$ unit, $\beta = 90^\circ$, $\theta_{B\max i} = \theta_{C\max i}$ (i=1, 2,…,6), $L_{\min i} = 30$ unit, $L_{\max i} = 50$ unit, D_i = 5 unit(i=1, 2,…,6).

Nonsingular orientation-workspace is graphically represented as shown in Fig.4, when the mobile platform at a certain position, (X,Y,Z)=(0,0,40) unit. It means that the manipulator can arrive at any orientation and will never be singular inside the surface namely the boundary of the orientation-workspace.



(a) $\theta_{B\max i} = \theta_{C\max i} = 45^\circ$ (b) $\theta_{B\max i} = \theta_{C\max i} = 30^\circ$

Fig. 4. Nonsingular orientation-workspace at position (0,0,40) unit

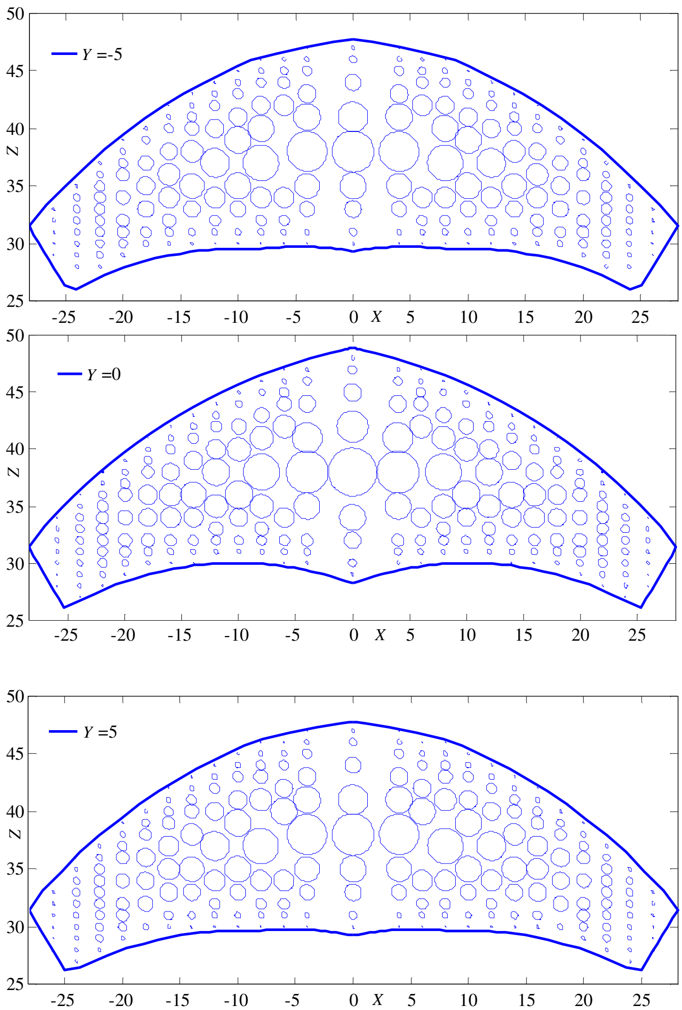


Fig. 5. Practical orientation-capability R versus position parameters (X, Z) for three different over the whole position-workspace (a) for Y = -5; (b) for Y = 0; (c) for Y = 5

Fig. 5 shows the practical orientation-capability analysis of this special class of the Stewart-Gough manipulators versus position parameters X, Z for three different value of Y =-5, 0, 5 unit. In the visualization of the practical orientation-capability of the manipulator over the whole position-workspace, the thick solid curve ( ) indicates the horizontal cross-section of the position-workspace boundary of the manipulator when the mobile platform of the manipulator is kept at the “initial orientation”, i.e., the orientation (x_1, x_2, x_3)=(0, 0, 0), the legend on the top left corner of the plot represents the plane Y=*const* from which boundary of the position-workspace of the manipulator is cut. The discrete circles ( ) with the same radius R of the inscribed sphere C represent the practical orientation-capabilities of the manipulator at the corresponding positions (X, Y, Z) over the whole horizontal cross-section of the position-workspace of the present manipulator. For clarity of the plot, only a discrete number of circles are chosen to represent orientation-capabilities of the manipulator at corresponding positions (X, Y, Z) in the position-workspace of the manipulator under consideration.

From Fig.5, it can be seen clearly that as the mobile platform of the manipulator approaches the boundary of the position-workspace of the manipulator the practical orientation-capabilities decreases gradually and it will be equal to zero when the mobile platform of the manipulator being located at the boundary of the position-workspace.

5 Conclusions

In this paper, the orientation-singularity and the orientation-workspace analyses of the manipulator is addressed using unit quaternion. The orientation-singularity expression is firstly deduced with respect to unit quaternion and a graphical example is given to illustrate it when the mobile platform is at a certain position. Then, the concept and algorithm of nonsingular orientation-workspace at a certain position is proposed. The nonsingular orientation-workspace can guarantee the manipulator is nonsingular in the whole orientation-workspace taking the limitations of active and passive joints and the movement interference of actuators into consideration. The nonsingular orientation-workspace is always a nonregular shape. Therefore, the practical orientation-workspace that is the minimum inscribed sphere of the nonsingular orientation-workspace's boundary surface is put forward. The radius of the sphere namely practical orientation-capability is used to evaluate the practical orientation-workspace. The practical orientation-capability versus positions within position-workspace of "initial orientation" is graphically described.

It is very important and significant that the result in this paper, which may be some useful to some problems as follows. For example, when the manipulator is used as the main structure of a machine tool, it should be known that whether the parallel manipulator is singular locating at one position within the position-workspace, and whether the mobile platform can arrive at the expected orientation with nonsingularity.

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