Gait Analysis of the Passive Dynamic Walker with Knees

Peijie Zhang, Yantao Tian*, and Zhenze Liu

College of Communication Engineering, Jilin University, Changchun 130025, China
mczpj2007@yahoo.cn, tianyt@jlu.edu.cn

Abstract. A systematic study to the passive gaits of the passive dynamic walker with knees is presented in the paper. The motion models are derived firstly and normalized for numerical simulation. A time based limit cycle is employed to represent the hybrid dynamics of periodic passive dynamic walking. The evaluation of gait descriptors under variations of parameters of the robot is studied based on the Poincaré first return map which captures the periodic dynamics of passive gaits. The stability property is analyzed and the effect of knees is discussed. The research of the paper is much helpful to the study of passive dynamic walkers with complex structures, the theoretical analysis and control strategy design.

Keywords: Passive Dynamic Walker, Limit Cycle, Poincaré Map.

1 Introduction

Passive dynamic walking is a new research field initiated by T.McGeer in 1990's [1], with the aim to gain high efficiency and natural gaits in biped walking by exploring the natural dynamics of two legged-machines. After first passive dynamic walker made by T.McGeer, many researchers have performed studies from theoretic studies of passive dynamic walking model and development of prototypes of passive biped walkers [2], [3], [4], [5]. The simple four-legged two dimensional passive walkers are successfully are developed. Among the different passive dynamic models, the compass gait model is the most commonly used model and A.Goswami presented a systematic study about this model [6], [7]. But the compass-like biped model has the problem of 'foot-scuffing', which can be avoided by some special design. But the introducing of knees can give the walker a more anthropomorphic look and it turns out, avoiding the foot scuffing problem [8], [9].

Passive walkers have been implemented with great success some groups [10]. But compared with the successful experiments, the researches of theoretical models are a little lagged, because of the complexity of passive dynamic models. Vanessa F. Hsu Chen [11] built a kneed passive gait model without torso and developed a corresponding walking machine. The exploring of passive dynamic walking can greatly enhance the mechanism design and control method design of efficient biped robot [12], [13].

The passive dynamic walkers with knees can also perform stable passive gaits and some kneed models with or without a torso are studied. Like the compass gait biped,

* Corresponding author.

C. Xiong et al. (Eds.): ICIRA 2008, Part I, LNAI 5314, pp. 992–1002, 2008.

the kneed model has the similar dynamics. A deep understanding of kneed passive walker will be a valuable aid for the construction of passive biped mechanism. In this paper, the kneed passive model is deeply studied to gain some insight to kneed passive walking. The motion model is derived using the Lagrange formula and numerically simulated by a Matlab program. In order to explore the kernel properties of the passive dynamic walking, the kneed model is scaled, of which the result is that the walking dynamics is independent of the specific leg lengths and masses. The energy balance and specific curve of kinetic and potential energy are analyzed. The evaluation of gait descriptors under slope angle variation is studied, as wall as when the other structure parameters vary.

2 The Passive Dynamic Walking Model with Knees

2.1 Model of Passive Gait on a Slope

The diagram of the kneed passive dynamic walkers and the physical parameters are shown in Fig.1. All the masses of the different parts are assumed to be point masses, m_t for the thigh and m_s for the shank. A point mass m_H at hip are used to represent the upper body. Two legs of the robot are the same, with the same length L and the same mass distribution $L = l_t + l_s$, where l_t is the length of the upper leg and l_s is that of the lower leg. $l_s = a_1 + b_1$, where a_1 is the distance from the center of mass of shank to the end point of the leg. $l_t = a_2 + b_2$, where a_2 is the distance from the center of mass of thigh to the knee point. In this paper, the model assumption used is the similar to that in [11]. The motion model is re-derived and some mistakes in [11] are corrected.

The stable gait of the kneed passive gait is a combination of the continuous dynamics in swing stages and the discrete transition in the collision stages. However, there are four different stages, as is shown in the Fig. 2 including two continuous swing stages and two collision stages, different from that of the compass gait.

In stage 1 (Fig. 2.a), the system is governed by its unlocked knee dynamics. In stage 2 (Fig. 2.b), the swing legs comes to the straight out position and knee-strike occurs. The knee-strike is assumed to be an instantaneous $\dot{x}_2 = f_2(x_2) + g_2(x_2)u_2$ and the angular momentum is preserved for the entire system about the stance foot and for the swing leg about the hip. In stage 3 (Fig. 2.c), the robot is constrained to be a double-link pendulum, and the dynamics is similar to that of the compass-like biped robot. In stage 4 (Fig. 2.d), the swing leg collides with the ground and two legs swamp their roles. The heel-strike is also modeled as an instantaneous collision too, and the angular momentum is conserved for the entire system about the colliding foot and for the stance leg before impact about the hip.

The model of the kneed walker walking on an inclined plane is a hybrid dynamic system, which can be described by the hybrid automation shown in Fig. 3 [14], [15]. There are two locations and two transitions,

$$\dot{x}_1 = f_1(x_1) + g(x_1)u_1, \quad \dot{x}_2 = f_2(x_2) + g_2(x_2)u_2 . \tag{1}$$

The first model is the three-link pendulum dynamics in the unlocked-knee swing phase and the second one is the governing equation of the locked-knees swing phase.

For the two swing stages, the governing equations can be derived by the Lagrange formula,

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + g(\theta) = B(\theta)u\,, \tag{2}$$

where $\theta = \begin{bmatrix}\theta_1 & \theta_2 & \theta_3\end{bmatrix}^T$, with $\theta_1, \theta_2,\ \theta_3$ being the angles of stance leg, the thigh and the shank of the swing leg with the vertical (counterclockwise positive).

The states transitions in two collision stages can be modeled as algebraic equations, which can be constructed by the angular momentum conservation laws. $x_2^+ = \Delta_1(x_1^-)$ and $x_1^+ = \Delta_2(x_2^-)$ are the transition equations of knee-strike and heel-strike respectively. $\varphi_1(x_1) = 0$ and $\varphi_2(x_2) = 0$ are the super-plane of knee-strike and heel-strike respectively, acting as the guards of collision events and states transition.

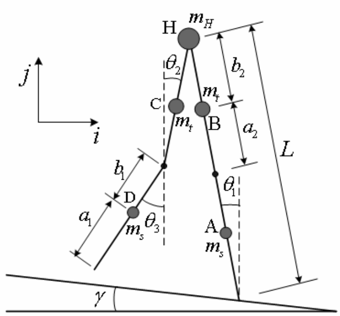
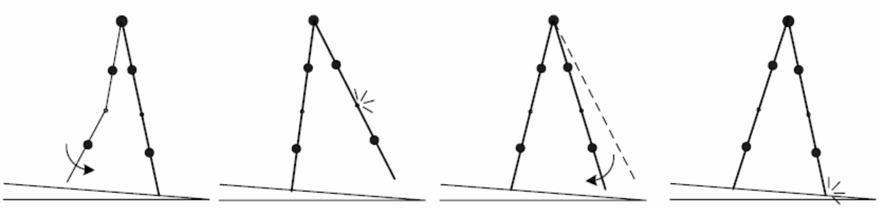


Fig. 1. The kneed passive dynamic walker



(a) Stage 1 (b) Stage 2 (c) Stage 3 (d) Stage 4

Fig. 2. A period of the stable passive gait

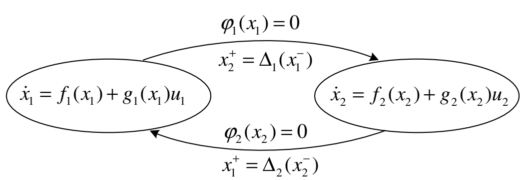


Fig. 3. Dimension-variant hybrid automation of the stable passive gait

2.2 Simulation of the Kneed Passive Dynamic Walker

In the dynamic model of the kneed biped robot, there are eight or more physical parameters making it hard to find the internal principle of kneed passive dynamic walking and the relations between the parameter and gait property. In our research, the kneed model is normalized. As a result, the dynamics of the passive gait is independent of the specific mass and length parameters of the model, such as L, l_t, l_s, m_H, m_t, m_s, a_1, b_1, a_2, b_2. The scaling of the model is done by dividing two sides of the motion model by m_H and L. The main parameters appearing in the motion model become,

$$\mu_{sH} = \frac{m_s}{m_H},\ \mu_{tH} = \frac{m_t}{m_H},\ \beta_s = \frac{l_s}{L},\ \beta_{ls} = \frac{a_1}{l_s},\ \beta_{lt} = \frac{a_2}{l_t}. \tag{3}$$

By setting L=1 and m_H=1, the model is normalized and main parameters are calculated as

$$\begin{aligned} &m_s = \mu_{sH} m_H,\quad m_t = \mu_{tH} m_H,\quad l_s = \beta_s L,\quad l_t = (1-\beta_s)L,\quad a_1 = \beta_{ls}\beta_s L \\ &b_1 = (1-\beta_{ls})\beta_s L,\quad a_2 = \beta_{lt}(1-\beta_s)L,\quad b_2 = (1-\beta_{lt})(1-\beta_s)L. \end{aligned} \tag{4}$$

The dynamics of a stable passive gait can be represented by a limit cycle on the phase plane. A limit cycle is a figure of one state and it's time derivative, but limit cycle is a phase plot independent of time. That is to say, the time evaluation of the trajectory is disabled. In order to present the time measurement of the trajectory, the time based limit cycle is used. The so called time based limit cycle is constructed by appointing areas with different width to different parts of the limit cycle, shown as the grey area along limit cycle in Fig. 4. The different parts of the grey area represent the time consumed by the corresponding section of phase trajectory with unit length. Wider the area is, more time the phase trajectory of unit length consumes.

The time based limit cycle is only a relative qualitative description of the time evaluation of the trajectory. From the figure, it can be seen that the states with most large velocity sustain longer time, just opposite to the intuitive understanding of the passive gaits.

There is no external torque applied on the biped during swing stages except gravity force, so the mechanical energy is conserved, shown as the level lines in Fig. 6. The down-jumps correspond to the two discrete events of knee-strike and heel-strike, where the smaller down-jump is the energy loss for the knee-strike and the larger down-jump is the energy loss for the heel-strike. Fig. 7 and Fig. 8 show the evaluations of kinetic energy and potential energy respectively. The changing trends of the two trajectories are opposite to each other, proving the conservation of mechanical energy during the continuous motion stages.

Large amount of knowledge can be gained from a kinetic energy vs. potential energy plot as shown in Fig. 5. The energy trajectory of a full gait cycle starts from point A and follows the trajectory *ABCDEF*. As a result of the mechanical energy conversation in the two swing stage, the energy trajectories of swing stages are straight lines making a *135°* angle with the kinetic energy axis, where *ABC* is the energy trajectory for the knee-unlocked three-link swing stage and DE for the knee-locked double-link swing stage. Point *C* is the point where knee-strike occurs, where the kinetic energy loss is shown by the horizontal line segment *CD* in the diagram. At point *E*, the swing leg collides with the ground and the kinetic energy loss is represented by the level line *EF*. Total loss of potential energy is given by the line segment *AF*. For a stable passive limit cycle, the total loss of potential energy is exactly the total loss of kinetic energy, then it is clear that *AF=CD+EF*.

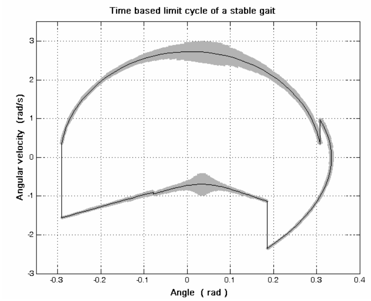


Fig. 4. The time based limit cycle

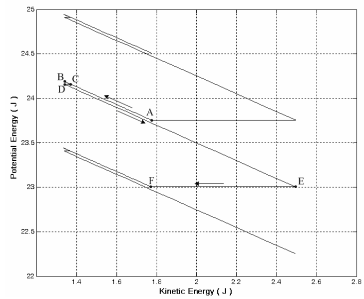


Fig. 5. Kinetic vs. potential energy plot

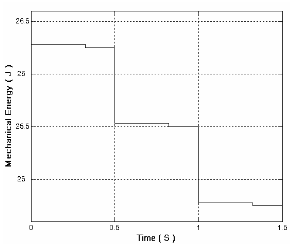


Fig. 6. Mechanical energy

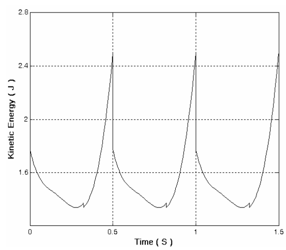


Fig. 7. Kinetic energy

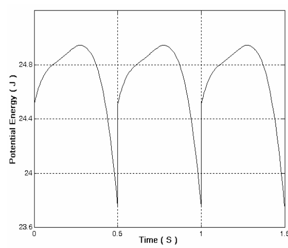


Fig. 8. Potential energy

Table 1. Physical parameters used in the simulation experiments

Parameters	ϕ	μ_{sH}	μ_{tH}	β_s	β_{ls}	β_{lt}
value	3°	0.05	1	0.5	0.75	0.5

But it should be noted that for the robot with some specific physical parameters, point *B* will approach point *C* or become one single point, meaning that the knee-strike occurs before the swing thigh move forward to the largest angle. The physical parameters used in the simulation are listed in table 1.

3 Numerical Experiments of Nonlinear Kneed-Biped Model

A small change in parameter will result in small changes in the robot's gait and the gait descriptors. In this section, the characteristics of the steady limit cycles under parameter variations are studied. The plots show the evaluation of gait descriptors as a function of physical parameters when the robot parameters vary. A gait descriptor is an observed value that can't be directly changed, but can be indirectly influenced by the physical parameters.

The gait descriptors studied includes the angular velocity of stance leg at the beginning of a step $\dot{\theta}_1^0$, stride time or the gait period T, time of knee-strike T_k in a period (scaled by T), the half inter-leg angle α_k at knee-strike and α_h at heel-strike, the mechanical energy E, and the ratio of energy loss ΔE_k at knee-strike and ΔE_h at heel-strike (both scaled by E) and the average progress speed v. The scaled model is used in simulation experiments, since that the dynamics is independent of the specific values of physical parameters. The scaled parameters used are the mass ratio of the upper leg to the lower leg, the mass ratio of the leg to the hip and the length ratio of the upper leg to the lower leg, which are defined as

$$\mu_{ts} = \frac{m_t}{m_s}, \quad \mu_{st-H} = \frac{m_s + m_s}{m_H}, \quad \beta_{ts} = \frac{l_t}{l_s}, \tag{5}$$

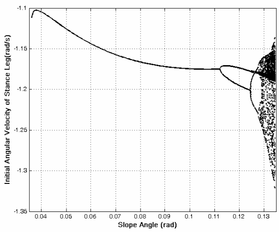
where μ_{ts} is the mass ratio of the upper leg to the lower leg, μ_{st-H} is the mass ratio of the leg to the lower leg and β_{ts} is the length ratio of thigh and shank. Then, other main parameters are calculated as, (while it is assumed m_H=m_t+m_s)

$$\mu_{sH} = \frac{\mu_{st-H}}{1+\mu_{ts}}, \quad \mu_{sH} = \frac{\mu_{ts}\mu_{st-H}}{1+\mu_{ts}}, \quad \beta_s = \frac{1}{1+\beta_{ts}}. \tag{6}$$

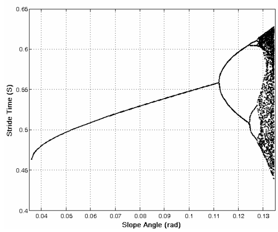
3.1 Influence of the Slope Angle

The Evaluation of gait descriptors under variation of slope angles are shown in Fig. 9 and the different limit cycles are shown in Fig. 10. In simulation, it is found that the kneed biped model can present gaits for different scale of slope angle, approximately from 2.07° to 7.7°, contrast to that of the compass-like biped, which is approximately from 0.2° to 5.19°. The angle region of passive gait for the two biped robots is of the same size. As the slope angle become longer, period-doubling bifurcation occurs in the gaits of the kneed biped and can be studied by means of the Poincaré first return map. As a consequence of the period-doubling bifurcation, the symmetric limit cycle becomes asymmetric 2-periodic gait with a shorter and longer step. On further increasing the slope angle, the robot gait may experience further period-doubling, giving rise to 4-period and 8-period limit cycle. The sequence of period-doubling is called a period doubling cascade which leads to chaotic gaits finally.

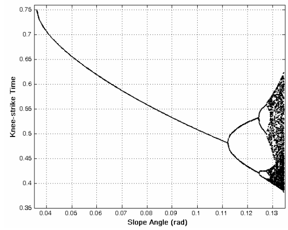
It is confirmed that the introducing of knees to the passive biped model has a great effect to the dynamics of the robot. One of the most different characteristics is the gait period T, which is smaller than that of the compass-like biped. This can be interpreted as that the free rotation of the lower leg increases the equivalent inertia of the swing leg. The passive gait for the kneed-biped has a bigger half inter-leg angle α and more larger mechanical energy.



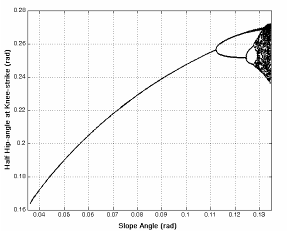
(a) initial angular velocity of stance leg



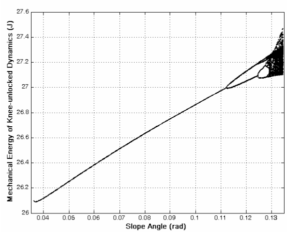
(b) Step period



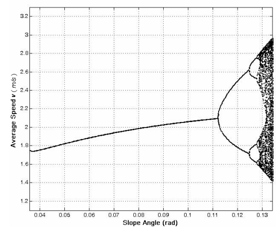
(c) knee-strike time (scaled by step period)



(d) Hip angle at knee-strike



(e) Mechanical energy



(f) Average speed

Fig. 9. Evaluation of gait descriptors under variation of slope angles; (Energy loss at knee-strike and heel-strike are scaled by the mechanical energy of three-link dynamics)

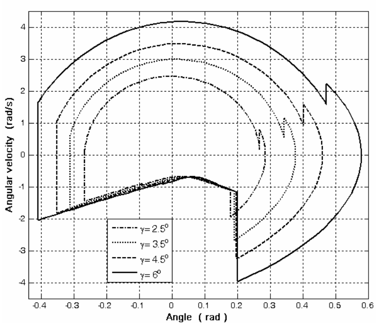


Fig. 10. Limit cycles under slopes with different angles

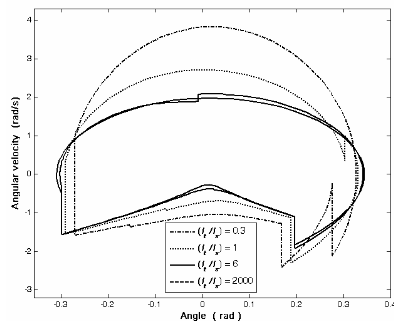


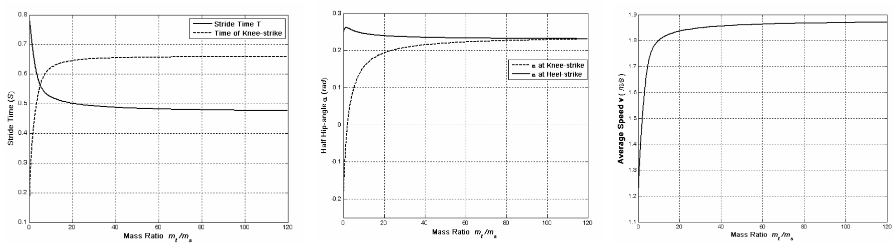
Fig. 11. Limit cycles under different physical parameter μ_{ts}

3.2 Influence of the Mass Distribution

The mass distribution can change the passive gait of the kneed biped. The mass ratio of the leg to the hip μ_{st-H} and the upper leg to the lower leg μ_{ts} are selected. Fig. 11 shows the different limit cycles when μ_{ts} increases from 0.27 to 50 and the time evaluations of the gait descriptors are shown in Fig. 12. For different μ_{ts}, the time of knee-strike is effected significantly. When μ_{ts} very small, knee-strike occurs shortly after the step period begins.

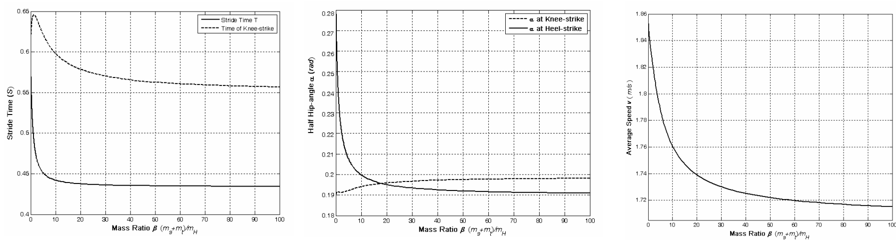
When μ_{ts} very small, knee-strike occurs until shortly before the step period begins. So, the limit cycles can be seen much different from each other. And the gait descriptors tend to a single value as the μ_{ts} becoming larger.

Fig. 13 shows the time evaluations of the gait descriptors when $(m_t+m_s)/m_H$ increases from 0.0001 to 100 and the different limit cycles are shown in Fig. 14. The



(a) Step period and scaled knee-strike time (b) Hip angle at knee-strike and heel-strike (c) Average speed

Fig. 12. Evaluation of gait descriptors under variation of physical parameter μ_{ts}



(a) Step period and scaled knee-strike time (b) Hip angle at knee-strike and heel-strike (c) Average speed

Fig. 13. Evaluation of gait descriptors under variation of parameter μ_{st-H}

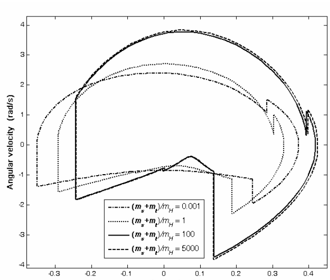


Fig. 14. Limit cycles under different physical parameter μ_{st-H}

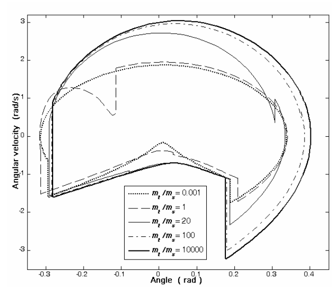


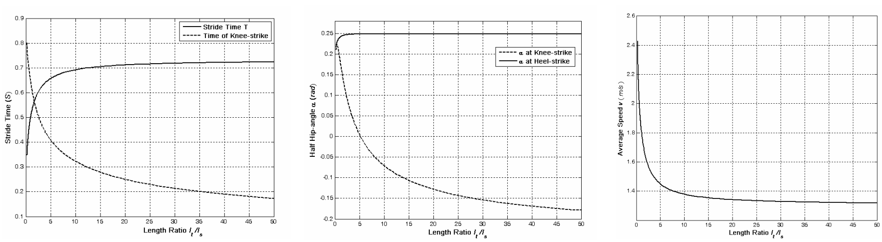
Fig. 15. Limit cycles under different physical parameter β_{ts}

value of $(m_t+m_s)/m_H$ can effect the time of knee-strike, but less significantly. However, the limit cycles under different $(m_t+m_s)/m_H$ change a lot, too.

3.3 Influence of the Length Distribution

Fig. 15 shows the different limit cycles when the length ratio of the upper leg to the lower leg l_t/l_s increases from 0.27 to 50 and the evaluation of the gait descriptors is shown in Fig. 16. For different β_{ts} , much different limit cycles can be obtained. When the value of is too small or too large, knee-strike can't be explicitly found.

The changes of gait descriptors in above figures are summarized as Table 2. It should be noted that in the simulation work, the 'foot-scuffing' is neglected. And no bifurcation and chaotic gait is found when the physical parameters vary in a large region.



(a) Step period and scaled knee-strike time

(b) Hip angle at knee-strike and heel-strike

(c) Average speed

Fig. 16. Evaluation of gait descriptors under variation of physical parameter β_{ts}

Table 2. Evaluation of gait descriptors when the scaled physical parameters vary

Gait descriptors	$\dot{\theta}_1^0$	T	T_k	α_k	α_h	E	ΔE_k	ΔE_h	v
$\phi \nearrow$	↘	↗	↘	↗	↗	↗	↗	↗	↗
$\mu_{ts} = m_t / m_s \nearrow$	↗	↘	↗	↗	↘	↗	↗	↘	↗
$\mu_{st-H} = (m_t+m_s)/m_H \nearrow$	↗	↘	↘	↗	↘	↗	↗	↘	↘
$\beta_{ts} = l_t/l_s \nearrow$	↗	↗	↘	↘	↗	↘	↘	↗	↘

4 Conclusions and Future Work

In this paper, a deep research of passive dynamics of the kneed-biped is presented, in order to give more insight to the passive dynamic walking of biped robot with knees. The energy evaluation in a step period is analyzed. It is revealed that the free rotating knee decreases the actuation of thigh to shank and brings some new dynamic characteristics to the passive gait. The limit cycle under different physical parameters are studied by simulation experiments. The work of the paper is helpful to the theoretical analysis and control strategy design of passive walker. The gait cycle of passive gait is a period dynamics and new criterion of stability should be developed and new control method based on passive dynamic walking should be studied in future work.

Acknowledgement

This work is supported by National High Technology Research and Development Program of China (863 Program), grant No. 2006AA04Z251, and Science and Technology Development Program of Jilin Province, grant No. 20070524.

References

1. McGeer, T.: Passive dynamic walking. International Journal of Robotics Research 9(2), 62–82 (1990)
2. Garcia, M., Chatterjee, A., Ruina, A., Coleman, M.: The simplest walking model: stability, complexity, and scaling. ASME Journal of Biomechanical Engineering 120(2), 281–288 (1998)
3. Spong, M.W., Bullo, F.: Controlled symmetries and passive walking. IEEE Transactions on Automatic Control 50(7), 1025–1031 (2005)
4. Wisse, M., Schwab, A.L., van der Helm, F.C.T.: Passive dynamic walking model with upper body. Robotica 22(6), 681–688 (2004)
5. Liu, Z.Z., Zhou, C.J., Zhang, P.J., Tian, Y.T.: Anti-phase Synchronization Control Scheme of Passive Biped Robot. In: Robot, Weld, Intellige & Automation. LNCIS, vol. 362, pp. 529–539 (2007)
6. Goswami, A., Thuilot, B., Espiau, B.: A study of the passive gait of a compass-like biped robot: symmetry and chaos. International Journal of Robotics Research 17(12), 1282–1301 (1998)
7. Goswami, A.: Limit cycles in a passive compass gait biped and passivity-mimicking control laws. Journal of Autonomous Robots 4(3), 273–286 (1997)
8. Bhatia, G., Spong, M.: Hybrid control for smooth walking of a biped with knees and torso. In: Proceedings of 2004 Institute of Electrical and Electronics Engineers Conference on Control Applications (2004)
9. Collins, S.H., Wisse, M., Ruina, A.: A 3-D passive-dynamic walking robot with two legs and knees. Int. J. Robot. Res. 20(7), 607–615 (2001)
10. Collins, S., Ruina, A., Tedrake, R., Wisse, M.: Efficient bipedal robots based on passive-dynamic walkers. Science Magazine 307, 1082–1085 (2005)
11. Chen, V.F.H.: Passive Dynamic Walking with Knees: A Point Foot Model. Master thesis. Massachusetts Institute of Technology (2007)
12. Hobbelen, D.G.E., Wisse, M.: A Disturbance Rejection Measure for Limit Cycle Walkers: The Gait Sensitivity Norm. IEEE Transactions on robotics 23(6), 1213–1224 (2007)
13. Olensek, A., Matjacic, Z., Jarm, T., Kramar, P., Županič, A.: Two-level control of bipedal walking model. In: Medicon 2007, IFMBE Proceedings, vol. 16, pp. 673–676. Springer, Heidelberg (2007)

14. van der Schaft, A.J., Schumacher, H.: An introduction to hybrid dynamical systems. Lecture notes in control and information science, vol. 251. Springer, London (2000)
15. Grizzle, J.W., Plestan, F., Abba, G.: Poincaré's method for systems with impulse effects: application to mechanical biped locomotion. In: 38th Conference on Decision & Control, Phoenix, Arizona, pp. 3869–3876 (1999)