An Iterative Kalman Filter Approach to Camera Calibration

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Abstract. An iterative camera calibration approach is presented in this paper. This approach allows computing the optimal camera parameters for a given set of data. If non linear estimation process is done, a risk of reaching a local minimum exists. With this method this risk is reduced and a best estimation is achieved. By one hand, an iterative improving of the estimated camera parameters is done maximizing a posteriori probability density function (PDF) for a given set of data. To resolve it, a Kalman filter is used based on the Bayesian standpoint. Each update is carried out starting with a new set of data, its covariance matrix and a previous estimation of the parameters. In this case, a different management of the input data is done to extract all its information. By the other hand, apart from the calibration algorithm, a method to compute an interval which contains camera parameters is presented. It is based on computing the covariance matrix of the estimated camera parameters.

Keywords: camera calibration, Kalman filter, sampling, interval computation.

1 Introduction

Camera calibration is an important step in 3D computer vision. Accurate calibration of the camera should be done in order to compute quantitative measurements, depth from stereoscopy, or motion from images. Camera calibration process estimates the internal and external camera parameters. They are represented in a pinhole camera model. Existing camera calibration techniques can be classified into photogrammetric calibration and self-calibration. Photogrammetric calibration is carried out observing a well known 3D pattern. The term "well known" means that the spatial coordinates of some points in the pattern and their uncertainties are known with very high precision. Usually, this pattern should contain points in the X, Y, Z planes and therefore, two or three planar objects orthogonal to each other are used [2] [6]. Self-calibration techniques do not need any special 3D pattern in the scene. Using the rigidity of the scene, it provides constrains which should be satisfied by the internal parameters.

Therefore, just taking different images of the scene with fixed internal parameters are sufficient to compute them. Afterwards, external parameters can be recovered starting with the intrinsic ones [1] [5]. Another camera calibration technique which lies between the photogrammetric calibration and self-calibration is the one based on a planar pattern [7] [9]. In this case, it uses 2D metric information rather than 3D and

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the camera only needs to observe a planar pattern showed at least at three different orientations. Compared with classical techniques, this method can be considered more flexible and also, it gains significant degree of robustness [6].

From the point of view of the computation method, calibration algorithms minimize a cost function. The close-form solution is followed by a nonlinear refinement based on the minimization of the Euclidean or Mahalanobis distance. In all cases, the camera parameters are recovered from a single set of data. This data corresponds to the points coordinates in the scene and its correspondences in one or several images from one or more camera positions. Since a parameter fitting is done, all camera calibration algorithms are sensitive to observation noise. Moreover, if non linear refinement is done the starting parameter values are important in order to achieve an absolute minimum. In some situations these two factors interact with each other. Large observation noise might create more local minima which can easily trap iterative optimization methods such as Levenberg-Marquardt or steepest-descendent. In this case, in order to achieve the absolute minima a good initial guess of the calibration parameters is needed.

In this paper, a new camera calibration approach is defined in order to achieve a more accurate result. In this case, an optimal filter is applied to the camera calibration process. An iterative method is used in which the update rule is derived from the Bayesian standpoint. The principle of the maximum a posteriori probability estimation is put into practice assuming the observation noise is Gaussian. The optimal filter updates the initial estimation using a new set of data. This set of data can be obtained before the calibration algorithm is carried out. From the point of view of number of data, all camera calibration algorithms need a minimum number of data to be solvable. The improvement of the result is related with the amount of data. If the amount of data is bigger, better results are computed. However, if the amount of data is bigger than a certain number, the result improvement is less significant. This certain number depends on the algorithm and normally it is determined by simulation. Therefore, instead of using all data in order to resolve the close form solution followed by a non linear refinement, a certain amount of data can be used first to obtain an initial estimation and afterwards the remaining data is used with Kalman filter in order to improve the result. Kalman filter can be put into practice with any photogrammetry or self-calibration methods. In this case the camera calibration method based on planar pattern [7] [9] has been used. This method has been chosen because new set of data can be obtained just by taking a new image of the calibration pattern from a different position of the previous ones. In case of photogrammetric methods, an image of an over dimensioned calibration object with a lot of interesting points should be used in order to obtain a lot of data which will be used in several steps of the Kalman filter. From a practical point of view this could be useless. Another solution to use the Kalman filter with photogrammetric methods could be to take new images of the same calibration object from the same position. This way seems more useful.

The paper is organized as follow. Section 2 is a brief description of Kalman filter. Section 3 describes the Kalman filter applied to the camera calibration method based on planar template and section 4 shows experimental results. A comparison with the Levenberg-Marquardt non linear optimization method has been done in order to test the results of the Kalman filter. Better results have been obtained.

2 Kalman Filter

Kalman filter is based on Bayes' theorem which calculates conditional probabilities. In this case the aim is to maximize a posteriori PDF of the camera parameter given a set of data. The set of data are points in the scene and its correspondences in the image. Maximizing the PDF, most probable camera parameters will be computed according to given data. This index is called maximum a posteriori probability estimation. Kalman filter can be summarized as follow. If a_α is a m-vector datum, the aim is to estimate an n-vector u of parameters that satisfies the hypothesis $f^{(k)}(a_\alpha,u)=0$.The optimal value of the parameter vector $u_{\alpha+1}$ is computed with an initial estimation of the parameter vector u_α together with a new set of data. This process is repeated iteratively with new data. A covariance matrix of estimated parameter vector is necessary to use it in the next iteration. The new value of the parameter vector is given by

$$u_{\alpha+1} = u_\alpha - \left(V(u_\alpha)^- + P_u S_\alpha P_u\right)^- P_u t_\alpha \tag{1}$$

Its covariance matrix is defined by

$$V(u_{\alpha+1}) = \left(V(u_\alpha)^- + P_u S_\alpha P_u\right)^- \tag{2}$$

Where

$$S_\alpha = \sum_{k,l=1}^{L}\left(W_\alpha^{(kl)} \cdot \nabla_u f_\alpha^{(k)} \cdot \nabla_u f_\alpha^{(l)^T}\right) + 2 f_\alpha^{(k)} \cdot \nabla_u W_\alpha^{(kl)} \cdot \nabla_u f_\alpha^{(l)^T} \tag{3}$$

$$t_\alpha = \sum_{k,l=1}^{L}\left(W_\alpha^{(kl)} \cdot f_\alpha^{(k)} \cdot \nabla_u f_\alpha^{(l)^T} + 0.5 f_\alpha^{(k)} \cdot f_\alpha^{(l)} \cdot \nabla_u W_\alpha^{(kl)}\right) \tag{4}$$

$$E_\alpha = \sum_{k,l=1}^{L}\left(W_\alpha^{(kl)} \cdot f_\alpha^{(k)} \cdot f_\alpha^{(l)}\right) \tag{5}$$

$$W^{(kl)}(u) = \left(\nabla_a f^{(k)}(a_\alpha,u)^T \cdot V(a_\alpha) \cdot \nabla_a f^{(l)}(a_\alpha,u)\right)^- \tag{6}$$

An exhaustive explanation of Kalman filter can be found for example at [4].

3 Camera Calibration Using Kalman Filter

First a brief description of camera calibration based of planar template is done.

3.1 Camera Calibration with a Planar Template

In [7] [9] a novel camera calibration method based on the homographies between a calibration pattern and the image of it from several camera positions and orientations was presented. The method assume a planar calibration pattern situated at $z=0$ of the world coordinate system. Denoting r_i as the i^{th} column of the rotation matrix R and t the translation vector of the camera in the scene coordinates, the homography H which defines the coordinates of a point p in the image of a point q in the planar calibration pattern is $p=Hq$ where

$$p=\begin{bmatrix}u\\v\\1\end{bmatrix}=A\begin{bmatrix}r_1 & r_2 & r_3 & t\end{bmatrix}\begin{bmatrix}x\\y\\0\\1\end{bmatrix}=A\begin{bmatrix}r_1 & r_2 & t\end{bmatrix}\begin{bmatrix}x\\y\\1\end{bmatrix}=H\cdot q \qquad (7)$$

Matrix A contains internal camera parameters. Vector t represent the translation of the camera in the scene and matrix R its rotation. Both are the external camera parameters. Given a homography two constrains arise because r_1 and r_2 are two orthonormal vectors. This constrains affect to the matrix A which contains the internal camera parameters. They are

$$\begin{aligned}&h_1^T A^{-T}A^{-1}h_2=0\\&h_1^T A^{-T}A^{-1}h_1=h_2^T A^{-T}A^{-1}h_2\end{aligned} \qquad (8)$$

Given that $A^{-T}A^{-1}$ is a symmetric matrix, a 6D vector is defined as $b=[\,B_{11}, B_{12}, B_{22}, B_{13}, B_{23}, B_{33}]$ where B_{ij} represents the element ij of the symmetric matrix $A^{-T}A^{-1}$. Using vector b, constraints (9) can be written as $h_i^T B\cdot h_j=v_{ij}^T b$ where

$$v_{ij}^T=\begin{bmatrix}h_{i1}h_{j1} & h_{i1}h_{j2}+h_{i2}h_{j1} & h_{i2}h_{j2} & h_{i3}h_{j1}+h_{i1}h_{j3} & h_{i3}h_{j2}+h_{i2}h_{j3} & h_{i3}h_{j3}\end{bmatrix} \qquad (9)$$

The constraints (9) for a given homography can be rewritten as two homogeneous equations in b as follows:

$$\begin{bmatrix}v_{12}^T\\v_{11}^T-v_{22}^T\end{bmatrix}b=0 \qquad (10)$$

If n images of the calibration object are observed, by staking n such equations as (10) we have $Vb=0$ where V is a $2n\ x\ 6$ matrix. At least 3 images are necessary in order to obtain a unique solution. The closed-form solution is given by the eigenvector of V^TV associated with the smallest eigenvalue. Once b is estimated the camera internal parameters can be computed. When A is known, the external parameters for each image can be computed knowing the corresponding homography.

3.2 Camera Calibration with Kalman Filter

The closed-form solution gives an initial estimation of b which can be refined through the Kalman filter described above. In this case the a posteriori PDF of the vector b given a set of homographies H is wanted to be maximized. Hypotheses to be satisfied are given by the equation (10). Homographies are represented by the vector v_{ij}. In this case the number L of hypotheses are two which are given by:

$$f^{(1)}(\bar{h},b)=h_1h_2B_{11}+(h_4h_2+h_1h_5)B_{12}+h_4h_5B_{22}+(h_7h_2+h_1h_8)B_{13}+(h_7h_5+h_4h_8)B_{23}+h_7h_8B_{33}=0 \qquad (11)$$

$$f^{(2)}(\bar{h},b)=(h_1^2-h_2^2)B_{11}+2(h_4h_1-h_5h_2)B_{12}+(h_4^2-h_5^2)B_{22}+2(h_7h_1-h_2h_8)B_{13}+2(h_7h_4-h_5h_8)B_{23}+(h_7^2-h_8^2)B_{33}=0 \qquad (12)$$

In order to compute the update increment for the parameter vector b, equation (2) should be used. This equation has five terms which should be defined in this case. P_u is the projection matrix along the domain in which the parameter vector b is defined. Since b is a 6D homogeneous vector, P_u is an identity matrix in which the element (6,6) is equal to zero. S_α, t_α are computed according to the equations (4) and (5). In this case $W^{(kl)}$ is computed with the equation (7) and the subscript α refers to the value evaluated with the new set of data and the current estimation of the parameter vector b. Terms $\nabla_h f^{(1)}$, $\nabla_h f^{(2)}$, $\nabla_b f^{(1)}$, $\nabla_b f^{(2)}$, and $\nabla_h W^{(kl)}$ are easily defined using (11) and (12).

3.3 Covariance Matrix of Initial Parameters Estimation and Homographies

Terms $V(b_\alpha)$ in (2) $V(h_\alpha)$ in (7) are not defined yet. It is necessary to define the covariance matrix of the current estimation of the parameter vector b and the covariance matrix of the input data which are the elements of the homographies. Term $V(b_\alpha)$ can be computed with equation (3) for successive iterations. The difficulty is to compute it for the first one and also to compute the covariance matrix $V(h)$ for all homographies which are used as input data. The method of propagating the covariance matrix described by Haralick in [3] has been used in this case. Random perturbation of points coordinates measurements are represented with a covariance matrix. This method allows propagating the covariance matrix of points coordinates through the computation of the elements of the homography H and afterwards, the elements of the vector b. It just needs that the scalar function has finite second partial derivative and that the random perturbation is small enough so that the scalar function evaluated at noise input can be approximated sufficiently well by a first order Taylor series.

First the covariance matrix of each homography will be computed propagating the random perturbation of the points coordinates. In this case, point coordinates are arranged into a vector $p^T=[x\ y\ 1\ u\ v\ 1]$ where x, y are the coordinates of the point in the planar patternn calibration an u, v their correspondence in the image. $V(p)$ is a diagonal matrix formed with the covariance matrix of the point coordinates of the calibration pattern and the covariance matrix of the image coordinates. In order to compute the homography the following restrictions are assumed.

$$g^{(1)}(h,p) = x \cdot h_1 + y \cdot h_2 + h_3 - u \cdot x \cdot h_7 - u \cdot y \cdot h_8 - u \cdot h_9 = 0 \tag{13}$$

$$g^{(2)}(h,p) = x \cdot h_4 + y \cdot h_5 + h_6 - v \cdot x \cdot h_7 - v \cdot y \cdot h_8 - v \cdot h_9 = 0 \tag{14}$$

In this case vector h contains the elements of the matrix H arranged in a column. Therefore if $V(p)$ is the covariance matrix of the point coordinates, which is propagated to the elements of the homography H through the two restrictions (13) and (14) as

$$V(h) = \left(\nabla^2_{h,h} g^{(1)}\right)^{-} \nabla^2_{h,p} g^{(1)} V(p) \nabla^2_{h,p} g^{(1)} \left(\nabla^2_{h,h} g^{(1)}\right)^{-} + \left(\nabla^2_{h,h} g^{(2)}\right)^{-} \nabla^2_{h,p} g^{(2)} V(p) \nabla^2_{h,p} g^{(2)} \left(\nabla^2_{h,h} g^{(2)}\right)^{-} \tag{15}$$

Likewise, the covariance matrix $V(h)$ is propagated to the elements of vector b through the restrictions (11) and (12). Covariance matrix $V(b)$ is defined as follow:

$$V(b) = \left(\nabla^2_{b,b} f^{(1)}\right)^{-} \nabla^2_{b,h} f^{(1)} V(h) \nabla^2_{b,h} f^{(1)} \left(\nabla^2_{b,b} f^{(1)}\right)^{-} + \left(\nabla^2_{b,b} f^{(2)}\right)^{-} \nabla^2_{b,h} f^{(2)} V(h) \nabla^2_{b,h} g^{(2)} \left(\nabla^2_{b,b} f^{(2)}\right)^{-} \tag{16}$$

3.4 Proposed Calibration Algorithm

Camera calibration process using the Kalman filter has the following steps:

1. Take n images of the planar calibration object
2. Compute the homografy matrix H between each image and the calibration object together with the covariance matrix $V(h)$ of its elements. Use equation (13), (14) and (15)

3. Compute vector b and also the covariance matrix $V(b)$ of its elements with equations (10) and (16)
4. Take a new set of data. In this case a new set of data means a set of homographies together with its covariance matrices.
5. Calculate the values of S_α and t_α using equations (4) and (5)
6. Update the initial estimation of vector b using equation (2)
7. Compute new covariance matrix $V(b)$ with (3)
8. Go to step 4

Finally, in order to resolve all details related with the Kalman filter applied to camera calibration with planar pattern, it is necessary to take into account the new set of data mentioned in step 4. It is. In this case, the input data of the algorithm for estimating vector b are homographies between the calibration object and images of it from different camera positions. Attending to experimental results of the closed-form solution (Fig. 2 of [9]), vector b is not significantly improved if more than three homographies are used in the estimation process. Therefore, an initial estimation of vector b can be done just using three homographies and the remaining ones are used as a new data in the Kalman filter. Moreover, since just one homography is necessary in order to compute an update of vector b, n-3 iterations can be done if n images of the calibration object have been taken. However, in order to improve the quality of the input data in the Kalman filter, several homographies could be used. This allows combining n homographies in groups of three and taking the first group to compute an initial estimation and the remaining ones to perform several update iterations using the optimal filter. The number of improving iterations will be given by the combinations of n elements taking 3 by 3. It is $n!/3!(n-3)!$. So using a small set of images more iteration can be carried out.

4 Experimental Results

In order to evaluate this calibration process, simulated and real experiments has been done. First, the closed form solution is computed using only 3 homographies followed by the Kalman filter with the remaining images. Second, in order to compare the results with classical methods, the closed-form solution is computed with all homographies followed by a nonlinear refinement using Levenberg-Mardquart method. Next figures show the result of the internal parameter called principal point (U_0, V_0) only. Similar behaviour has been obtained with the remaining parameters.

4.1 Simulated Experiments

The simulated camera has the following properties: α=980, β=980, γ=1, u_0=320, v_0=240. The model plane is a checker with 100 corner points (10x10) of size 20x20 cm^2. The distance of the camera to the origin is always 1 m and the images are taken from different angles and orientation of the projection of the optical axis with respect to the X axis. Several situations have been simulated.

Performance w.r.t. the noise level. Gaussian noise level of mean 0 and standard deviation σ is added to the coordinates of the image points and calibration pattern points. The standard deviation has been changed in order to test how the Kalman filter works depending on the noise of the input data. 100 simulated calibrations have been done and the mean of errors are shown in figure 1. Ten images of the planar patter have been used. Since real value is known, it is compared with the estimated one. A comparison with the result of the Levenberg-Mardquart method is done. Better results are computed with kalman filter.

Performance w.r.t. the number of planes. Figure 2 shows the results changing the number of images from 3 to 13. In this case, a constant standard deviation of noise level of 1 mm in the pattern points coordinates and 1 pixel in the image points is added. Also, a mean of 100 simulated calibrations is shown in the figure. If the number of images is small, errors are smaller with Levenberg-Mardquart optimization because Kalman filter has no new data to iterate. When the number of images increases, the number of iterations with Kalman filter increase and it allows improving the result.

Computing an interval which contains the camera parameter. Since it is possible to estimate an interval which contains the camera parameter, several simulations has been done to test its behaviour. Figure 3 shows the result of computing the interval of camera parameter changing the noise level. Figure 4 shows the evolution of the interval if the number of homographies increase. Straight line represents the real value. Dotted lines represent the interval which contains the parameter.

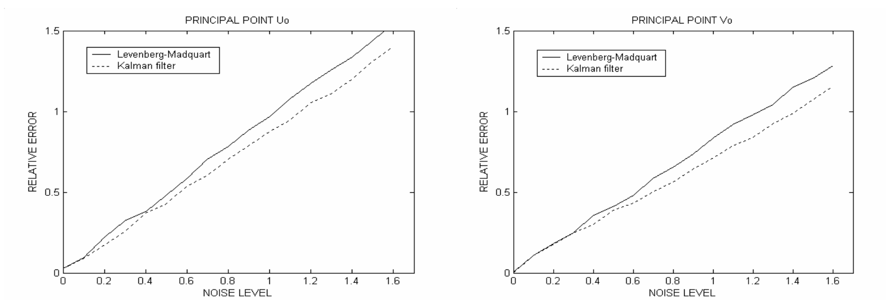

Fig. 1. Errors versus noise level

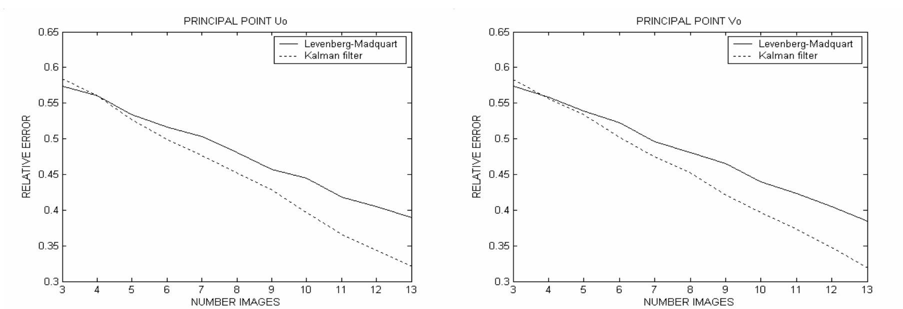

Fig. 2. Errors versus noise level

4.2 Experiments with Real Data

In this case a webcam has been used. Its resolution is *640x480*. Two calibration images are shown in figure 5. It has 63 points. As before, several images have been taken approximately from the same positions as simulated experiments, but now the distance of the camera to the calibration pattern is 40 cm. Table 1 shows the results. First column (CF) is the result of the initial closed form solution for each internal camera parameter. Second column (KF) shows the result using Kalman filter and the third one is the solution of the Levenberg-Madquart (LM) algorithm. Last row show the value RMS. It indicates the root of mean squared distances in pixels between detected image points and projected ones. Kalman filter improves this result.

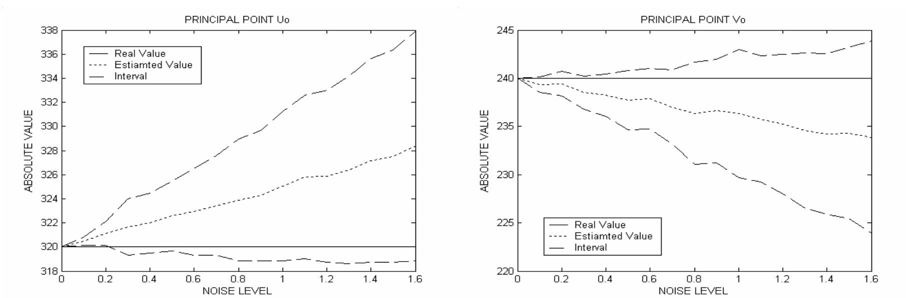

Fig. 3. Absolute value with the interval versus noise level

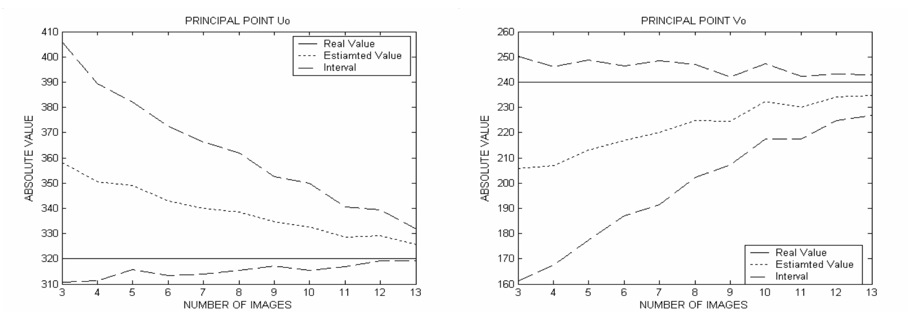

Fig. 4. Absolute value with the interval versus number of images

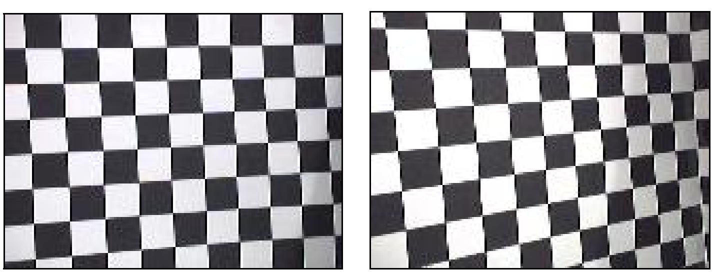

Fig. 5. Two images of planar calibration template for real experiments

Table 1. Results with real data

	5 images			10 images		
	CF	KF	LM	CF	KF	LM
α	982.3	990.5	987.4	979.5	985.2	988.6
β	975.8	992.8	988.4	980.4	982.4	988.4
u_0	309.5	315.7	319.4	315.4	322.7	316.8
v_0	229.7	234.8	238.7	232.7	238.9	242.1
RMS	0.845	0.554	0.587	0.935	0.425	0.567

If the number of images is small, the Kalman filter algorithm does not have degrees of freedom to iterate. Then improvement is not significant. However, if the number of images increase it allows to increase the number of iterations and then better results is computed than if we use Levenberg-Madquart optimization algorithm.

5 Conclusions

A new approach to camera calibration process has been done. In this case a Kalman filter has been used to maximize the probability density function of the camera parameters for a given set of points coordinates. Moreover, to make the calibration algorithm less sensitive to measurement noise, a different management of the input data is done. This allows improving the computed parameters with the information of new data which has not participated in the estimation process before. Non linear refinement methods use always the same information and they could fall into a local minimum if starting parameters are not very good. With the Kalman filter new information is added to the estimation algorithm and it allows finding optimal values. Furthermore, the covariance matrix of the input data has been taken into account as a measurement of credibility of the input data. This fact allows the algorithm improve the estimated parameters depending on the accuracy of the input data. In addition, if the input data fits with the current estimated values, the optimal filter decreases the covariance matrix of the estimation.

One disadvantage of this calibration method is the estimation of the non linear camera parameters. These are (lens and CCD distortions). If Levenberg-Madquart optimization method is used, non linear relations between parameters can be taken into account. In this case, only linear relations are estimated. Future work is oriented to resolve this task.

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