A New Expression to Construct Jacobian Matrix of Parallel Mechanism

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Abstract. Based on the velocity relationship between two points on the mobile platform, the new expression to construct the forward Jacobian matrix of parallel mechanism by its branched chains is deduced. This method is able to be used for the automatic generation of Jacobian matrix of parallel mechanism or hybrid mechanism, the correctness of presented method is validated by analyzing the planar 5R parallel mechanism. The multiply operations in the new expression are (36fp+6fa) times less than that in the Jacobian matrix deduced by Monsarrat and Gosselin in literature five while the flexible linkages are omitted by comparing two expressions.

Keywords: Parallel Mechanism, Jacobian Matrix, Serial Mechanism, Hybrid Mechansim.

1 Introduction

Jacobian matrix plays an important role in robotics, it includes the important information, such as accuracy, dexterity and manipulability etc.., which utilized in robotic kinematics, dynamics and control. Generally, Jacobian matrix becomes redundant with the increase of robot DOF, the computational efficiency[1] turns into the obstacle for real time control, thereby, some researchers tries to establish the nonlinear mapping relationship between kinematic parameters through neurofuzzy selflearning thoery rather than though Jacobian matrix[2]. Nevertheless, to establish Jacobian matrix is still the basic task of robotics research.

The main methods to obtain Jacobian matrix include the direct differentiation method based on the inverse kinematics, the construction method that consists of differential movement method and vector product method, recursion method and screw theory[3]etc.. Sometimes two method can be combined to use, for examples, to apply the screw theory with vector product method [4] or with differential movement method [5] etc.

Compared to serial mechanism, parallel mechanism has the features of high stiffness, stable structure, high accuracy and compact work volume etc.. Generally the

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forward Jacobian matrix of parallel mechanism can not be obtained directly, in order to achieve the forward Jacobian matrix of parallel mechanism, researchers commonly solve the inverse kinematic equations first, then differentiate the inverse kinematic equations. Through differentiating the inverse kinematic equation in the form of equation (1), researchers can obtain the inverse Jacobian matrix of parallel mechanism H(X) [6].

$$L=f(X)$$
$$\mathrm{d}L = H(X)\mathrm{d}X \tag{1}$$

where, L the vector of legs, viz. the vector of actuators displacements, X position and attitude of mobile platform,$H(X)$ inverse Jacobian matrix.

In literature 7, the influence coefficient method is used to solve the velocity of reference point on mobile platform, then the Jacobian matrix of parallel mechanism is constructed based on the results. In literature 8,9, the inverse Jacobian matrix is figured out by projecting the differential results of forward vector equations to the directions of generalized coordinates, that is the connecting lines between hinge points on base frame and mobile platform.

On basis of the velocity relationship between two points on a rigid body, this paper presents a new method and a new expression to construct the forward Jacobian matrix of parallel mechanism by the Jacobian matrices of its branched chains.

2 Velocity Analysis of Mobile Platform of Parallel Mechanism

As shown in figure 1, P denotes the mobile platform of parallel mechanism, B the base frame, b is the numbers of branched chains, f_i is the degrees of freedom of the i-th branched chain, i=1,2,…,b. The terminal link of each leg is the mobile platform, the Denavit-Hartenberg co-ordinate systems are established on every branched chain, and the original point of terminal link of each leg is made the same as the former co-ordinate system, this point is the hinge point Pi between the i-th branched chain and mobile platform. In manipulating space, the vector pointing to $\mathrm{P_i}$ from the reference point A on mobile platform is $\vec{r}_\mathrm{i}$, i=1,2,…,b, $\vec{\omega}$ is the angular velocity of mobile platform. The velocities of the reference point A and the point $\mathrm{P_i}$ on mobile platform are $\vec{V}_\mathrm{A}$, $\vec{\omega}_\mathrm{A}$, $\vec{V}_\mathrm{Pi}$ and $\vec{\omega}_\mathrm{B}$ respectively, then the relationship between the velocities of reference point A and the point $\mathrm{P_i}$ on mobile platform are as the following.

$$\begin{cases} \vec{V}_{p_\mathrm{i}} = \vec{V}_A + \vec{\omega}\times\vec{r}_\mathrm{i} \\ \vec{\omega}_{\mathrm{p_i}} = \vec{\omega} \end{cases} \tag{2}$$

Or

$$\begin{cases} \vec{V}_A = \vec{V}_{P_i} - \vec{\omega}\times\vec{r}_i \\ \vec{\omega}_A = \vec{\omega} \end{cases} \quad i=1,2,\dots,\mathrm{b} \tag{3}$$

Let $\overline{V}_A = \left[\vec{\mathrm{V}}_\mathrm{A}, \vec{\omega}_\mathrm{A}\right]^\mathrm{T}, \overline{V}_{p_i} = \left[\vec{\mathrm{V}}_{\mathrm{p_i}}, \vec{\omega}_{\mathrm{p_i}}\right]^\mathrm{T}$, equation (2) can be rewritten as

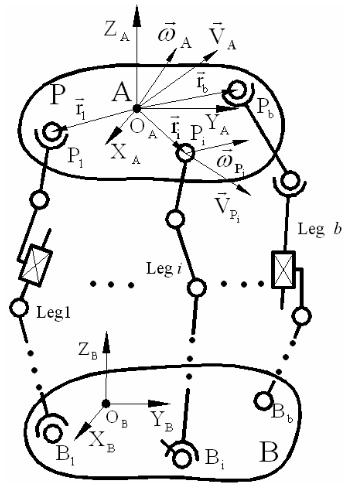

Fig. 1. Parallel mechanism scheme

$$\overline{V}_{p_i} = \begin{bmatrix} \vec{V}_{p_i} \\ \vec{\omega}_{p_i} \end{bmatrix} = \begin{bmatrix} \vec{V}_A - \vec{r}_\mathrm{i} \times \vec{\omega} \\ \vec{\omega}_A \end{bmatrix} = \begin{bmatrix} I_{3\times3} & R_i \\ O_{3\times3} & I_{3\times3} \end{bmatrix} \overline{V}_A \tag{4}$$

$$R_i = \begin{bmatrix} 0 & r_{iz} & -r_{iy} \\ -r_{iz} & 0 & r_{ix} \\ r_{iy} & -r_{ix} & 0 \end{bmatrix}$$

where, $I_{3\times3}$ 3×3 identity matrix, $0_{3\times3}$ 3×3 null matrix, R_i ——3×3 cross matrix.

$R_\mathrm{i}\,\vec{\omega} = -\vec{r}_\mathrm{i} \times \vec{\omega}$, R_i is determined by the vector $\vec{r}_\mathrm{i}$ describing hinge point P_i in coordinate system { O_A,X_A,Y_A,Z_A} fixed on the mobile platform and the rotation matrix from mobile platform to base frame.

Let $C_i = \begin{bmatrix} I_{3\times3} & R_i \\ O_{3\times3} & I_{3\times3} \end{bmatrix}$, C_i is named as transition matrix, thus equation (4) is

$$\overline{V}_{p_i} = C_i \overline{V}_A \tag{5}$$

More over, equation (3) can be expressed as

$$\overline{V}_A = \begin{bmatrix} \vec{V}_A \\ \vec{\omega}_A \end{bmatrix} = \begin{bmatrix} \vec{V}_{P_i} + \vec{r}_\mathrm{i} \times \vec{\omega} \\ \vec{\omega}_{p_i} \end{bmatrix} = \begin{bmatrix} I_{3\times3} & -R_i \\ O_{3\times3} & I_{3\times3} \end{bmatrix} \overline{V}_{p_i} \tag{6}$$

Let $D_i = \begin{bmatrix} I_{3\times3} & -R_i \\ O_{3\times3} & I_{3\times3} \end{bmatrix}$, since $C_i D_i = I$ and $D_i C_i = I$, $D_i = C_i^{-1}$ or $C_i = D_i^{-1}$, equation (6) can be written as

$$\overline{V}_A = D_i \overline{V}_{p_i} = C_i^{-1} \overline{V}_{p_i} \tag{7}$$

The following is to find the relationship between $\overline{V}_{\mathrm{A}}$ and the Jacobian matrices of branched chains. Based on equation (5), we obtain

$$\sum_{i=1}^{b}\overline{V}_{p_i}=\sum_{i=1}^{b}(C_i\overline{V}_A) \tag{8}$$

To supersede $\overline{V}_{\mathrm{Pi}}$ in above equation by $\overline{V}_{\mathrm{Pi}}=J_i\dot{\overline{q}}_i$ that is the velocity of point $\mathrm{P_i}$ on mobile platform, viz. the original point of terminal link in *i*-th branched chain, thus yields

$$\begin{bmatrix}J_1 J_2 & \cdots & J_b\end{bmatrix}\begin{bmatrix}\dot{\overline{q}}_1\\ \dot{\overline{q}}_2\\ \vdots\\ \dot{\overline{q}}_b\end{bmatrix}=(\sum_{i=1}^{b}C_i)\overline{V}_A \tag{9}$$

where, J_i Jacobian matrix of point $\mathrm{P_i}$ on mobile platform, viz. the original point of the Denavit- Hartenberg co-ordinate system fixed on the terminal link in the *i*-th branched chain of parallel mechanism, $\dot{\overline{q}}_i$ generalized velocity vector of the *i*-th branched chain, $\dot{\overline{q}}_i=\begin{bmatrix}\dot{q}_{i1} & \dot{q}_{i2} & \cdots & \dot{q}_{ifi}\end{bmatrix}^T$, *i*=1,2,…,*b*.

Let $C_{\mathrm{s}}=\sum_{i=1}^{b}C_{\mathrm{i}}$, $\dot{\overline{q}}=\begin{bmatrix}\dot{\overline{q}}_1^T & \dot{\overline{q}}_2^T & \cdots & \dot{\overline{q}}_b^T\end{bmatrix}^T$,then

$$C_{\mathrm{s}}=\begin{bmatrix}bI_{3\times3} & \sum_{i=1}^{b}R_i\\ 0 & bI_{3\times3}\end{bmatrix},$$

and $\dot{\overline{q}}$ is the generalized velocity vector of all branched chains of parallel mechanism, equation (9) can be written as

$$C_{\mathrm{s}}\overline{V}_A=\begin{bmatrix}J_1 & J_2 & \cdots & J_b\end{bmatrix}\dot{\overline{q}} \tag{10}$$

Solve for $\overline{V}_{\mathrm{A}}$ from above equation, yields

$$\overline{V}_A=D_{\mathrm{s}}\begin{bmatrix}J_1 & J_2 & \cdots & J_b\end{bmatrix}\dot{\overline{q}} \tag{11}$$

$$D_{\mathrm{s}}=C_{\mathrm{s}}^{-1},\quad D_{\mathrm{s}}=\begin{bmatrix}\frac{1}{b}I_{3\times3} & -\frac{1}{b^2}\sum_{i=1}^{b}R_i\\ 0 & \frac{1}{b}I_{3\times3}\end{bmatrix}$$

3 To Construct the Jacobian Matrix of Parallel Mechanism

Let the degrees of freedom of parallel mechanism is f_a, it can be figured out through Grübler formula

$$f_a=6(l-n-1)+f_{\mathrm{t}} \tag{12}$$

where, l Link numbers including frame, N Numbers of all joints, f_{t} Degrees of freedom of all joints, $f_{\mathrm{t}}=\sum_{\mathrm{i=1}}^{\mathrm{b}}f_{\mathrm{i}}$, *i*=1,2,…,*b*.

To make parallel mechanism has determined movement, the degrees of freedom f_a should be selected as active inputs, and the degrees of freedom $f_p=f_t-f_a$ as the passive inputs.

Without loss of generality, on the assumption that there are f_{ia} active inputs and $f_{ip}=f_i-f_{ia}$ passive inputs in i-th branched chain, i=1,2,…,b, thus

$$f_a=\sum_{i=1}^{b} f_{ia},\qquad f_p=\sum_{i=1}^{b} f_{ip} \tag{13}$$

For convenience, to reorder the active and passive general velocity inputs and their corresponding submatrices in $\overline{V}_{\mathrm{p_i}}=J_i\dot{\overline{q}}_i$, yields

$$\overline{V}_{p_i}=\begin{bmatrix}J_{ia} & J_{ip}\end{bmatrix}\begin{bmatrix}\dot{\overline{q}}_{ia}\\ \dot{\overline{q}}_{ip}\end{bmatrix}\quad i=1,2,\dots b \tag{14}$$

where, J_{ia},J_{ip} Corresponding submatrix in J_i to active and passive general velocity input vectors $\dot{\overline{q}}_{ia}$ and $\dot{\overline{q}}_{ip}$ respectively.

Considering equation (14), equation (11) can be rewritten as

$$\overline{V}_A=D_\mathrm{s}\begin{bmatrix}J_{1a} & J_{2a} & \cdots & J_{ba} & J_{1p} & J_{2p} & \cdots & J_{bp}\end{bmatrix}\begin{bmatrix}\dot{\overline{q}}_{1a}\\ \dot{\overline{q}}_{2a}\\ \vdots\\ \dot{\overline{q}}_{ba}\\ \dot{\overline{q}}_{1p}\\ \dot{\overline{q}}_{2p}\\ \vdots\\ \dot{\overline{q}}_{bp}\end{bmatrix}$$

That is

$$\overline{V}_A=D_\mathrm{s}\begin{bmatrix}J_a & J_p\end{bmatrix}\begin{bmatrix}\dot{\overline{q}}_a\\ \dot{\overline{q}}_p\end{bmatrix} \tag{15}$$

$$J_a=\begin{bmatrix}J_{1a} & J_{2a} & \cdots & J_{ba}\end{bmatrix}$$

$$J_p=\begin{bmatrix}J_{1p} & J_{2p} & \cdots & J_{bp}\end{bmatrix}$$

$$\dot{\overline{q}}_a=\begin{bmatrix}\dot{\overline{q}}_{1a}{}^T & \dot{\overline{q}}_{2a}{}^T & \cdots & \dot{\overline{q}}_{ba}{}^T\end{bmatrix}^T$$

$$\dot{\overline{q}}_P=\begin{bmatrix}\dot{\overline{q}}_{1P}{}^T & \dot{\overline{q}}_{2P}{}^T & \cdots & \dot{\overline{q}}_{bP}{}^T\end{bmatrix}^T$$

where, J_a, J_p Corresponding matrix in parallel mechanism to active and passive general velocity input vectors $\dot{\overline{q}}_a$ and $\dot{\overline{q}}_\mathrm{p}$ respectively.

The following is to solve for the relationship between the passive general velocity input vector $\dot{\overline{q}}_\mathrm{p}$ and active general velocity input vector $\dot{\overline{q}}_a$. On basis of equations (7) and (14), the following equations can be written out while i=1,2,…b.

$$\begin{cases} \overline{V_A} = D_1\overline{V_{P1}} = D_1\left(J_{1a}\dot{\overline{q}}_{1a} + J_{1p}\dot{\overline{q}}_{1p}\right) \\ \overline{V_A} = D_2\overline{V_{P2}} = D_2\left(J_{2a}\dot{\overline{q}}_{2a} + J_{2p}\dot{\overline{q}}_{2p}\right) \\ \cdots \\ \overline{V_A} = D_b\overline{V_{Pb}} = D_b\left(J_{ba}\dot{\overline{q}}_{ba} + J_{bp}\dot{\overline{q}}_{bp}\right) \end{cases} \tag{16}$$

Let two adjacent equations are equal to each other, yields

$$\begin{cases} D_1\left(J_{1a}\dot{\overline{q}}_{1a} + J_{1p}\dot{\overline{q}}_{1p}\right) = D_2\left(J_{2a}\dot{\overline{q}}_{2a} + J_{2p}\dot{\overline{q}}_{2p}\right) \\ D_2\left(J_{2a}\dot{\overline{q}}_{2a} + J_{2p}\dot{\overline{q}}_{2p}\right) = D_3\left(J_{3a}\dot{\overline{q}}_{3a} + J_{3p}\dot{\overline{q}}_{3p}\right) \\ \cdots \\ D_{b-1}\left(J_{b-1,a}\dot{\overline{q}}_{b-1,a} + J_{b-1,p}\dot{\overline{q}}_{b-1,p}\right) = D_b\left(J_{ba}\dot{\overline{q}}_{ba} + J_{bp}\dot{\overline{q}}_{bp}\right) \end{cases} \tag{17}$$

Equation (17) can be rewritten as the matrix form

$$A_a\dot{\overline{q}}_a = B_p\dot{\overline{q}}_p \tag{18}$$

$$A_a = \begin{bmatrix} D_1J_{1a} & -D_2J_{2a} & 0 & 0 & \cdots & 0 & 0 \\ 0 & D_2J_{2a} & -D_3J_{3a} & 0 & \cdots & 0 & 0 \\ 0 & 0 & D_3J_{3a} & -D_4J_{4a} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & D_{b-1}J_{b-1,a} & -D_bJ_{ba} \end{bmatrix}_{6(b-1)\times f_a}$$

$$B_p = \begin{bmatrix} -D_1J_{1P} & D_2J_{2P} & 0 & 0 & \dots & 0 & 0 \\ 0 & -D_2J_{2P} & D_3J_{3P} & 0 & \dots & 0 & 0 \\ 0 & 0 & -D_3J_{3P} & D_4J_{4P} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & -D_{b-1}J_{b-1,P} & D_bJ_{bP} \end{bmatrix}_{6(b-1)\times f_P}$$

Given the active input vector $\dot{\overline{q}}_a$, the passive input vector $\dot{\overline{q}}_{\mathrm{p}}$ can be solved from equation (18)

$$\dot{\overline{q}}_p = \left(B_p\right)^+ A_a\dot{\overline{q}}_a \tag{19}$$

where, $(B_p)^+$ Moore-Penrose pseudo-inverse of matrix B_p.

To rewrite equation (15) yields

$$\overline{V_A} = D_{\mathrm{s}}\left(J_a\dot{\overline{q}}_a + J_p\dot{\overline{q}}_p\right) \tag{20}$$

To substitute the equation (19) into above equation yields

$$\overline{V_A} = D_{\mathrm{s}}[\mathrm{J}_a + \mathrm{J}_p(\mathrm{B}_p)^+\mathrm{A}_a]\dot{\overline{q}}_a \tag{21}$$

Let $\dot{\overline{q}} = \dot{\overline{q}}_a$ the active general velocity vector and

$$J_{\mathrm{PM}} = D_{\mathrm{s}}[\mathrm{J}_a + \mathrm{J}_p(\mathrm{B}_p)^+\mathrm{A}_a]_{6\times f\mathrm{a}} \tag{22}$$

Equation (21) is rewritten as

$$\overline{V_A} = J_{\mathrm{PM}}\dot{\overline{q}} \tag{23}$$

The equation (22) is the Jacobian matrix J_{PM} of parallel mechanism which is constructed by the Jacobian matrices J_{i} of all branched chains.

Compared to the Jacobian matrix of parallel mechanism deduced by Monsarrat and Gosselin in literature 10 on the basis of equation (3) while the flexible links are omitted in it, the new Jacobian matrix expression of parallel mechanism presented in this paper is equivalent to it. But the process of generating J_a and J_p from J_i includes no multiply operation, the multiply operations between inverse transition matrix and the rest in new expression will be carried out only after the adding operations of inverse transition matrices D_i are carried out, the multiply operations in the new expression are $(36f_p+6\ f_a)$ times less than the former. This is a new method to construct the Jacobian matrix of parallel mechanism and is able to increase the operation efficiency of automatic generation of Jacobian matrix of parallel mechanism or hybrid mechanism.

4 Correctness Verification by Instance

In order to verify the correctness of the new method presented in this paper by comparing the Jacobian matrices in direct viewing, we take the planar five-bar linkage as an example. As shown in figure 2, the planar 5R mechanism has 2 degrees of freedom, if the link P_1P_2 are regarded as the mobile platform, the link B_1B_2 as the base frame, θ_1 and θ_4 as active inputs, the planar five-bar linkage is the planar 5R parallel mechanism which has two branched chains B_1CP_1 and B_2P_2, the vectors from reference point A on mobile platform P_1P_2 to hinge points P_1 and P_2 are $\vec{r}_1$ and $\vec{r}_2$ respectively.

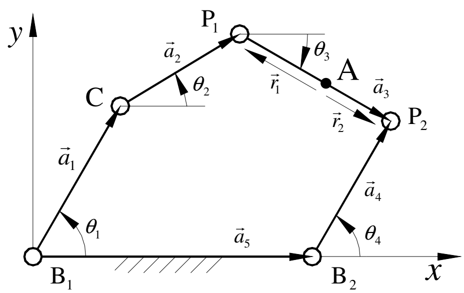

Fig. 2. The planar 5R parallel mechanism scheme

Firstly, the Jacobian matrix of reference point A on mobile platform P_1P_2 is deduced manually by using vector analytical method, the result is

$$J_5=\begin{bmatrix} \begin{array}{l} -a_1\sin\theta_1+a_1(\sin\theta_2\sin(\theta_3-\theta_1)a_3 \\ +r_1\sin\theta_3\sin(\theta_1-\theta_2))/a_3/\sin(\theta_3-\theta_2) \end{array} & \begin{array}{l} a_4(\sin\theta_2\sin(\theta_4-\theta_3)a_3 \\ -r_1\sin\theta_3\sin(\theta_4-\theta_2))/a_3/\sin(\theta_3-\theta_2) \end{array} \\ \\ \begin{array}{l} a_1\cos\theta_1-a_1(\cos\theta_2\sin(\theta_3-\theta_1)a_3 \\ +r_1\cos\theta_3\sin(\theta_1-\theta_2))/a_3/\sin(\theta_3-\theta_2) \end{array} & \begin{array}{l} -a_4(\cos\theta_2\sin(\theta_4-\theta_3)a_3 \\ -r_1\cos\theta_3\sin(\theta_4-\theta_2))/a_3/\sin(\theta_3-\theta_2) \end{array} \\ \\ -a_1\sin(\theta_1-\theta_2)/a_3/\sin(\theta_3-\theta_2) & a_4\sin(\theta_4-\theta_2)/a_3/\sin(\theta_3-\theta_2) \end{bmatrix} \quad (24)$$

Secondly, based on the method presented, the program for automatic generating the Jacobian matrix of planar 5R parallel mechanism by constructing the Jacobian matrices of points P_1 and P_2 on branched chains B_1CP_1 and B_2P_2 is composed for verifying the correctness of the new method presented in this paper, the block diagram of program is shown in figure 3. The following simply describes the calculating process and results.

To break the mobile platform, viz. the link 3 in figure 2 yields two branched chains of planar 3R and 2R mechanisms, as shown in figure 4, the numbers of branched chains b=2, in program, the angle θ is denoted by st, the length of link is denoted by a, for example, st1 denotes θ_1, a1 denotes a_1, r1 denotes r_1, the rest are by analogy of above means.

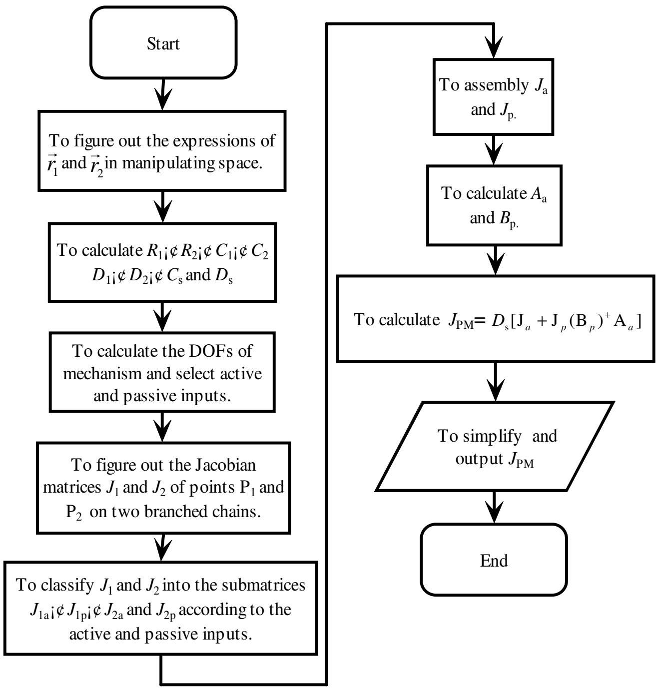

Fig. 3. Block diagram of program

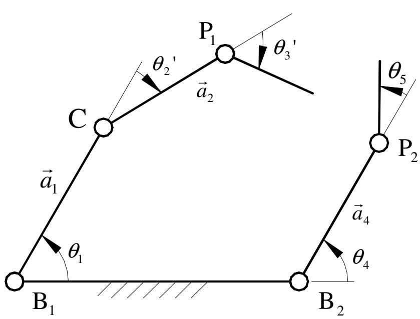

Fig. 4. Branched chains of planar 3R and 2R

For the planar mechanism, the cross matrices are the simple 2×1 column vectors.

$$R_1 = [r1*\sin(st3),-r1*\cos(st3)]^T$$

$$R_2 = [(-a3+r1)*\sin(st3),(a3-r1)*\cos(st3)]^T$$

Thereby, C_i and D_i, i=1,2, C_s and D_s are as the following

$$C_1 = \begin{bmatrix} 1 & 0 & r1*\sin(st3) \\ 0 & 1 & -r1*\cos(st3) \\ 0 & 0 & 1 \end{bmatrix} \quad C_2 = \begin{bmatrix} 1 & 0 & (-a3+r1)*\sin(st3) \\ 0 & 1 & (a3-r1)*\cos(st3) \\ 0 & 0 & 1 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 1 & 0 & -r1*\sin(st3) \\ 0 & 1 & r1*\cos(st3) \\ 0 & 0 & 1 \end{bmatrix} \quad D_2 = \begin{bmatrix} 1 & 0 & -(-a3+r1)*\sin(st3) \\ 0 & 1 & (-a3+r1)*\cos(st3) \\ 0 & 0 & 1 \end{bmatrix}$$

$$C_s = \begin{bmatrix} 2 & 0 & \sin(st3)*(2*r1-a3) \\ 0 & 2 & -\cos(st3)*(2*r1-a3) \\ 0 & 0 & 2 \end{bmatrix} \quad D_s = \begin{bmatrix} 1/2 & 0 & -1/4*\sin(st3)*(2*r1-a3) \\ 0 & 1/2 & 1/4*\cos(st3)*(2*r1-a3) \\ 0 & 0 & 1/2 \end{bmatrix}$$

The general coordinates θ_1 and θ_4 on branched chain B_1CP_1 and B_2P_2 are selected as the active inputs, the general velocity vectors of active and passive inputs $\dot{\bar{q}}_a$ and $\dot{\bar{q}}_p$ in parallel mechanism are $\dot{\bar{q}}_a = [\dot{\theta}_1,\dot{\theta}_4]^T$ and $\dot{\bar{q}}_P = [\dot{\theta}_2',\dot{\theta}_3',\dot{\theta}_5]^T$ respectively. For the convenience of comparing symbolic results, the relationship between the relative angle coordinates in figure 4 and the absolute angle coordinates in figure 2 should be noticed.

For instance, in branched chain 3R mechanism, $\theta_2' = \theta_2 - \theta_1$, $\theta_3' = \theta_3 - \theta_2$, and in branched chain 2R mechanism, $\theta_5 = \pi - (\theta_4 - \theta_3)$.

Thus the Jacobian submatrices[4] corresponding to the active and passive inputs of two branched chains respectively are

$$J_{1a} = [-a2*\sin(st2)-a1*\sin(st1),\ a2*\cos(st2)+a1*\cos(st1),1]^T$$

$$J_{1p} = \begin{bmatrix} -a2*\sin(st2) & 0 \\ a2*\cos(st2) & 0 \\ 1 & 1 \end{bmatrix}$$

$$J_{2a} = [-a4*\sin(st4),a4*\cos(st4),1]^T$$

$$J_{2p} = [0,0,1]^T$$

To assembly J_a and J_p by using above submatrices yields

$$J_a = \begin{bmatrix} -a2*\sin(st2)-a1*\sin(st1) & -a4*\sin(st4) \\ a2*\cos(st2)+a1*\cos(st1) & a4*\cos(st4) \\ 1 & 1 \end{bmatrix} \quad J_p = \begin{bmatrix} -a2*\sin(st2) & 0 & 0 \\ a2*\cos(st2) & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Then, to solve for A_a and B_p yields

$$A_a = \begin{bmatrix} -a2*\sin(st2)-a1*\sin(st1)-r1*\sin(st3) & a4*\sin(st4)+(-a3+r1)*\sin(st3) \\ a2*\cos(st2)+a1*\cos(st1)+r1*\cos(st3) & -a4*\cos(st4)-(-a3+r1)*\cos(st3) \\ 1 & 1 \end{bmatrix}$$

$$B_p = \begin{bmatrix} a2*\sin(st2)+r1*\sin(st3) & r1*\sin(st3) & -(-a3+r1)*\sin(st3) \\ -a2*\cos(st2)-r1*\cos(st3) & -r1*\cos(st3) & (-a3+r1)*\cos(st3) \\ -1 & -1 & 1 \end{bmatrix}$$

According to equation (22), the program generates the Jacobian matrix J_{PM} which is constructed by the Jacobian matrices of two branched chains, the result is

$$J_{pm}=\begin{bmatrix} \begin{array}{l}\text{1/2*(-a1*a3*cos(st1+st3-st2)+a1*a3*cos(st1-st3-st2)}\\ \text{-a1*r1*cos(st1-st3-st2)+a1*r1*cos(st1+st3-st2))/a3/sin(-st3+st2)}\end{array} & \begin{array}{l}\text{1/2*(a3*a4*cos(st4-st3+st2)-a4*a3*cos(-st4+st3+st2)}\\ \text{-a4*r1*cos(-st4-st3+st2)+a4*r1*cos(-st4+st3+st2))/a3/sin(-st3+st2)}\end{array} \\ \begin{array}{l}\text{1/2*(-a1*a3*sin(st1+st3-st2)-a1*a3*sin(st1-st3-st2)}\\ \text{+a1*r1*sin(st1+st3-st2)+a1*r1*sin(st1-st3-st2))/a3/sin(-st3+st2)}\end{array} & \begin{array}{l}\text{1/2*(a3*a4*sin(st4-st3+st2)-a4*a3*sin(-st4+st3+st2)}\\ \text{+a4*r1*sin(-st4+st3+st2)+a4*r1*sin(-st4-st3+st2))/a3/sin(-st3+st2)}\end{array} \\ \text{a1*sin(-st2+st1)/a3/sin(-st3+st2)} & \text{a4*sin(st2-st4)/a3/sin(-st3+st2)} \end{bmatrix} \tag{25}$$

Through the simplification operating on equations (24) and(25) such as summation and product convert operations of trigonometric function, the Jacobian matrix of parallel mechanism generated by the new method presented by this paper is identical with that deduced manually by vector analytical method, this proves the correctness of the method presented in this paper.

5 Conclusions

The new method to construct the forward Jacobian matrix of parallel mechanism by the Jacobian matrices of branched chains is presented in this paper, and the new expression to construct the Jacobian matrix of parallel mechanism is deduced. This method is able to be used for the automatic symbolic generation of Jacobian matrix of parallel mechanism or hybrid mechanism. Compared to the Jacobian matrix of parallel mechanism deduced by Monsarrat and Gosselin on the basis of equation (3) while the flexible links are omitted in it, the new Jacobian matrix expression of parallel mechanism presented in this paper is equivalent to it, because the process of generating J_a and J_p from J_i includes no multiply operation, the multiply operations between inverse transition matrix and the rest in new expression will be carried out only after the adding operations of inverse transition matrix D_i are carried out, the multiply operations in the new expression are $(36f_p+6\,f_a)$ times less than the former. This is a new method to construct the Jacobian matrix of parallel mechanism and is able to increase the operation efficiency of automatic generation of Jacobian matrix of parallel mechanism or hybrid mechanism.

The Jacobian matrix of planar 5R parallel mechanism is generated automatically by the program composed on the basis of presented method, it is identical with the result deduced manually by vector analytical method through comparing, it validates the correctness of presented method.

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