14

Unified Limit Analysis of a Wellbore

14.1 Introduction

A wellbore structure is usually used for underground engineering. The wellbore should be kept stable as it is subjected to earth stress in the mining engineering. A petroleum wellbore sustains the earth stress around the rock as well as the internal pressure of the oil. The stability of the wellbore is of great importance for the successful drilling.

Wellbore stress study in rock and soil engineering usually employs the expansion theory for a thick cylinder. One main aspect of wellbore stability analysis is the selection of an appropriate rock failure criterion, as indicated by Al-Ajmi and Zimmerman (2006), and Al-Ajmi (2006). The commonly used criterion for brittle failure of rocks is the Mohr-Coulomb criterion. This criterion involves only the maximum and minimum principal stresses, σ_1 and σ_3, and therefore assumes that the intermediate principal stress has no influence on rock strength. In contrast to the predictions of the Mohr-Coulomb criterion, much evidence has been accumulating to suggest that intermediate principal stress σ_2 does indeed have a strengthening effect. Wellbore-stability prediction by use of a modified Lade criterion was reported by Ewy (1999). The stability analysis of vertical boreholes using the Mogi-Coulomb failure criterion was presented by Al-Ajmi and Zimmerman (2006). A detailed report is given by Al-Ajmi (2006). Luo and Li (1994) used the twin-shear strength theory (Yu, 1985) to derive the gradually damaged behavior for thick bores in rock and soil. Jian and Shen (1996) used the unified strength theory (Yu, 1991; 1994; 2004) to analyze the expansion trait by considering the strain-softening characteristic of rock and soil. A unified solution for stability analysis of vertical boreholes was derived by Li and Yu (2001; 2002), Xu et al. (2004), and Xu and Hou (2007).

In this chapter the unified strength theory is applied to analyze the stress distribution of rock around wellbore and the limit load of the wellbore.

14.2 Unified Strength Theory

The unified strength theory (Yu et al., 1991; 1992; 2004)

$$F = \sigma_1 - \frac{1}{1+b}(b\sigma_2 + \sigma_3) = \sigma_t \quad \text{when} \quad \sigma_2 \leqslant \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha}, \tag{14.1a}$$

$$F' = \frac{1}{1+b}(\sigma_1 + b\sigma_2) - \sigma_3 = \sigma_t \quad \text{when} \quad \sigma_2 \leqslant \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha}, \tag{14.1b}$$

can be expressed as

$$\begin{aligned} F &= \left[\sigma_1 - \frac{1}{1+b}(b\sigma_2 + \sigma_3)\right] + \left[\sigma_1 + \frac{1}{1+b}(b\sigma_2 + \sigma_3)\right] \sin\varphi_0 \\ &= 2c_0 \cos\varphi_0, \\ &\quad \text{when} \quad \sigma_2 \leqslant \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \sin\varphi_0, \end{aligned} \tag{14.2a}$$

$$\begin{aligned} F' &= \left[\frac{1}{1+b}(\sigma_1 + b\sigma_2) - \sigma_3\right] + \left[\frac{1}{1+b}(\sigma_1 + b\sigma_2) + \sigma_3\right] \sin\varphi_0 \\ &= 2c_0 \cos\varphi_0, \\ &\quad \text{when} \quad \sigma_2 \geqslant \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \sin\varphi_0, \end{aligned} \tag{14.2b}$$

where the unified strength theory parameter b $(0 \leqslant b \leqslant 1))$ is a yield criterion parameter to reflect the relative effect of the intermediate principal stress σ_2. c_0 and φ_0 are the internal cohesion and the angle of internal friction respectively.

The relations of c_0 and φ_0 to other commonly used material parameters are

$$\alpha = \frac{1 - \sin\varphi_0}{1 + \sin\varphi_0}, \quad \sigma_t = \frac{2c_0 \cos\varphi_0}{1 + \sin\varphi_0},$$

where α is the tensile and compressible strength ratio of a material, i.e., $\alpha = \sigma_t/\sigma_c$.

For plane strain problems a coefficient m $(0 < m \leqslant 1)$ should be introduced. When the considered material is incompressible, m is approximately 1. For simplicity, m is defined as 1 in the following analysis. The yield function can be expressed as

$$\frac{\sigma_1-\sigma_3}{2}=-\frac{2(1+b)\sin\varphi_0}{2(1+b)+mb(\sin\varphi_0-1)}\frac{\sigma_1+\sigma_3}{2}+\frac{2(1+b)c_0\cos\varphi_0}{2(1+b)+mb(\sin\varphi_0-1)}. \tag{14.3a}$$

It can be rewritten as (Yu et al., 1997; 2001)

$$\frac{\sigma_1-\sigma_3}{2}=-\frac{\sigma_1+\sigma_3}{2}\sin\varphi_{\text{uni}}+C_{\text{uni}}\cos\varphi_{\text{uni}}, \tag{14.3b}$$

where the unified strength parameters C_{uni} and φ_{uni} were proposed by Yu et al. in 1997 and 2001.

These two unified strength parameters are referred to as the unified effective cohesion and unified effective internal friction angle respectively, with regard to the unified strength theory (UST). Their relations to the material constants C_0 and φ_0 can be written as (Yu et al., 1997; 2001; 2006)

$$\begin{aligned}\sin\varphi_{\text{uni}}&=-\frac{2(1+b)\sin\varphi_0}{2(1+b)+mb(\sin\varphi_0-1)},\\ C_{\text{uni}}&=\frac{2(1+b)\cos\varphi_0}{2(1+b)+mb(\sin\varphi_0-1)}\cdot\frac{C_0}{\cos\varphi_{\text{uni}}},\end{aligned} \tag{14.4}$$

where C_{uni} and φ_{uni} are the unified internal cohesion and the unified angle of internal friction, Eq.(14.3) gives the failure criterion for plane strain problems (Yu et al., 1997; 2006).

14.3 Equations and Boundary Conditions for the Wellbore

The plan of a wellbore is shown in Fig.14.1. The cylindrical coordinate system is used, where the z-axis is along the wellbore axis.

Assuming that the wellbore radius is R_0, the internal liquid pressure is p_0, R_∞ ($R_\infty >> R_0$) represents an infinite radius at which the liquid pressure is p_∞ and the rock lateral pressure is $\sigma_{r\infty}$. The parameter β is the effective void ratio, k is the seepage ratio, E is the elastic modulus, and ν is the Poisson's ratio. D_e and D_p represent the elastic and plastic zones of the surrounding rock respectively. The radius $r = R_d$ gives the boundary of the elastic and plastic zones.

14.3.1 Strength Analysis for Wellbore

The stress state of the rock around the wellbore is plane strain and axisymmetrical. It is assumed that the lateral stress $\sigma_{r\infty}$ around the rock is a constant and the rock is isotropic. The normal stresses σ_r, σ_θ, and σ_z in the

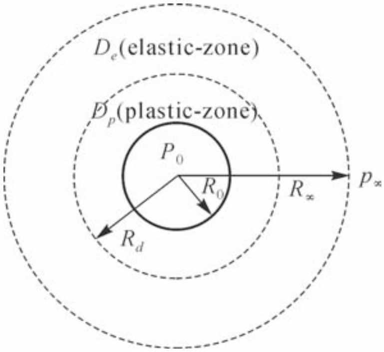

Fig. 14.1. Wellbore subjected to the pore pressure and seepage

radial, circumferential and axial directions are the principal stresses, and the associated shear stress components are zero.

For the present plane strain problem, $\sigma_z = \nu(\sigma_r + \sigma_\theta)$ and there is $\sigma_3 = \sigma_r \leqslant \sigma_z \leqslant \sigma_\theta = \sigma_1$.

When the drilling is finished, the rock around the wellbore is softened. The modulus, internal cohesion, and angle of internal friction will decrease. Denoting that c_1 and φ_1 are the softened internal cohesion and the angle of internal friction respectively, and providing that the rock obeys the failure criterion given in Eq.(14.3), in the initial stage of drilling there is

$$\frac{\sigma_r - \sigma_\theta}{2} = -\frac{\sigma_r + \sigma_\theta}{2}\sin\varphi_{t0} + c_{t0}\cos\varphi_{t0}, \tag{14.5a}$$

where

$$\sin\varphi_{t0} = \frac{2(1+b)\sin\varphi_0}{2+b+b\sin\varphi_0},\ c_{t0} = \frac{2(1+b)c_0\cos\varphi_0}{2+b+b\sin\varphi_0}\cdot\frac{1}{\cos\varphi_{t0}}.$$

The parameters c_{t0} and φ_{t0} represent the effective internal cohesion and the effective angle of internal friction respectively in the original stage with regard to the unified strength theory. When the drilling is finished, the failure condition is

$$\frac{\sigma_r - \sigma_\theta}{2} = -\frac{\sigma_r + \sigma_\theta}{2}\sin\varphi_{t1} + c_{t1}\cos\varphi_{t1}, \tag{14.5b}$$

where

$$\sin\varphi_{t1} = \frac{2(1+b)\sin\varphi_1}{2+b+b\sin\varphi_1},\ c_{t1} = \frac{2(1+b)c_1\cos\varphi_1}{2+b+b\sin\varphi_1}\cdot\frac{1}{\cos\varphi_{t1}},$$

and c_{t1} and φ_{t1} represent the corresponding effective internal cohesion and effective angle of internal friction in the softening stage.

14.3.2 Pore Pressure Analysis

According to the Darcy's law, the pore pressure distribution along the radius is

$$q = \frac{2\pi r k}{\eta}\frac{\mathrm{d}p}{\mathrm{d}r}, \tag{14.6}$$

where η is the liquid viscosity, r is the radius of the wellbore, p is the pressure at the inner surface of the wellbore, q is the liquid flux per unit length in the wellbore, and k is the seepage.

The boundary conditions are $p\,|_{r=R_0}= p_0$ and $p\,|_{r=R_\infty}= p_\infty$. The pressure distribution along the radius direction is derived as

$$p = p_0 + (p_0 - p_\infty)\left(\ln\frac{r}{R_0}/\ln\frac{R_0}{R_\infty}\right), \quad R_0 \leqslant r \leqslant R_\infty. \tag{14.7}$$

Equilibrium equation for the rock by considering the seepage effect is

$$\frac{\mathrm{d}\sigma_r}{\mathrm{d}r} - \chi\frac{\mathrm{d}p}{\mathrm{d}r} + \frac{\sigma_r - \sigma_\theta}{r} = 0. \tag{14.8}$$

At the inner surface of the wellbore, the stress boundary condition is

$$\sigma_r|_{r=R_0} = \sigma_{r0} = -p_0(1-\chi). \tag{14.9a}$$

At R_∞,

$$\sigma_r|_{r=R_\infty} = \sigma_{r\infty} = \sigma_k + \chi p_\infty, \tag{14.9b}$$

$$\sigma_z|_{r=R_\infty} = \sigma_{z\infty} = p_{\infty b} + \chi p_\infty, \tag{14.9c}$$

where χ is the effective void ratio, $p_{\infty b}$ and σ_∞ are the vertical pressure and horizontal stress caused by the above rock weight.

14.4 Elastic and Plastic Analysis

14.4.1 Elastic Phase

In the elastic stage the stress distribution can be obtained from Eqs.(14.7), (14.8) and (14.9),

$$\sigma_r = \sigma_{r\infty} + (\sigma_{r\infty} - \sigma_{r0})\frac{R_0^2}{R_\infty^2 - R_0^2}\left(1 - \frac{R_\infty^2}{r^2}\right) - \frac{\chi(p_0 - p_\infty)}{2(1-\mu)} \times \left[\frac{R_0^2}{R_\infty^2 - R_0^2}\left(\frac{R_0^2}{r^2} - 1\right) + \left(\ln\frac{R_0}{r}\right) / \left(\ln\frac{R_0}{R_\infty}\right)\right], \tag{14.10a}$$

$$\sigma_\theta = \sigma_{r\infty} + (\sigma_{r\infty} - \sigma_{r0})\frac{R_0^2}{R_\infty^2 - R_0^2}\left(1 + \frac{R_\infty^2}{r^2}\right) - \frac{\chi(p_0 - p_\infty)}{2(1-\nu)} \times \left[-\frac{R_0^2}{R_\infty^2 - R_0^2}(\frac{R_0^2}{r^2} + 1) + \left(\ln\frac{R_0}{r} + 1 - 2\nu\right) / \left(\ln\frac{R_0}{R_\infty}\right)\right]. \tag{14.10b}$$

Based on the plane strain assumption, i.e., $\varepsilon_z = 0$, the following expression can be obtained

$$\sigma_z = \nu(\sigma_r + \sigma_\theta), \tag{14.10c}$$

where σ_r and σ_θ are expressed in Eqs.(14.10a) and (14.10b).

14.4.2 Plastic Limit Pressure

When the pressure p_0 increases to the elastic limit, the rock material around the wellbore falls into the plastic stage. In the plastic zone D_p where $R_0 < r < R_d$, from Eq.(14.5) the relation of σ_r and σ_θ to c_{t1} and φ_{t1} is derived as

$$\frac{\sigma_\theta - \sigma_r}{2} = (c_{t1}\cot\varphi_{t1} - \sigma_r)\frac{\sin\varphi_{t1}}{1+\sin\varphi_{t1}}. \tag{14.11}$$

From Eqs.(14.11), (14.7), (14.8) and the boundary condition in Eq.(14.9), the pressure at the elastic-plastic boundary can be determined as

$$p_d = p_0 + (p_0 - p_\infty)\ln\frac{R_d}{R_0}\Big/\ln\frac{R_0}{R_\infty}, \tag{14.12}$$

and the stress distribution in the plastic region D_p can be obtained as

$$\sigma_r = -(1-\chi)p_0\left(\frac{r}{R_0}\right)^{-\frac{2\sin\varphi_{t1}}{1+\sin\varphi_{t1}}} + \left(c_{t1} - D\frac{2\sin\varphi_{t1}}{1+\sin\varphi_{t1}}\right) \times \left[1 - \left(\frac{r}{R_0}\right)^{-\frac{2\sin\varphi_{t1}}{1+\sin\varphi_{t1}}}\right], \tag{14.13a}$$

$$\sigma_\theta = \frac{2c_{t1}\cos\varphi_{t1}}{1+\sin\varphi_{t1}} + \frac{1-\sin\varphi_{t1}}{1+\sin\varphi_{t1}}\sigma_r, \tag{14.13b}$$

where

$$D = \chi(p_0 - p_\infty) \Big/ \ln \frac{R_0}{R_\infty}.$$

If R_0 is substituted by R_d in Eq.(14.10a) and Eq.14.10(b), the elastic stress distribution in the elastic zone D_e $(r > R_d)$ can be obtained from Eq.(14.10a) and Eq.14.10(b).

14.4.3 Elastic-plastic Boundary

With $R_\infty >> R_0$ and $R_\infty >> R_d$, substituting σ_{r0}, R_0 and p_0 in Eqs.(14.10a) and (14.10b) with σ_{rd}, R_d and p_d, the stress distribution can be deduced from Eqs.(14.12) and (14.13) at the elastic-plastic boundary of $r = R_d$,

$$\sigma_{rd} = -[(1-\chi)p_0 + c_{t1}\cot\varphi_{t1}]\left(\frac{R_d}{R_0}\right)^{-\frac{2\sin\varphi_{t1}}{1+\sin\varphi_{t1}}} + c_{t1}\cot\varphi_{t1}, \tag{14.14a}$$

$$\sigma_{\theta d} = 2\sigma_{r\infty} - \sigma_{rd} + \chi\frac{p_0 - p_\infty}{1-\nu}. \tag{14.14b}$$

The relation of pressure p_0 at the wellbore surface to the plastic damaged radius R_d is

$$\begin{aligned} &-[(1-\chi)p_0 + c_{t1}\cot\varphi_{t1}]\left(\frac{R_d}{R_0}\right)^{-\frac{2\sin\varphi_{t1}}{1+\sin\varphi_{t1}}} + c_{t1}\cot\varphi_{t1} \\ &= (1+\sin\varphi_{t0})\left[\sigma_{r\infty} + \frac{\chi(p_0 - p_\infty)}{2(1-\nu)}\right] + c_{t0}\cot\varphi_{t0}. \end{aligned} \tag{14.15}$$

When the wellbore surface goes into the plastic yield stage, that is $R_d = R_0$, the maximum radial pressure for retaining the wellbore elastic stabilization can be deduced from Eq.(14.15),

$$p_{e0} = -\frac{(1+\sin\varphi_{t0})\left[\sigma_{r\infty} + \frac{\chi(p_0-p_\infty)}{2(1-\nu)}\right] + c_{t0}\cot\varphi_{t0}}{1 - \chi\frac{1-\sin\varphi_{t0}-2\nu}{2(1-\nu)}}. \tag{14.16}$$

When $b = 0$, the elastic limit pressure for the Mohr-Coulomb strength theory can be obtained from Eq.(14.16).

When the rock material is completely in a plastic state, i.e., $R_d >> R_0$, the maximum radial pressure for the retaining wellbore stabilization can be obtained from Eq.(14.15),

$$p_{p0} = -\frac{2(1-\mu)(\sigma_{r\infty} + c_{t0}\cos\varphi_{t0} - c_{t1}\cos\varphi_{t1})}{\chi(1+\sin\varphi_{t0})} + p_\infty. \tag{14.17}$$

When $b = 1$, the limit load p_{p0} derived from Eq.(14.17) is the limit plastic load based on the Mohr-Coulomb criterion reported by Li (1998). The maximum plastic radius R_d can be obtained from Eqs.(14.15) and (14.17) with reference to the stability of the wellbore.

14.4.4 Example

For an oil drilling wellbore (Li and Li, 1997; Liu et al., 1995) with the radius and the oil pressure of R_0 and p_0, at R_∞ ($R_\infty >> R_0$), the void pressure p_∞ in the rock is 5 MPa, the radial stress $\sigma_{r\infty}$ is 43.4 MPa, the effective void ratio χ is 25%, the seepage ratio k is $100 \times 10^{-3} \mu\text{m}^2$, the elastic modulus E is 1300 MPa, the Poisson's ratio ν is 0.15. The initial yield internal cohesion c_0 and angle of internal friction φ_0 are 0.179 MPa and 31.4° respectively. The softened internal cohesion c_1 and angle of internal friction φ_1 are 0.154 MPa and 25.2°, respectively.

According to the above derivation, the relation of oil pressure on the surface of the wellbore to the plastic radius is shown in Fig.14.2. The elastic and plastic limit pressure for the stability of the oil wellbore are given in Fig.14.3 and Fig.14.4, where $\bar{p}_0 = p_0/p_\infty$, $\bar{p}_{e0} = p_{e0}/p_\infty$, $\bar{p}_{p0} = p_{p0}/p_\infty$.

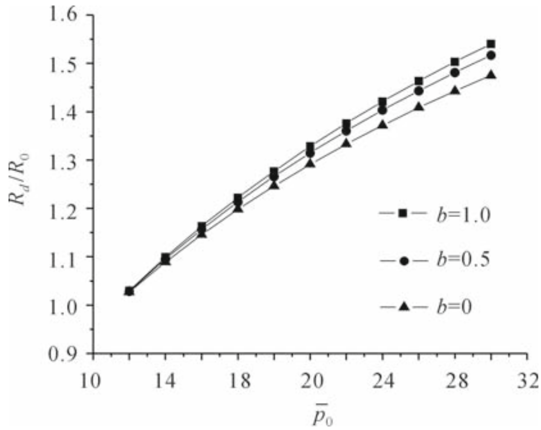

Fig. 14.2. Relation curves of p_0 to R_d

It is shown that the unified strength parameter b influences the plastic radius and the limit pressures. Fig.14.2 shows that the plastic radius increases with the increase of oil pressure in the wellbore, which means that more rock material around the wellbore will enter the plastic phase when the oil pressure on the wellbore surface increases. For a given oil pressure on the wellbore surface, the plastic radius increases with the increase of the parameter b.

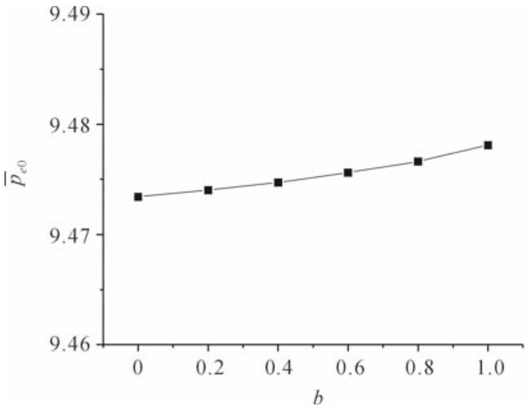

Fig. 14.3. Relation of p_{e0} to the unified strength theory parameter b

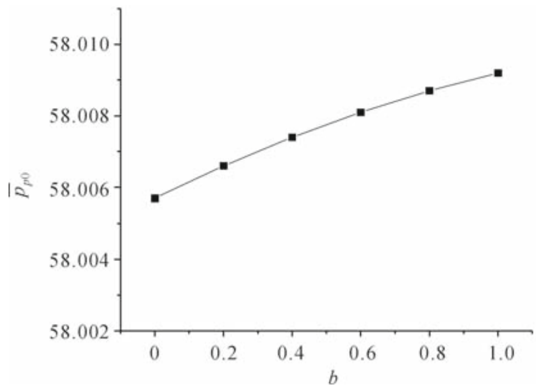

Fig. 14.4. Relation of p_{p0} to the unified strength theory parameter b

Figs.14.3 and 14.4 show the elastic and plastic limit pressures versus the unified strength theory parameter b.

14.4.5 Limit Depth for Stability of a Shaft

Analysis of the stability of a shaft (Fig.14.5) taking into consideration the effect of intermediate principal stress is presented by Xu and Hou (2007).

On the basis of the unified strength theory, a stability analysis of a circular shaft was carried out. The stability formula for the limit depth of the shaft can be expressed as

$$Z_{\max} = \frac{2+2b}{2+b} \cdot \frac{\cos\varphi}{1-\sin\varphi} \cdot \frac{c}{\gamma}. \tag{14.18}$$

It can be seen from Eq.(14.18) that the influence of the unified strength theory parameter b and friction angle φ (i.e. the intermediate principal stress

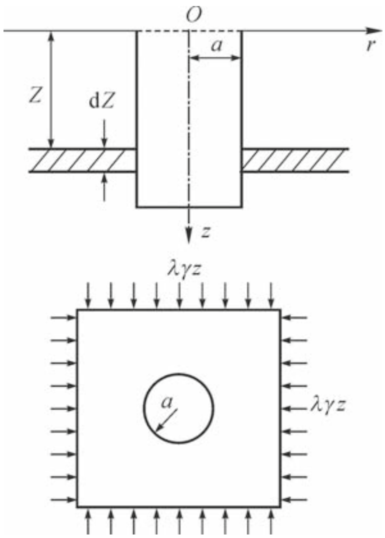

Fig. 14.5. Scheme of a shaft under internal pressures

effect and the strength-differential effect) on the limit depth of the shaft are given. A special case for $b = 0$ can be obtained from Eq.(14.18) as

$$Z_{\max} = \frac{\cos\varphi}{1-\sin\varphi} \cdot \frac{c}{\gamma}. \tag{14.19}$$

It is the same as the result of the Mohr-Coulomb single-shear theory. The serial results for limit depth $Z_{\max}\gamma/c$ are listed in Table 14.1, which can also be found in Fig.14.6.

Table 14.1. The limit depth of the shaft with the unified strength theory parameter b and φ

φ°	$b=0$	$b=1/4$	$b=1/2$	$b=3/4$	$b=1$
0°	1.00	1.11	1.20	1.27	1.33
15°	1.30	1.45	1.56	1.66	1.74
20°	1.43	1.59	1.71	1.82	1.90
25°	2.09	2.13	2.16	2.18	2.20
30°	2.53	2.54	2.55	2.55	2.56
35°	2.81	2.82	2.83	2.83	2.84

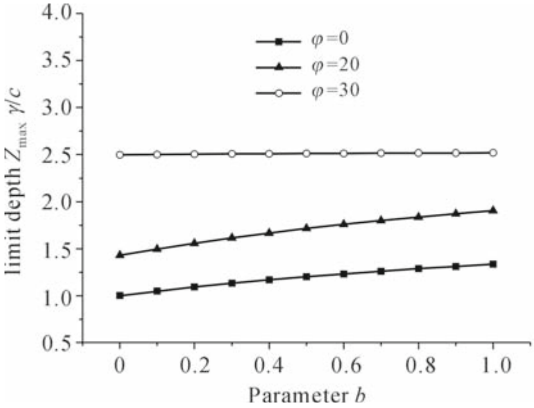

Fig. 14.6. The limit depth of the shaft with the parameters b and φ

The results show that the limit depth of the shaft will increase when the strength-differential effect and the intermediate principal stress effect are considered.

14.5 Summary

Based on the unified strength theory, the elastic and plastic analysis has been carried out for the rock material around the wellbore. The stress distribution of the rock, the elastic and plastic limit loads for the stability of the wellbore and the maximum plastic radius are obtained. The analysis of the stability of a shaft taking into consideration the effect of intermediate principal stress is also discussed.

The analysis results show that the plastic radius increases with the increase of the pressure on the wellbore and the unified strength theory parameter b. It influences the elastic and plastic limit pressures and limit stability depth of the shaft. The analysis results can cover the solutions obtained by other traditional failure conditions, such as the Mohr-Coulomb criterion, the twin shear strength theory.

14.6 Problems

Problem 14.1 Compare the solutions of the elastic and plastic limit pressures of the wellbore.

Problem 14.2 Determine the elastic and plastic limit pressures of the wellbore by using the Mohr-Coulomb criterion ($b = 0$).

Problem 14.3 Determine the elastic limit pressures of the wellbore by using the Mohr-Coulomb criterion ($b = 0$).

Problem 14.4 Determine the elastic limit pressures of the wellbore by using the unified strength theory with $b = 0.5$.

Problem 14.5 Determine the elastic limit pressures of the wellbore by using the unified strength theory with $b = 0.8$.

Problem 14.6 Determine the elastic limit pressures of the wellbore by using the unified strength theory with $b = 1.0$.

Problem 14.7 Determine the plastic limit pressures of the wellbore by using the Mohr-Coulomb criterion ($b = 0$).

Problem 14.8 Determine the plastic limit pressures of the wellbore by using the unified strength theory with $b = 0.5$.

Problem 14.9 Determine the plastic limit pressures of the wellbore by using the unified strength theory with $b = 0.8$.

Problem 14.10 Determine the plastic limit pressures of the wellbore by using the unified strength theory with $b = 1.0$.

Problem 14.11 Compare the solutions of limit depth of the shaft with different criteria.

Problem 14.12 Determine the limit depth of the shaft using the unified strength theory with $b = 0.5$.

Problem 14.13 Determine the limit depth of the shaft using the Mohr-Coulomb criterion ($b = 0$).

Problem 14.14 Determine the limit depth of the shaft using the unified strength theory with $b = 0.8$.

Problem 14.15 Determine the limit depth of the shaft using the unified strength theory with $b = 1.0$.

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