

Intuitionistic Fuzzy Clustering with Applications in Computer Vision

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Abstract. Intuitionistic fuzzy sets are generalized fuzzy sets whose elements are characterized by a membership, as well as a non-membership value. The membership value indicates the degree of belongingness, whereas the non-membership value indicates the degree of non-belongingness of an element to that set. The utility of intuitionistic fuzzy sets theory in computer vision is increasingly becoming apparent, especially as a means to coping with noise. In this paper, we investigate the issue of clustering intuitionistic fuzzy image representations. To achieve that we propose a clustering approach based on the fuzzy c-means algorithm utilizing a novel similarity metric defined over intuitionistic fuzzy sets. The performance of the proposed algorithm is evaluated for object clustering in the presence of noise and image segmentation. The results indicate that clustering intuitionistic fuzzy image representations can be more effective, noise tolerant and efficient as compared with the conventional fuzzy c-means clustering of both crisp and fuzzy image representations.

1 Introduction

Clustering provides an unsupervised approach to organize vector-coded objects into groups and it comprises an invaluable tool in computer vision. Its usefulness spans a wide range of applications including image segmentation, categorization and retrieval [1-3].

A rather popular approach to clustering based on fuzzy sets theory [4] is Fuzzy C-Means (FCM) [5]. FCM considers soft boundaries between the clusters, allowing each sample of the dataset being clustered to belong to different clusters by a certain degree. The degree of membership of a sample to a cluster is usually considered as a function of its distance from the cluster centroids or from other representative vectors of the cluster. Through the years many variants of the FCM algorithm have been proposed, including variants dedicated to image segmentation [6,7], or generalizations applicable to various domains [8].

In the middle of the eighties, Attanassov proposed a generalization of the fuzzy sets called intuitionistic fuzzy sets [9-13]. These sets comprise of elements

characterized by both a membership and a non-membership value. The membership value indicates the degree of belongingness of that element to the set, whereas the non-membership value indicates the degree to which that element does not belong to the set. The non-membership value need not necessarily be symmetric to the membership value, and this poses a challenge when it comes to expressing the intuitionistic notion of hesitancy in the belongingness of an element to a fuzzy set in the context of a real world application.

Recent studies indicate that expressing images in terms of intuitionistic fuzzy sets can lead to novel perspectives in computer vision [14-17]. In [14] an intuitionistic fuzzy image representation was defined by combining both crisp and fuzzy histogram information. This representation was later used for image hesitancy equalization, which is a contrast enhancement technique similar to histogram equalization [15]. Its improved performance has been attributed to the way intuitionistic fuzzy sets cope with uncertainty. In another study [16] mathematical morphology operations, including dilation and erosion, have been defined over intuitionistic fuzzy image representations. Further research and application perspectives are discussed in [17].

The increasing amount of applications of the intuitionistic fuzzy sets in computer vision, pose the need of developing specialized approaches capable of clustering intuitionistic fuzzy data. As a first attempt to this direction we propose a fuzzy clustering approach capable of grouping intuitionistic fuzzy image representations. The proposed approach is based on the FCM algorithm and incorporates a novel similarity metric defined over the respective membership and non-membership values. Its performance is investigated for object clustering and image segmentation.

The rest of this paper consists of four sections. Section 2 provides a short introduction on the intuitionistic fuzzy sets and describes the intuitionistic fuzzy image representation considered. The proposed approach is described in Section 3, and the experimental results are apposed in Section 4. The last section, Section 5, summarizes the conclusions of this study.

2 Intuitionistic Fuzzy Image Representation

The theory of intuitionistic fuzzy sets (IFS) is based on a sound mathematical framework with foundations from the theory of fuzzy sets [4], which has been extended by Attanassov in [9-13], and still has many potentials worth investigating [19]. In the following we provide the required background on IFSs and on the intuitionistic fuzzy image representation considered.

2.1 Intuitionistic Fuzzy Sets Theory

An intuitionistic fuzzy set [9-13] is defined as a generalization of a fuzzy set [4].

Definition 1. Let a set E be fixed. A fuzzy set on E is an object \tilde{A} of the form

$$\tilde{A} = \left\{ \left\langle x, \mu_{\tilde{A}}(x) \right\rangle \mid x \in E \right\} \quad (1)$$

where $\mu_{\tilde{A}} : E \rightarrow [0,1]$ defines the degree of membership of the element $x \in E$ to the set $\tilde{A} \subset E$. For every element $x \in E$, $0 \leq \mu_{\tilde{A}}(x) \leq 1$.

Definition 2. An intuitionistic fuzzy set A is an object of the form

$$A = \left\{ \left\langle x, \mu_A(x), \gamma_A(x) \right\rangle \middle| x \in E \right\} \tag{2}$$

where $\mu_A : E \rightarrow [0,1]$ and $\gamma_A : E \rightarrow [0,1]$ define the degree of membership and non-membership, respectively, of the element $x \in E$ to the set $A \subset E$. For every element $x \in E$, it holds that $0 \leq \mu_A(x) \leq 1$, $0 \leq \gamma_A(x) \leq 1$ and

$$0 \leq \mu_A(x) + \gamma_A(x) \leq 1 \tag{3}$$

For every $x \in E$, if $\gamma_A(x) = 1 - \mu_A(x)$, A represents a fuzzy set. The function

$$\pi_A(x) = 1 - \mu_A(x) - \gamma_A(x) \tag{4}$$

represents the degree of *hesitancy* of the element $x \in E$ to the set $A \subset E$.

For every two intuitionistic fuzzy sets A and B the following operations and relations are valid [9-11]

$$A \subset B \text{ iff } \forall x \in E, \mu_A(x) \leq \mu_B(x) \text{ and } \gamma_A(x) \geq \gamma_B(x) \tag{5}$$

$$A = B \text{ iff } A \subset B \text{ and } B \subset A \tag{6}$$

$$A^C = \left\{ \left\langle x, \gamma_A(x), \mu_A(x) \right\rangle \middle| x \in E \right\} \tag{7}$$

$$A \cap B = \left\{ \left\langle x, \min(\mu_A(x), \mu_B(x)), \max(\gamma_A(x), \gamma_B(x)) \right\rangle \middle| x \in E \right\} \tag{8}$$

$$A \cup B = \left\{ \left\langle x, \max(\mu_A(x), \mu_B(x)), \min(\gamma_A(x), \gamma_B(x)) \right\rangle \middle| x \in E \right\} \tag{9}$$

$$A @ B = \left\{ \left\langle x, \frac{1}{2}(\mu_A(x) + \mu_B(x)), \frac{1}{2}(\gamma_A(x) + \gamma_B(x)) \right\rangle \middle| x \in E \right\} \tag{10}$$

$$\bigoplus_{i=1}^n A_i = \left\{ \left\langle x, \frac{1}{n} \left(\sum_{i=1}^n \mu_{A_i}(x) \right), \frac{1}{n} \left(\sum_{i=1}^n \gamma_{A_i}(x) \right) \right\rangle \middle| x \in E \right\} \tag{11}$$

2.2 Derivation of an Intuitionistic Fuzzy Image Representation

A pioneering approach to intuitionistic fuzzy representation of images was defined in [14] by utilizing crisp and fuzzy histogram information extracted from the images.

Definition 3. An image P of size $a \times b$ pixels having q gray levels θ ranging between 0 and $q-1$, is represented as the following IFS

$$\Phi = \left\{ \left\langle \theta_{ij}, \mu_{\Phi}(\theta_{ij}), \gamma_{\Phi}(\theta_{ij}) \right\rangle \middle| i = 1, \dots, a, j = 1, \dots, b \right\} \tag{12}$$

where θ_{ij} is the value of P at position (i, j) , and $\mu_{\Phi}(\theta_{ij})$ and $\gamma_{\Phi}(\theta_{ij})$ are the membership and the non-membership of θ_{ij} to P , respectively.

Based on that definition as a membership function $\mu_{\Phi}(\theta)$, we consider the probability of occurrence of grey level $\theta \in [0, q-1]$ in an image,

$$\mu_{\Phi}(\theta) = \frac{h(\theta)}{a \cdot b}, \quad \forall \theta \in [0, q-1] \tag{13}$$

where $h(\theta)$ is the crisp histogram of the pixel values.

The degree of belongingness of a grey level value θ in an image, as expressed by $\mu_{\Phi}(\theta)$, is subject to uncertainty due to noise. In order to model such a situation, a penalty factor $1 - p(\theta)$ is introduced so that θ belongs less to the image if $h(\theta)$ diverges more from the fuzzy histogram $\tilde{h}(\theta)$. The fuzzy histogram, originally proposed in [18], is defined as

$$\tilde{h}(\theta) = \left\| \left\{ (i, j) \in P \mid \mu_{\theta}(\theta_{ij}), i = 1, \dots, a, j = 1, \dots, b \right\} \right\| \tag{14}$$

with

$$\mu_{\tilde{\theta}}(x) = \max \left(0, 1 - \frac{|x - \theta|}{\psi} \right) \tag{15}$$

where the parameter ψ controls the span of the fuzzy number $\tilde{\theta}: R \rightarrow [0, 1]$ representing a fuzzy grey level. As ψ increases the fuzzy histogram becomes smoother and more insensitive to noise than the corresponding crisp histogram.

In accordance with [14], a simple and intuitive formulation of the non-membership of θ to an image can be expressed as

$$\gamma_{\Phi}(\theta) = (1 - \mu(\theta)) \cdot (1 - p(\theta)) \tag{16}$$

The term $p(\theta)$ is chosen to be proportional to the distance between the crisp $h(\theta)$ and the fuzzy histogram $\tilde{h}(\theta)$, so that Eq. (3) is satisfied

$$p(\theta) = \lambda \cdot \frac{|h(\theta) - \tilde{h}(\theta)|}{\max_{\theta} (|h(\theta) - \tilde{h}(\theta)|)} \tag{17}$$

where $\lambda \in [0, 1]$ is constant and the denominator facilitates normalization purposes. The physical meaning of this non-membership function is that the non-belongingness of a grey level value θ to an image increases by a factor that is proportional to the smoothness of the crisp histogram. So, as the noise levels in the image increase, the crisp histogram becomes coarser and the hesitancy in the determination of the grey level value θ increases.

3 Clustering Based on Intuitionistic Fuzzy Intersection

Conventional, as well as contemporary clustering approaches are incapable of clustering feature vectors with components characterized by both membership and non-membership values. Therefore, they are not directly applicable to clustering intuitionistic fuzzy image representations. To cope with this problem we introduce a

novel similarity metric in the FCM algorithm so that it becomes capable of clustering intuitionistic fuzzy image representations.

3.1 Intuitionistic Fuzzy Intersection

The similarity metric proposed in this paper is motivated by the histogram intersection technique [21], which has been generalized to measure the similarity of two IFSs.

Definition 4. Let $A=(M_A, \Gamma_A)$ and $B=(M_B, \Gamma_B)$ be two intuitionistic fuzzy sets, where $M_A, M_B, \Gamma_A, \Gamma_B \in F(E)$ and $E=\{x_1, x_2, \dots, x_n\}$ is a finite universe, the *intuitionistic fuzzy intersection* similarity Z_1 between the intuitionistic fuzzy sets A and B is calculated by the following equation

$$Z_1(A, B) = \frac{z_1(M_A, M_B) + z_1(\Gamma_A, \Gamma_B)}{2} \tag{18}$$

where

$$z_1(A', B') = \begin{cases} \frac{\sum_{i=1}^n \min(A'(x_i), B'(x_i))}{\sum_{i=1}^n \max(A'(x_i), B'(x_i))}, & A' \cup B' \neq \emptyset \\ 1, & A' \cup B' = \emptyset \end{cases} \tag{19}$$

and $F(E)$ is the set of all fuzzy subsets of E , and $M_A = \{\mu_A(x)\}$, $\Gamma_A = \{\gamma_A(x)\} \forall x \in E$.

The intuitionistic fuzzy intersection is a similarity metric since [19] $Z_1(A, B) \in [0, 1]$, $Z_1(A, B) = 1 \Leftrightarrow A = B$, $Z_1(A, B) = Z_1(B, A)$, and $Z_1(A, C) \leq Z_1(A, B)$ and $Z_1(A, C) \leq Z_1(B, C)$, for $A \subset B \subset C \in F(E)$.

3.2 Fuzzy c-Means Clustering of Intuitionistic Fuzzy Image Representations

Let U be a $c \times N$ -dimensional matrix of reals $u_{ik} \in [0, 1], \forall 1 \leq i \leq c$ and $1 \leq k \leq N$ denoting a fuzzy partition, where c and N represent the number of clusters and the cardinality of the feature vectors, respectively. Reformulating the fuzzy c-means clustering problem [5], we seek to minimize the objective function

$$J_m^{IFS}(U, V) = \sum_{i=1}^c \sum_{k=1}^N (u_{ik})^m |x_k - v_i|_{IFS}, \text{ with } \sum_{i=1}^c u_{ik} = 1, \quad 0 < \sum_{k=1}^N u_{ik} < N \tag{20}$$

where V is a $(p \times c)$ -dimensional matrix with c cluster centroids, p is the dimensionality of the feature vectors, $|x_k - v_i|_{IFS} = 1 - Z_1$ is the distance between data vector x_k and cluster centroid v_i , and $m \in [1, \infty)$ is a weighting exponent controlling the fuzziness of the clusters.

Solving the minimization problem in accordance with [5], $J_m^{IFS}(U, V)$ may be minimized only if

$$\forall_{\substack{1 \leq i \leq c \\ 1 \leq k \leq N}} u_{ik} = \begin{cases} \frac{\left(|x_k - v_i|_{IFS} \right)^{\frac{1}{1-m}}}{\sum_{j=1}^c \left(|x_k - v_j|_{IFS} \right)^{\frac{1}{1-m}}}, & I_k = \emptyset, \\ \begin{cases} 0, & i \notin I_k \\ \sum_{i \in I_k} u_{ik} = 1, & i \in I_k, \end{cases} & I_k \neq \emptyset, \end{cases} \quad (21)$$

and the centroids are computed by

$$\forall_{1 \leq i \leq c} v_i = \frac{\sum_{k=1}^N (u_{ik})^m x_k}{\sum_{k=1}^N (u_{ik})^m}. \quad (22)$$

It is worth noting that the distance between two vectors is computed solely upon the membership and the non-membership values of the vector components, after the computation of the centroids by Eq. (22) and before the next iteration, where the u_{ik} memberships to the new clusters are updated. Therefore the proposed FCM clustering approach requires an additional step for the estimation of the memberships and the non-memberships of the new centroids.

4 Results

Two sets of experiments were performed to evaluate the performance of the proposed intuitionistic image clustering approach, especially in the presence of noise. The first investigates clustering of real-world objects, whereas the second set provides indicative results from its application to image segmentation. In all the experiments, the same parameters ($\varepsilon=0.00001$, $m=2.0$) and initialization conditions were used.

4.1 Clustering Objects in the Presence of Noise

A reference set of 40 images originating from the COIL-20 database was used in this first set of experiments. The complete database comprises of a total of 20 color objects represented by 72 images of 192×192 pixels size, obtained for different rotations translations and scales [22]. For the purposes of this study 10 images of four representative objects were selected. The images have been selected so that the objects are rotated within a narrow range of angles in the third dimension and are scaled by a factor between 60 and 100%. Considering that the image representation described in Section 2 is intensity-based, the color information was discarded. In order to investigate clustering under noisy conditions, eight sets of images were produced by distorting the original ones using different levels of additive Gaussian noise with zero mean and variance (σ^2) ranging from 2 to 16. The corresponding PSNRs were ranging from 20 to 10dB).

Clustering experiments were performed on the original and on the produced sets of images. In all experiments four target clusters were considered. The results of the

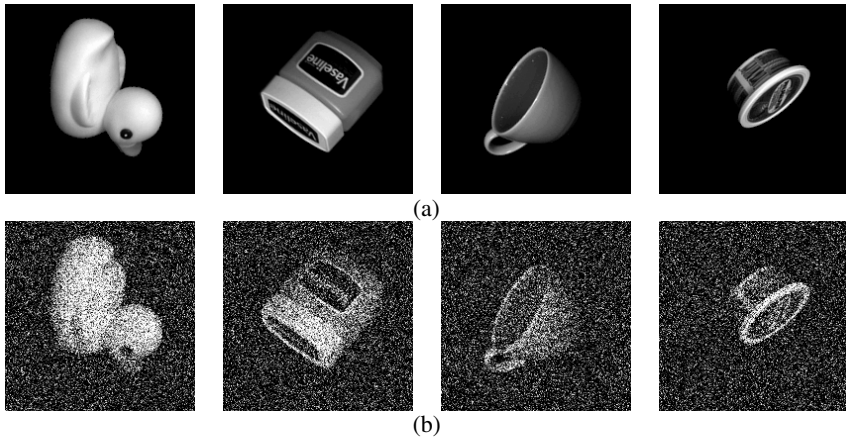


Fig. 1. Images of four objects from the COIL-20 database, (a) without noise, (b) with noise (PSNR=10dB)

proposed intuitionistic fuzzy clustering approach in comparison with the results obtained using the conventional FCM with fuzzy and crisp histogram image representations are illustrated in Fig. 2. As the noise level increases, it can be observed that clustering intuitionistic fuzzy image representations results in higher and more stable classification accuracy than the compared methods. Another interesting observation is that the introduction of noise benefits the accuracy of the particular clustering task.

Figure 3 provides comparative results on the number of iterations required for the clustering experiments of Fig. 2 to converge. It can be noticed that the proposed clustering approach requires always less iterations than the two other approaches compared. Moreover, unlike the other two approaches the number of iterations required for convergence remains almost constant as the noise level increases.

4.2 Image Segmentation Experiments

Segmentation experiments were performed on a set of twenty images of ancient monuments provided by the Foundation of Hellenic World, which maintains a rich repository of texts, images and multimedia data collections of Greek historical items and art [23]. Representative examples of the images used are illustrated in Fig. 4. The quality and the size of the images vary as they have been inconsistently acquired from different sources, and most of them have been captured in uncontrolled environments and stored using a lossy compression format resulting in a PSNR between 30 and 50dB.

The images were converted to greyscale and sampled using sliding windows of 32×32 pixels in a raster scanning fashion with a step of 8 pixels. The intuitionistic fuzzy, the fuzzy and the crisp representations of the samples were clustered into four target clusters producing respective image segmentations into four regions.

The algorithm iterations required for the convergence of the proposed approach was ranging between 6 and 12, whereas for the conventional FCM with the fuzzy and

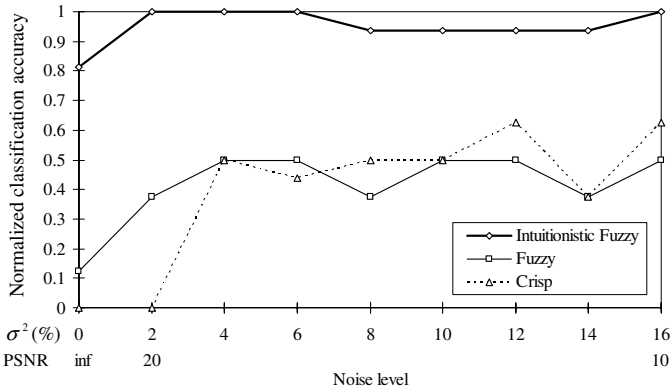


Fig. 2. Comparative results in terms of normalized classification accuracy

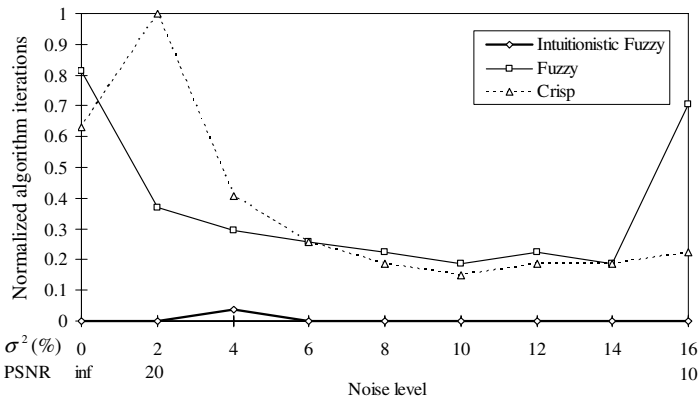


Fig. 3. Comparative results in terms of algorithm iterations required for convergence

the crisp histogram representations the iterations were ranging between 13 and 114. Indicative segmentations obtained from the three clustering approaches used are illustrated in Fig. 5. It can be observed that the segmented images obtained are better than those of the conventional methods, in the sense that the image regions corresponding

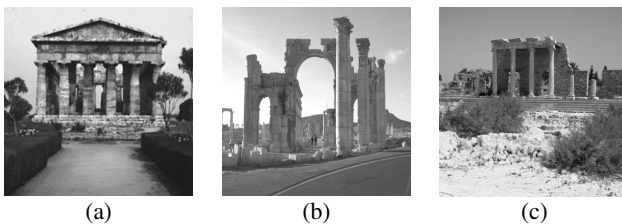


Fig. 4. Representative images used in the segmentation experiments

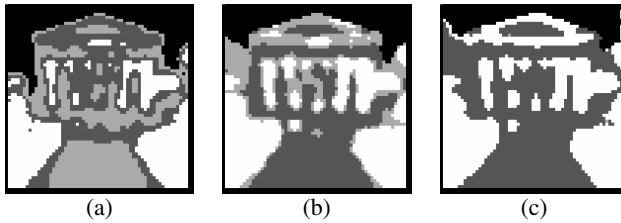


Fig. 5. Example segmentation of Fig. 4a based on clustering of (a) intuitionistic fuzzy image representations, (b) fuzzy histograms, and (c) crisp histograms

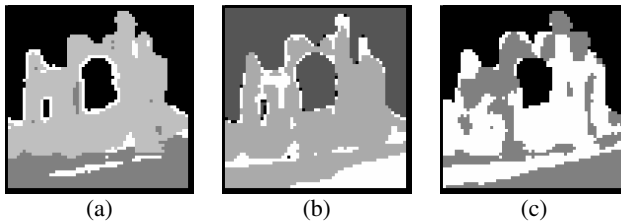


Fig. 6. Example segmentation of Fig. 4b based on clustering of (a) intuitionistic fuzzy image representations, (b) fuzzy histograms, and (c) crisp histograms

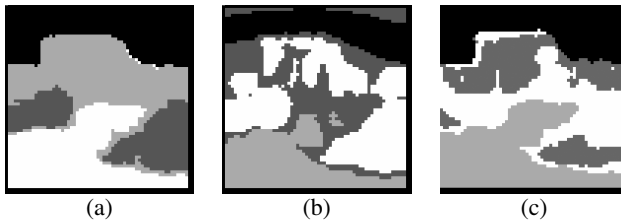


Fig. 7. Example segmentation of Fig. 4c based on clustering of (a) intuitionistic fuzzy image representations, (b) fuzzy histograms, and (c) crisp histograms

to the different objects are better discriminated. However, the segmentation performance is still limited by the variations of the intensity due to illumination and the sliding window technique, with which the detail of the segmentation and the definition of the object boundaries are subject to the selection of the window size and the sampling step.

5 Conclusion

A novel approach to clustering images represented as IFSs has been proposed and investigated. It is based on fuzzy c-means and it utilizes a novel similarity metric to measuring the distance between two IFSs. To this end, an intensity-based intuitionistic fuzzy image representation was considered. The experimental evaluation of the

proposed approach focused on object clustering especially in the presence of noise and image segmentation.

Three major conclusions can be derived about the intuitionistic fuzzy image clustering investigated:

1. It can lead to more accurate clustering than the conventional FCM clustering of crisp and fuzzy histograms;
2. It is noise tolerant, providing more stable performance in the presence of noise;
3. It converges in less iterations than the conventional FCM.

Future research includes systematic and thorough evaluation of the proposed approach in comparison with other clustering schemes, as well as derivation and utilization of alternative intuitionistic fuzzy image representations. As the intuitionistic fuzzy theory is becoming more and more popular, the intuitionistic fuzzy clustering has increasing potential of extending its applications to the broader domains of pattern recognition and data mining.

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