15

Unified Solution of Shakedown Limit for Thick-walled Cylinder

15.1 Introduction

Correct prediction of the load-bearing capacity of structures is a crucial task in the analysis and design of engineering structures. The plastic limit load of structures from limit analysis or slip-line analysis is usually used as an index of the load-bearing capacity of the structure, subjected to a monotonic loading. When the loading is a repeated loading, the structures fail at a load which is lower than the plastic limit load. This is due to gradual deterioration caused by the alternating plasticity or by the incremental plasticity instead of sudden collapse.

If the load does not exceed the critical value, the structure subjected to the repeated loading may behave plastically at first and then elastically. No further plastic deformation takes place in the structure. The structure shakes down due to the repeated loading. If the load exceeds the critical value the structure does not shake down and fails due to the alternating plasticity or the incremental plasticity. This critical load level is called the shakedown load. The shakedown load is usually regarded as the load-bearing capacity of the structure subjected to the repeated loading.

Many engineering structures or components are subjected to mechanical or other loads varying with time. The shakedown condition should be guaranteed for the safety of such kinds of structures.

Shakedown theory of structures is usually applied for such kinds of problems. A structure in a non-shakedown or inadaptation condition under varying loads may fail by one of two failure modes, namely alternating plasticity or incremental plastic collapse. The structure will shake down if neither of the failure modes occurs (Symonds, 1951; Hodge, 1954; Kachanov, 1971; Martin, 1975; Zyczkowski, 1981; Chakrabarty, 1987, Mroz et al., 1995).

The concept and methods of shakedown analysis were initially addressed in the 1930s and developed in the 1950s. The pioneering works of shakedown include those by Bleich (1932), Melan (1936). Koiter (1956) proved two crucial shakedown theorems, i.e., the static shakedown theorem (Melan's theorem, the first shakedown theorem, or the lower bound shakedown theorem), and the dynamic shakedown theorem (Koiter's theorem, the second shakedown theorem, or the upper bound shakedown theorem), which constitute the fundamentals in the shakedown theory of elasto-plastic structures.

Accordingly, numerous existing methods for shakedown analysis can be divided into two classes, i.e., the static and the dynamic shakedown analysis methods. Shakedown theory has become a well-established branch of plasticity theory.

In recent years shakedown analysis of elasto-plastic structures has increasingly attracted attention from engineers due to the requirements of modern technologies such as in nuclear power plants, chemical industry, the aeronautical and astronautical, electrical and electronic industries. Shakedown theory has been applied with success in a number of engineering problems such as the construction of nuclear reactors, highways and railways and employed as one of the tools for structural design and safety assessment in some design standards, rules, and regulations. A study of the plastic shakedown of structures was made by Polizzotto (1993), and of some issues in shakedown analysis by Maier (2001) and Maier et al. (2000).

Long thick-walled cylinders are very often used as gun barrels and pressure vessels in engineering. They are usually subjected to repeated internal pressure. It is necessary to conduct shakedown analysis in order to determine the shakedown load of the cylinder. The solution to shakedown problem of cylinder is readily available in textbooks of the classical plasticity, and the analytical solution can be found in some published literature and in the Pressure Vessel Code, such as Cases of ASME Boiler and Pressure Vessel Code. However, the solution is based on the Tresca yield criterion, and the analytical solution based on the Huber-von Mises criterion is not readily derivable in most cases due to the nonlinear expression of the criterion. As we have discussed, the Tresca yield criterion considers the effects of only the first and the third principal stresses and ignores the compressive-tensile strength difference (SD) effect of materials. Thus, this classical solution can only be applied to the cylinder made of non-SD materials where the intermediate principal stress effect is negligible. It is of great importance to develop a new approach to cover the SD effect and the intermediate principal stress effect for more general applications. The influence of different strengths in tension and compression for the shakedown of thick-walled cylinders was studied by Feng and Liu (1995). A series of results were given by Feng et al. (1993-1999). An elasto-plastic model incorporating the Yu unified strength theory (UST) was suggested for shakedown analysis of a thick-walled cylinder by Xu and Yu (2005). A closed-form solution of shakedown load for cylinders will be presented in this chapter. The solutions involve the two parameters of Yu unified strength theory m and b and can reflect both the effect of intermediate principal stress and the SD effect in a quantitative manner. It is referred to as the unified solution including serial solutions. By choosing proper values for m and b, the solution is applicable to cylinders made of different materials. In addition, by applying the solution based on Yu unified strength theory, the effects of SD and the intermediate principal stress on the shakedown load of the thick-walled cylinder are evaluated.

15.2 Shakedown Theorem

Many engineering structures or components are subjected to mechanical or other loads varying with time. In many cases only the loading range within which the loads change can be estimated, while the loading path is unknown. It is important to guarantee the shakedown condition for the safety of such kinds of structures.

15.2.1 Static Shakedown Theorem (Melan's Theorem)

The static or Melan's shakedown theorem (Melan, 1936; Kachanov, 1971; Martin, 1975) indicates the necessary condition for the occurrence of shakedown: there exist time-independent fields of residual stresses $\overline{\sigma}_{ij}$ such that the sum $(\overline{\sigma}_{ij}+ \sigma^e_{ij})$ is admissible, where σ^e_{ij} are the elastic components of stresses. It implies that the stress field $(\overline{\sigma}_{ij}+ \sigma^e_{ij})$ is safe if no arbitrary load-variation in the prescribed limits causes the yield surface $f(\overline{\sigma}_{ij}+ \sigma^e_{ij})$ to be reached, i.e.,

$$f(\overline{\sigma}_{ij} + \sigma^e_{ij}) < 0. \tag{15.1}$$

The necessary condition is not obtained if there is no distribution of residual stresses for which $f\ (\overline{\sigma}_{ij}+ \sigma^e_{ij})<0$, and so shakedown cannot occur.

On the contrary, shakedown occurs if there is a fictitious residual stress field $\overline{\sigma}_{ij}$ that is independent of time. For any variations of loads within the prescribed limits, the sum of this field with the stress field σ^e_{ij} in a perfectly elastic body is safe (sufficient condition).

The residual stress field is expediently chosen such that the region of admissible load variation is the greatest. Melan's theorem serves as a low bound of the limit load.

15.2.2 Kinematic Shakedown Theorem (Koiter Theorem)

Koiter's theorem (1956), also called the kinematic inadaptation theorem, can be regarded as an extension of the upper bound theorem in limit analysis. The theorem is framed in terms of an admissible plastic strain rate cycle $\dot{\varepsilon}^{kp}_{ij}(s,t)$ for $0 < t < T$. In view of the principle of virtual work, the statement of Koiter's theorem can be interpreted as showing that if the external power of any admissible plastic strain rate cycle $\dot{\varepsilon}_{ij}^{kp}(s,t)$ can be found to exceed the power dissipated in the structure, i.e.,

$$\int_0^T \mathrm{d}t \iint_{S_T} p_j \dot{u}_j^k \mathrm{d}S > \int_0^T \mathrm{d}t \iint_{S_T} \sigma_{ij}^k \dot{\varepsilon}_{ij}^{kp} \mathrm{d}V, \tag{15.2}$$

shakedown will not occur, where σ_{ij}^k is the stress field associated with $\dot{\varepsilon}_{ij}^{kp}(s,t)$; $\dot{u}_j^k$ is the velocity field for a cycle by the loads p_j.

It should be noted that the static shakedown theorem and the kinematic non-shakedown theorem determine the lower and the upper bounds to the permissible loading range for the shakedown of a structure.

15.3 Shakedown Analysis for Thick-walled Cylinders

When considering a plane strain thick-walled cylinder under uniform internal pressure p with internal and external radii of r_i and r_e, respectively, for simplicity it is assumed that the material is incompressible and elastic-perfectly plastic with negligible Bauschinger effect. If the pressure p is moderate, the thick-walled cylinder is in an elastic state. The stress field of the cylinder is given by the Lame solutions,

$$\sigma_r = \frac{r_i^2 p}{r_e^2 - r_i^2}(1 - \frac{r_e^2}{r^2}), \tag{15.3a}$$

$$\sigma_\theta = \frac{r_i^2 p}{r_e^2 - r_i^2}(1 + \frac{r_e^2}{r^2}), \tag{15.3b}$$

$$\sigma_z = \frac{r_i^2 p}{r_e^2 - r_i^2}. \tag{15.3c}$$

From Eqs.(15.3a), (15.3b), and (15.3c), σ_θ is the major principal stress, σ_z the intermediate principal stress, σ_r the minor principal stress, and they satisfy

$$\sigma_z \leqslant \frac{m\sigma_\theta + \sigma_r}{m+1}. \tag{15.4}$$

Therefore, the unified strength theory can be expressed as

$$\sigma_\theta - \frac{1}{m(1+b)}(b\sigma_z + \sigma_r) = \sigma_t, \tag{15.5}$$

where m is the ratio of material strength in compression and in tension, for non-SD materials $m = \sigma_c/\sigma_t = 1$.

From Eq.(15.5) the maximum value for $\sigma_\theta - (b\sigma_z + \sigma_r)/(m+mb)$ occurs on the internal wall of the cylinder. Yielding starts from the internal wall of the cylinder when the internal pressure reaches

$$p_e = \frac{m(1+b)(r_e^2 - r_i^2)}{(m+1+mb)r_e^2 + (m-1)(1+b)r_i^2}\sigma_t, \tag{15.6}$$

where p_e is the elastic limit pressure of the cylinder.

When the internal pressure exceeds p_e, a plastic zone spreads out from the inner radius. If the plastic zone reaches the radius r_p, the cylinder can be divided into two parts: a plastic zone in the range of $r_i \leqslant r \leqslant r_p$, and a elastic zone of $r_p \leqslant r \leqslant r_e$. Using the Lame solution, the boundary condition $\sigma_r = 0$ at $r = r_e$ and at $r = r_p$, the yield condition in Eq.(15.5) is satisfied. The stress components in the elastic zone are derived as

$$\sigma_r = \frac{r_p^2 p_p}{r_e^2 - r_p^2}\left(1 - \frac{r_e^2}{r^2}\right), \tag{15.7a}$$

$$\sigma_\theta = \frac{r_p^2 p_p}{r_e^2 - r_p^2}\left(1 + \frac{r_e^2}{r^2}\right), \tag{15.7b}$$

$$\sigma_z = \frac{r_p^2 p_p}{r_e^2 - r_p^2}, \tag{15.7c}$$

where

$$p_p = \frac{m(1+b)(r_e^2 - r_p^2)}{(m+1+mb)r_e^2 + (m-1)(1+b)r_p^2}\sigma_t$$

is the associated radial pressure on the elasto-plastic interface under the internal pressure p.

According to the equilibrium equation

$$\frac{\mathrm{d}\sigma_r}{\mathrm{d}r} + \frac{\sigma_r - \sigma_\theta}{r} = 0, \tag{15.8}$$

the yield condition in Eq.(15.5), the boundary condition of $\sigma_r = p$ at $r = r_i$, the incompressible condition of materials, then the stress components in the plastic zone are derived as

$$\sigma_r = -\left(p + \frac{m\sigma_t}{m-1}\right)\left(\frac{r_i}{r}\right)^{\frac{2(m-1)(1+b)}{2m+2mb-b}} + \frac{m}{m-1}\sigma_t, \tag{15.9a}$$

$$\sigma_\theta = -\frac{(2+b)}{2m+2mb-b}\left(p + \frac{m\sigma_t}{m-1}\right)\left(\frac{r_i}{r}\right)^{\frac{2(1+b)(m-1)}{2m+2mb-b}} + \frac{m}{m-1}\sigma_t, \tag{15.9b}$$

$$\sigma_z = -\frac{1+m+mb}{2m+2mb-b}\left(p+\frac{m\sigma_t}{m-1}\right)\left(\frac{r_i}{r}\right)^{\frac{2(1+b)(m-1)}{2m+2mb-b}} + \frac{m}{m-1}\sigma_t. \quad (15.9c)$$

With reference to the continuity of σ_r across $r = r_p$, the relationship between the internal pressure p and the radius of the plastic zone r_p is obtained

$$p = \frac{m\sigma_t}{m-1}\left[\frac{(2m+2mb-b)r_e^2}{(m+1+mb)r_e^2+(m-1)(1+b)r_p^2}\left(r_p/r_i\right)^{\frac{2(m-1)(1+b)}{2m+2mb-b}} - 1\right]. \quad (15.10)$$

With the increase of the pressure p, the plastic zone expands further and the elastic-plastic interface moves gradually to the external wall of the cylinder. Setting $r_p = r_e$ in Eq.(15.10), the internal pressure becomes

$$p_s = \frac{m\sigma_t}{m-1}\left[\left(r_e/r_i\right)^{\frac{2(m-1)(1+b)}{2m+2mb-b}} - 1\right], \quad (15.11)$$

which is the plastic limit pressure of the cylinder.

If $p_e < p < p_s$, the cylinder is partially plastic. When the cylinder is unloaded there will be residual stress. If p is small the unloading process is purely elastic and the residual stress is derived by superposition of the elastic unloading stress and the elastic-plastic loading stress. The expressions for the residual stresses in the zone adjacent to the internal wall of the cylinder ($r_i \leqslant r \leqslant r_p$) can be written as

$$\begin{aligned}\sigma_r^r = &-\left(p+\frac{m\sigma_t}{m-1}\right)\left(\frac{r_i}{r}\right)^{\frac{2(m-1)(1+b)}{2m+2mb-b}} + \frac{m}{m-1}\sigma_t \\ &-\frac{r_i^2 p}{r_e^2-r_i^2}\left(1-\frac{r_e^2}{r^2}\right),\end{aligned} \quad (15.12a)$$

$$\begin{aligned}\sigma_\theta^r = &-\frac{(2+b)}{2m+2mb-b}\left(p+\frac{m\sigma_t}{m-1}\right)\left(\frac{r_i}{r}\right)^{\frac{2(m-1)(1+b)}{2m+2mb-b}} \\ &+\frac{m}{m-1}\sigma_t - \frac{r_i^2 p}{r_e^2-r_i^2}(1+\frac{r_e^2}{r^2}),\end{aligned} \quad (15.12b)$$

$$\begin{aligned}\sigma_z^r = &-\frac{1+m+mb}{2m+2mb-b}\left(p+\frac{m\sigma_t}{m-1}\right)\left(\frac{r_i}{r}\right)^{\frac{2(m-1)(1+b)}{2m+2mb-b}} \\ &+\frac{m}{m-1}\sigma_t - \frac{r_i^2 p}{r_e^2-r_i^2}.\end{aligned} \quad (15.12c)$$

Given $r = r_i$, the residual stresses on the internal wall of the cylinder are

$$\sigma_r^r = 0, \tag{15.13a}$$

$$\sigma_\theta^r = -\left[\frac{(2+2b)}{2m+2mb-b} + \frac{r_e^2+r_i^2}{r_e^2-r_i^2}\right]p + \frac{(2+2b)m}{2m+2mb-b}\sigma_t, \tag{15.13b}$$

$$\sigma_z^r = -\frac{1}{2}\cdot\left[\frac{(2+2b)}{2m+2mb-b} + \frac{r_e^2+r_i^2}{r_e^2-r_i^2}\right]p + \frac{(1+b)m}{2m+2mb-b}\sigma_t. \tag{15.13c}$$

It is seen that σ_r^r, σ_z^r, and σ_θ^r on the internal wall are the major principal stress, the intermediate principal stress, and the minor principal stress respectively, and the intermediate principal stress $\sigma_z^r \geqslant (m\sigma_r^r + \sigma_\theta^r)/(m+1)$. Therefore, the unified strength theory on the internal wall is

$$\frac{1}{1+b}(\sigma_r + b\sigma_z) - \frac{\sigma_\theta}{m} = \sigma_t. \tag{15.14}$$

From Eqs.(15.13) and (15.14), the internal wall of the cylinder yields when the internal pressure reaches

$$p_{\max} = \frac{2m(m+1)(1+b)(b+2)/(2m+2mb-b)/(2-mb+2b)}{(2+b)/(2m+2mb-b)+(r_e^2+r_i^2)/(r_e^2-r_i^2)}\sigma_t. \tag{15.15}$$

If $p < p_{\max}$, a secondary yielding does not take place at the internal wall of the unloaded cylinder. It can be demonstrated that the residual stress induced by the cycle of loading-unloading will not yield any new plastic deformation in the whole cross-section of the cylinder. Therefore the shakedown condition for a thick-walled cylinder under repeated loading and unloading is that the internal pressure p is less than the critical value $p_{\text{shakedown}}$ or p_{plastic}, i.e.,

$$p_{\max,\text{shakedown}} = \min\left\{\frac{2m(m+1)(1+b)(2+b)/(2m+2mb-b)/(2-mb+2b)}{(2+b)/(2m+2mb-b)+(r_e^2+r_i^2)/(r_e^2-r_i^2)}\sigma_t\right\}, \tag{15.16a}$$

$$p_{\max,\text{plastic}} = \min\left\{\frac{m\sigma_t}{m-1}\left[(r_e/r_i)^{\frac{2(m-1)(1+b)}{2m+2mb-b}} - 1\right]\right\}, \tag{15.16b}$$

which is the shakedown load of the thick-walled cylinder. Setting m=1 and b=0 in Eq.(15.16b), the shakedown load of the thick-walled cylinder has the form of

$$p_{\max} = \min\left\{\sigma_t(1-r_i^2/r_e^2),\ \sigma_t \ln(r_e/r_i)\right\}, \tag{15.17}$$

which is in agreement with the shakedown load of a cylinder from the classical plasticity based on the Tresca criterion.

The shakedown load given by Eq.(15.16a) is correlated with the compressive tensile strength ratio m, and the unified yield criterion parameter b. It can be said the present approach has the capability to reflect the SD effects and intermediate principal stress on the shakedown load of the cylinder quantitatively, which is ignored in the classical solution.

15.4 Unified Solution of Shakedown Pressure of Thick-walled Cylinders

In order to demonstrate the SD effects and intermediate principal stress on the shakedown load of a thick-walled cylinder, the results from the derived closed-form solution are depicted in Fig.15.1, in which the abscissa denotes the wall ratio of the cylinder r_e/r_i, and the ordinates is the shakedown load $p_{\max}/\sigma_t$.

From Fig.15.1, the effect of the intermediate principal stress on the shakedown load for non-SD materials (m=1) is obvious.

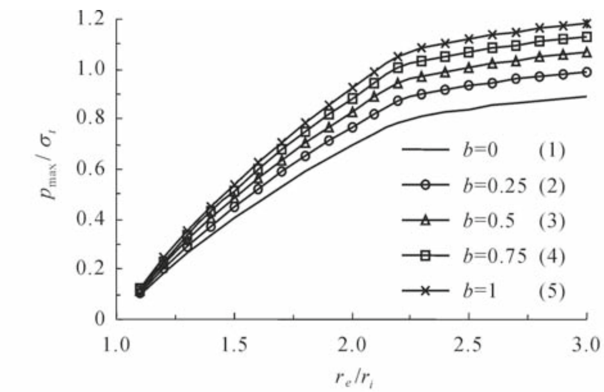

Fig. 15.1. Shakedown load for different values of the unified strength theory parameter b (m=1.0)

The curve (1) in Fig.15.1 (b=0 and m=1.0) is suitable for materials without both the SD and the intermediate principal stress effects, which is exactly the result of the classical solution based on the single-shear yield criterion. The present solution with m=1 and b=0.5 (curve (3) in Fig.15.1) is a close approximation to the result from the Huber-von Mises criterion. The curve (5) in Fig.15.1 (b=1.0 and m=1) is the same as the result from the twin-shear stress yield criterion.

Fig.15.2 (m=1.1) and Fig.15.3 (m=1.2) show the shakedown pressure for materials with SD effect. It is seen from these figures that the shakedown load is related to the unified strength theory parameter b which reflects the effect of the intermediate principal stress on material strength. The higher the parameter b, the higher the shakedown load $p_{\max}$. Consequently, for a given compressive-tensile strength ratio m, that of b=0 corresponding to the Tresca criterion or the Mohr-Coulomb criterion gives the lowest value of $p_{\max}$, that of b=1 corresponding to the twin-shear yield criterion or the generalized twin-shear criterion gives the highest value. Therefore, the shakedown load of the cylinder may be underestimated when the effect of the intermediate principal stress of materials is neglected, or an improper yield condition is applied.

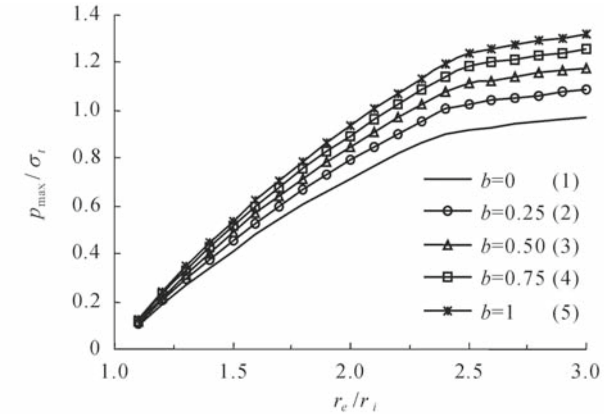

Fig. 15.2. Shakedown load for different values of the unified strength theory parameter b (m=1.1)

Fig.15.4 shows the SD effect on the shakedown load of a cylinder. The results with respect to b=0 are shown in Fig.15.4, which is the same as the result based on the Mohr-Coulomb criterion. It is suitable for materials with negligible intermediate principal stress effect. The curve (1) in Fig.15.4 (b=0 and m=1) is the result of the classical solution based on the Tresca criterion.

Fig.15.5 (corresponding to $b = 0.5$ of the unified strength theory) and Fig.15.6 (corresponding to $b = 1.0$, i.e., the twin-shear strength criterion) are suitable for materials with the intermediate principal stress effect.

From analysis and schematical illustrations of the results, the shakedown load depends on the compressive-tensile strength ratio m and the shakedown load will increase with increasing parameter m. Thus, the shakedown load of the cylinder may be underestimated when the SD effect of materials is ignored. The SD effect of materials on the shakedown load of the cylinder is insignificant when the wall ratio is small, whereas it is prominent when the

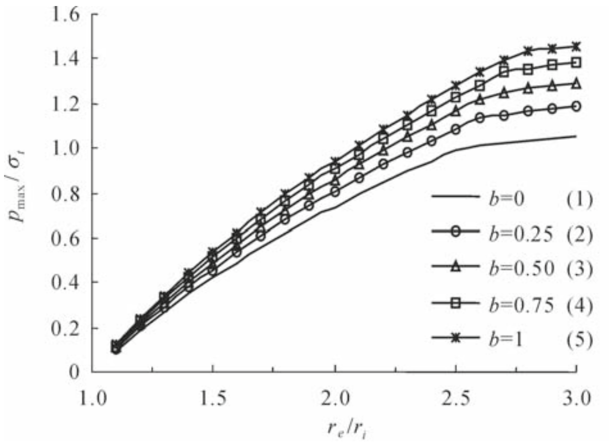

Fig. 15.3. Shakedown load for different values of the unified strength theory parameter b (m=1.2)

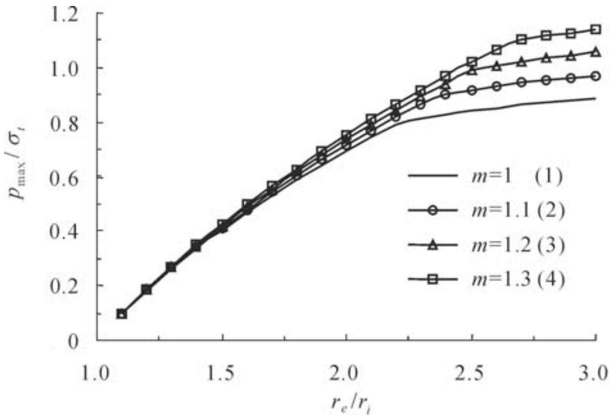

Fig. 15.4. Shakedown load for different values of parameter m (b=0)

wall ratio is high. Therefore, the SD effect of materials should be taken into account in shakedown analysis of the cylinder especially for a high wall ratio of the cylinder.

15.5 Connection between Shakedown Theorem and Limit Load Theorem

Based on the unified strength theory, shakedown analysis of a thick-walled cylinder under internal pressure is carried out and the unified analytical solution of shakedown load for a cylinder is derived in this chapter. This solution

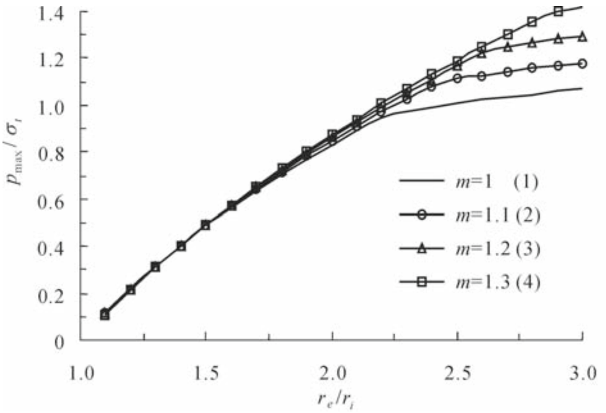

Fig. 15.5. Shakedown load for different values of parameter m (b=0.5)

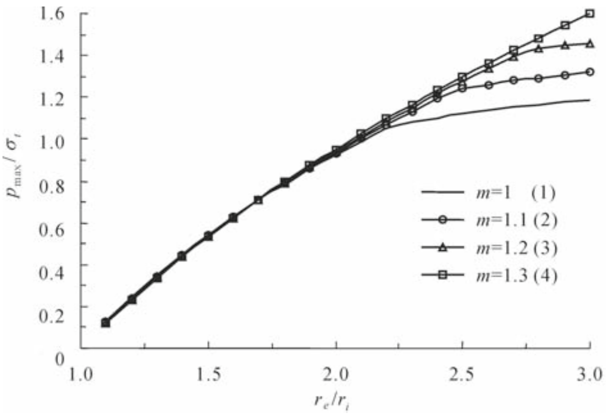

Fig. 15.6. Shakedown load for different values of parameter m (b=1.0)

includes not only the existing classical solution as its special case but gives a series of new results.

It is noted that this solution consists of two parts (Eqs.(15.16a) and (15.16b)): the limit pressure and shakedown pressure (Xu and Yu, 2004a; 2005b), i.e.,

$$p_{\max,\text{shakedown}} = \min\left\{\frac{2m(m+1)(1+b)(2+b)/(2m+2mb-b)/(2-mb+2b)}{(2+b)/(2m+2mb-b)+(r_e^2+r_i^2)/(r_e^2-r_i^2)}\sigma_t\right\}, \tag{15.18a}$$

$$p_{\max,\text{plastic}} = \min\left\{\frac{m\sigma_t}{m-1}\left[(r_e/r_i)^{\frac{2(m-1)(1+b)}{2m+2mb-b}}-1\right]\right\}. \tag{15.18b}$$

The relation between the shakedown pressure and the plastic limit pressure are shown in Fig.15.7 for different parameter b.

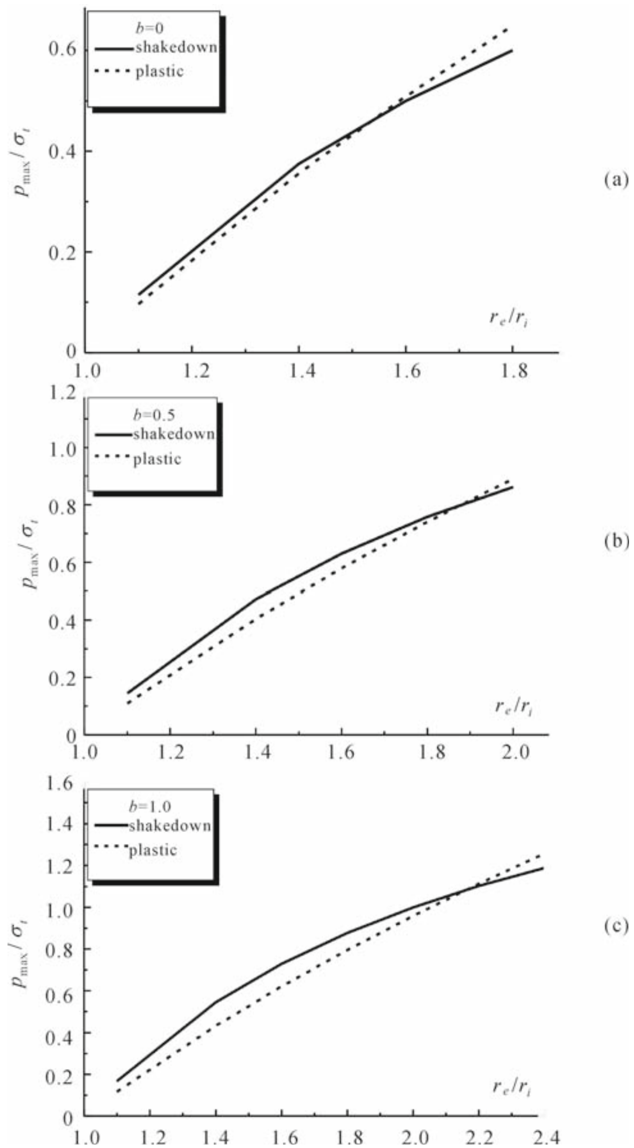

Fig. 15.7. Shakedown load and plastic limit load when m=1.5

It is seen that the two curves will intersect when the limit pressure equals the shakedown pressure, i.e.,

$$\frac{m\sigma_t}{m-1}\left[(r_e/r_i)^{\frac{2(m-1)(1+b)}{2m+2mb-b}}-1\right]=$$
$$\frac{2m(m+1)(1+b)(2+b)/(2m+2mb-b)/(2-mb+2b)}{(2+b)/(2m+2mb-b)+(r_e^2+r_i^2)/(r_e^2-r_i^2)}\sigma_t. \quad (15.19)$$

The current unified solution consists of the two parameters m and b to reflect both the SD and the intermediate principal stress effects of materials. With the variation of m and b, the present solution gives a series of values for the shakedown load that can be applied to materials with or without the SD and the intermediate principal stress effects.

In order to demonstrate more clearly the SD effects and intermediate principal stress on the shakedown load, the analytical solution is illustrated schematically. This shows that both the SD and the intermediate principal stress have influences on the shakedown load, and the more pronounced the two effects, the higher the shakedown load. Therefore, for the cylinder made of materials with the SD and/or the intermediate principal stress effect, the classical solution underestimates the shakedown load. It is therefore of significance for the shakedown analysis to take into account their effects.

It is worth mentioning that besides SD and intermediate principal stress, other important properties such as the Bauschinger effect, the strain-hardening effect, etc., should also be considered when their effects are prominent.

15.6 Shakedown Pressure of a Thick-walled Spherical Shell

Shakedown analysis of a thick-walled spherical shell was derived by Liu et al.(1997) using the Mohr-Coulomb criterion and Xu and Yu (2005b) using the UST. The shakedown limit pressure of a thick-walled spherical shell for SD material is

$$p_{\max,\text{shakedown}} = \min\left\{\frac{m}{m-1}\left[\left(\frac{r_e}{r_i}\right)^{\frac{2m-2}{m}}-1\right]\sigma_t, \frac{m(m+1)(r_e^3-r_i^3)}{(m-1)r_i^3+(0.5m+1)r_e^3}\sigma_t\right\}. \quad (15.20)$$

This result is the same as the solution obtained by using the Mohr-Coulomb strength criterion obtained by Feng and Liu (1995) and Liu et al. (1997) and the twin-shear strength criterion. The relationship of the shakedown limit pressure to the ratio of the strength of material in tension and compression is illustrated in Fig.15.8.

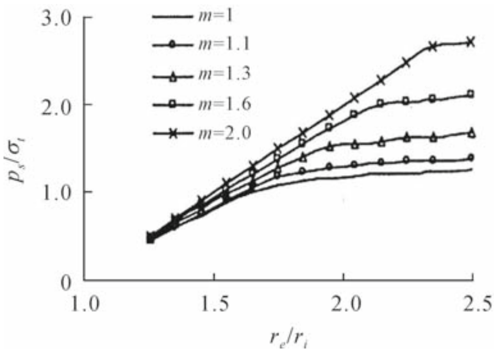

Fig. 15.8. Relationship of shakedown limit pressure to the ratio of material strength in tension and compression

15.7 Summary

The unified strength theory is used to derive unified solutions of the plastic limit and shakedown limit of a thick-walled cylinder. These results are applicable for a wide range of materials and engineering structures.

In the current solutions, the SD effect and the effect of intermediate principal stress acting on the plastic limit loads and shakedown loads of a thick-walled cylinder under uniform internal pressure are presented. By changing the two parameters $m\alpha$ and b (or m and b), a series of values for limit loads and shakedown loads can be obtained from the current solution, which includes both the results from classical plasticity and a series of new results. These solutions are suitable for materials with the SD effect and the intermediate principal stress effect.

Finally, the illustrations of alternatives to the analytical solution are presented to demonstrate graphically to examine the effects of strength difference and intermediate principal stress on the limit loads and the shakedown loads. They show that the limit loads and the shakedown loads depend on both the strength difference in tension and compression and the effect of intermediate principal stress. The limit loads and shakedown loads may be grossly underestimated if these two effects are simply neglected. The unified strength theory gives us a basic theory for use in the strength design of engineering structures. It also provides a tool for estimating accurately the admissible loads with an in-depth understanding of the material strength behavior so that a more economical and optimized design of structures can be achieved.

15.8 Problems

Problem 15.1 Compare the solutions of limit analysis and shakedown analysis.

Problem 15.2 Determine the shakedown load of a pressure cylinder by using the Tresca yield criterion (m=1 and b=0).

Problem 15.3 Determine the shakedown load of a pressure cylinder by using the Mohr-Coulomb strength theory(b=0).

Problem 15.4 Determine the shakedown load of a pressure cylinder by using the twin-shear yield criterion (m=b=1).

Problem 15.5 Determine the shakedown load of a pressure cylinder by using the twin-shear strength theory (b=1).

Problem 15.6 Determine the shakedown load of a cylinder under inter-pressure by using the unified strength theory with b=0.6.

Problem 15.7 Determine the shakedown load of a cylinder under tension and inter-pressure by using the twin-shear strength theory (b=1).

Problem 15.8 Determine the shakedown load of a cylinder under tension and inter-pressure by using the unified strength theory with b=0.6.

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