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Introduction

1.1 Background

Plasticity is one of the underlying principles in the design of structures, especially metal and reinforced concrete structures. Numerous textbooks and monographs on structural plasticity and plastic design have been published since the 1950s (Baker et al., 1956; Baker, 1956; Neal, 1956; Heyman, 1958; 1971; Hodge, 1959; 1963; Horne, 1964; 1978; Baker and Heyman, 1969; Save and Massonnet, 1972; Chen, 1975; 1982; Morris and Randall, 1979; Horne and Morris, 1981; Zyczkowski, 1981; König and Maier, 1981; König, 1987; Mrazik et al., 1987; Save et al., 1997; Nielsen, 1999). The European Recommendations for the design of steelwork and reinforced concrete structures apply widely the plastic behavior of materials (Horne and Morris, 1981).

The advantages of the plastic method for structural analysis were discussed by Massonnet, Beedle, Heyman and Chen as follows: "The method of plastic design represents reality better than the conventional elastic method; it must lead to better proportioned and more economical structures" (Heyman, 1960). "For plastic design to represent reality means that the collapse load computed from plastic theory can be closely observed in practice" (Heyman, 1960).

"Engineers and research workers have been stimulated to study the plastic strength of steel structures and its application to design for three principal reasons: (a) it has a more logical design basis; (b) it is more economical in the use of steel, and (c) it represents a substantial saving of time in the design" (Beedle, 1960).

"The calculation of load-carrying capacity by use of the limit theorems is much easier than the calculation of stress. Answers obtained are not only physically more meaningful but also simpler. The simplicity of limit analysis opens the way to limit design, to direct design as contrasted with the trial-and-error procedure normally followed in conventional design." "The estimation of the collapse load is of great value, not only as a simple check for a more refined analysis, but also as a basis for engineering design" (Chen, 1982).

The elastic and plastic limit loads are two important indicators for the overall robustness of a structure to resist external loads. Load-carrying capacity of a structure usually refers to the plastic limit load, which is also called collapse load.

The plastic limit analysis gives a straightforward approximation of the maximum strength and shakedown load of structures. It simplifies the analysis of structures and also helps to derive a cost-effective design.

The elastic and plastic limit loads of structures are often used for design and safety evaluation purposes. Although advanced computer software using numerical methods have been applied widely in engineering practices, analytical solutions still play an important role because they can provide explicit forms of the limit loads, stress and bending moment fields and deformation field of structures, which enables a preliminary structure design and validation of numerical results. The elastic and plastic limit solutions are simple to derive, especially for plate and rotationally symmetrical solids, whose governing equations contain only a few variables.

Conventionally the Tresca and Mohr-Coulomb criteria are adopted to derive the plastic limit load for structures because they have piecewise linear mathematical expressions, which makes the integration of the governing differential equations tractable. The plastic limit loads based on the Tresca criterion for metallic material structures and the Mohr-Coulomb criterion for geomaterial structures have been well applied in design and safety evaluation practice. However, these strength criteria ignore the influence of the intermediate principal stress on the material strength and may lead to improper or over-conservative design of structures.

On the other hand, another widely used criterion, the Huber-von Mises yield criterion has a nonlinear mathematical expression, which renders its application unstraightforward. The effect of different yield criteria on the estimation of the load-bearing capacity of structures has not been fully explored. Most of the textbooks on the plastic limit analysis introduce only solutions based on the Tresca criterion. However, these solutions give a lower bound of the plastic limit load of structures. They may underestimate the load-bearing capacity of structures, which will be addressed in the following chapters. To achieve a better design it is necessary to investigate accurately the plastic limit load of structures.

A unified strength theory was brought up and developed by Yu (1991; 1992; 2004), which is formulated by piecewise linear expression. The theory is based on the assumption that material yielding is dominated by the two larger principal shear stresses (or twin-shear stresses). A parameter b ranging from 0 to 1 is applied on the second principal shear stress, which adjusts the relatively low effect of the second principal shear stress compared to that of the first one. The parameter b reflects different material strength behaviors indicated by different relative shear strengths, which are supported by much experimental evidence. The unified strength theory can represent or approximate the most prevailing yield and strength criteria, such as the Tresca criterion, the Huber-von Mises criterion, the Mohr-Coulomb criterion, and the twin-shear stress criterion. Using the unified strength theory, the plastic limit load of structures made of different materials can be derived conveniently in a unified manner. This book presents the plastic limit and shakedown analyses of various plates and rotationally symmetrical structures based on the unified strength theory. The derived solutions demonstrate the effect of different yield and strength criteria on the plastic limit loads of structures.

This book is one of a series of books on the fundamentals, developments, and applications of the unified strength theory (Yu, 2004; Yu et al., 2006).

1.2 Unification of Yield and Strength Criteria

In general, a yield criterion refers to a simpler form of a strength criterion that is applied for metallic materials which are ductile and have identical tensile and compressive uniaxial strengths. A strength criterion is more general and applicable to both metallic and non-metallic materials, which may exhibit strength difference in tension and compression. Non-metallic materials, such as rock, concrete, and soils, are also referred as strength difference or SD materials. Correspondingly, the uniaxial tensile and compressive strengths of metallic materials are identified as those of non-SD materials. The unified strength theory consists of a unified strength theory, which is applicable to the SD materials, and a unified yield criterion to the non-SD materials. The unified yield criterion is a specific form of the unified strength theory when the strength difference in uniaxial tension and compression can be ignored.

For stable and isotropic metal materials, the Tresca criterion (or the single-shear criterion) is the lower bound yield criterion, and the twin-shear yield criterion (Yu, 1961; 1983) is the upper bound according to the convexity condition. The lowest and the highest load carrying capacities of metal structures are calculated with respect to these two criteria. Some nonlinear yield criteria, whose geometrical graphs lie between the two surfaces defined by the single-shear criterion and the twin-shear criterion, have also been proposed. However, their nonlinear mathematical expressions make the derivation of the closed-form solutions of plastic limit loads for structures very complicated.

The Yu unified yield criterion bounded by the single-shear criterion and the twin-shear criterion has the advantage of simple expression in giving the unified solution of the load-bearing capacity of structures. A series of different solutions can be derived by choosing the unified yield criterion parameter b from 0 to 1. It can be applied to various non-SD materials. The plastic limit loads estimated by the single-shear criterion (Tresca criterion), the Huber-von Mises criterion and the twin-shear stress criterion are special cases or close approximations of the solutions based on the unified yield criterion with specific values of strength parameter b.

The unified strength theory, which is a straightforward extension of the twin-shear yield criterion (Yu, 1961; 1983) and twin-shear strength theory (Yu et al., 1985), also has a piecewise linear mathematical expression. It gives the plastic limit solutions for structures of SD materials. The plastic limit load based on the Tresca criterion or the Mohr-Coulomb criterion is one of the specific forms of the solutions given by the unified strength theory. A series of new solutions can be derived by varying the unified strength theory parameter b from 0 to 1.

A detailed description of the unified strength theory can be found in the companion volumes of *Unified Strength Theory and Its Applications* and *General Plasticity* published by Springer in 2004 and 2006. A paper entitled "Remarks on Model of Mao-Hong Yu" was made by Altenbach and Kolupaev (2008). Reviews of "Unified Strength Theory and Its Applications" were made by Shen (2004) and Teodorescu (2006). The comments on the unified strength theory were made by Fan and Qiang (2001) and Zhang et al. (2001).

1.3 Plastic Limit Analysis

The traditional estimation of the load-carrying capacity of a structure under static loading was based on the "local" permissible stress condition. More realistic approaches must take plastic deformation into account. The simplest estimation of the load-bearing capacity is furnished by concept of the limit-carrying capacity. The related ideas date back to the 18th century (Gvozdev, 1938; Prager et al., 1951; Zyczkowski, 1981). The theorems of limit analysis which provide upper and lower bounds to the true collapse load were first presented by Gvozdev (1938) and independently proved by Hill (1951) for the rigid-perfectly plastic materials, and by Drucker et a1. (1952) for the elastic-perfectly plastic materials.

The circular plate is an important structural element in many branches of engineering. Reliable prediction of the load-bearing capacity of the circular plate is crucial in achieving an optimal structural design. Previous studies by other researchers have shown that the limit analysis is an effective measure in exploring the strength behaviors of a circular plate in the plastic limit state (Hodge, 1963). Hopkins and Prager (1953), Zaid (1958), and Hodge (1963) investigated the load-bearing capacity of circular and annular plates with limit analysis theorems and proved that an exact plastic limit solution for an axisymmetrical plate can be analytically derived if the material of the plate satisfies a linear yield criterion. Ghorashi (1994) derived the plastic limit solutions for circular plates subjected to arbitrary-rotationally symmetric loading. However, most of the reported results have been derived in terms of the maximum stress yield criterion or the Tresca criterion that takes account of the effect of only one or two principal stresses in the three dimensional stress space. Solutions in terms of these yield criteria may not reflect the real characteristics of a circular plate in the plastic limit state. Hopkins and Wang (1954) investigated the load-bearing capacities of circular plates with respect to the Huber-von Mises criterion and a parabolic criterion by iterative method. The percentage differences of the plastic limit loads with respect to the Huber-von Mises criterion and the Tresca criterion are approximately 8% and 10% for a simply supported and a clamped circular plate, respectively. These observations indicate that the plastic limit loads in terms of different yield criteria are different and the difference varies with the variation of the constraint conditions of the plate.

Limit analysis of structures applies only if the loading magnitude is less than the plastic collapse force. With impact or explosive blast loading the structures may be subjected to an intense but short duration pressure or force pulse that exceeds the plastic collapse force. The response of circular plates to pulse loading was presented by Florence (1966; 1977), Youngdagl (1971; 1987), Li and Jones (1994), Li and Huang (1989), etc. The dynamic plastic behavior of beams has been investigated in detail by Stronge and Yu (1993). However, it is difficult to get the analytical solution for plate and shell in a dynamic plastic deformation state because of the complicated constitutive formulation. On the other hand, the dynamic analytical solution for a circular plate is much simpler because of the axisymmetry. Exact theoretical solutions to the dynamic response of a rigid, perfectly plastic, simply supported circular plate have been explored initially by Hopkins and Prager (1954). In the past forty years a number of studies (Jones, 1980; 1989; Florence, 1966; 1977; Symonds, 1979; Li and Jones, 1994; Liu and Jones, 1996) have been done on this subject by introducing various boundaries, loading conditions, and plastic flow assumptions for a circular plate. So far all these studies are based on the Tresca yield criterion or the maximum stress yield criterion. The influence of different yield criteria on the dynamic plastic behavior of circular plates (Florence, 1966) has not been addressed. Recently the influence of the transverse shear force on the final central deflection of circular plates has attracted some attention (Liu and Stronge, 1996; Youngdagl, 1971; 1987; Shen and Jones, 1993; Woodward, 1987). Jones and Oliveira (1980) analyzed a simply supported circular plate subjected to an impulsive velocity uniformly distributed over the entire plate. Dynamic plastic behavior of annular plates with transverse shear effects was studied by Lellep and Torn (2006).

Circular plates are sometimes strengthened in the radial direction or the circumferential direction with stiffeners, which induces orthotropic yield moments. Material orthotropy can also arise from the cold forming process, which results in different yield strengths in different directions. Orthotropic yield criteria for those plates have been suggested by many researchers (Sawczuk, 1956; Markowitz and Hu, 1965; Save et al., 1997). They are mainly modifications of the Tresca criterion. Olszak and Sawczuk (1960) investigated the plastic limit behavior of an orthotropic circular plate in terms of the modified Tresca criterion. The results have been extended to various loading cases by Markowitz and Hu (1965). The plastic limit solution given in these studies satisfies both a statically admissible moment field and kinematically admissible velocity fields and thus, is an exact solution.

Many monographs concerning the limit analysis of a structure were published. It can be seen in Baker (1956), Baker, Horne and Heyman (1956), Neal (1956), Hodge (1959; 1963), Horne (1964), Baker and Heyman (1969), Heyman (1971), Save and Massonnet (1972), Chen (1975; 1981; 1998), Horne (1978), Morris and Randall (1979), Horne and Morris (1981), Zyczkowski (1981), Xu and Liu (1985), Mrazik, Skaloud and Tochacek (1987), Xiong (1987), Save, Massonnet and Saxce (1997), Huang and Zhen (1998), Nielsen (1999). The number of papers devoted to problems of the limit carrying capacity of various structures is enormous. The Tresca criterion, Huber-von Mises criterion and maximum normal stress criterion are used to obtain the plastic limit of structures in most books and papers. The literature concerning the plastic analysis of a structure was summarized by Zyczkowski (1981).

The twin-shear yield criterion was used for limit analysis of a structure. Some solutions were presented by Li (1988) and Huang and Zeng (1989). Fourteen problems using the twin-shear criterion were collected in the monograph of Huang and Zeng (1998). Application of the twin-shear strength theory in the strength calculation of gun barrels was reported by Liu, Ni, Yan et al. (1998). The twin-shear criterion was also used in axisymmetric identification of a semi-infinite medium (Zhao, Xu, and Yang et al., 1998) and mathematical solutions for forming mechanics of continuum (Zhao, 2004). The calculation of stable loads of strength-differential thick cylinders and spheres by the twin-shear strength theory was reported by Ni, Liu, and Wang (1998).

The unified yield criterion was used for plastic analysis of structures of non-SD materials by Ma, He and Yu (1993), Ma and He (1994) and Ma, Yu, Iwasaki et al. (1994; 1995). Following these results, Ma et al. presented a series of unified solutions for non-SD materials. The unified solutions of structures for SD materials were presented by Li and Yu (2000b), Wei and Yu (2001; 2002), Wang and Yu (2002; 2005), Xu and Yu (2004), and others.

1.4 Plastic Limit Analysis of Rotating Solids

Rotating discs are used widely as important structural elements in mechanical engineering. In a structural design procedure, it is inevitable that we must estimate the angular velocity and the stress distribution of a rotating disc in a fully plastic state. The theoretical study of a rotating disc was presented by Nadai and Donnell (1929), and since then numerous works involving plastic collapse speeds (Heyman, 1958; Lenard and Haddow, 1972), thermal effects (Thompson, 1946; Gamer and Mack, 1985), acceleration effects (Reid, 1972), and variable thickness influence (Güven, 1992; 1994), etc., have been reported. Most of them employed the Tresca criterion which results in an over-conservative estimation of the load-bearing capacity of a structure because it does not take account of the effect of the second intermediate principal stress on material yielding (Hill, 1950). As Gamer (1983; 1985) pointed out, when the Tresca criterion and its associated flow rule is used, the displacement across the elasto-plastic interface of a rotating disc is not continuous, and negative circumferential plastic strain is derived in the disc center area where the stresses are tensile. To solve this problem, Gamer (1984) suggested an additional strain-hardening region at the center area of the disc. The idea was extended by Güven (1994) in investigating the plastic limit of angular velocity of a solid rotating disc with variable thickness. Calladine (1969), on the other hand, explained that the singularities in the strain increments at the center could be interpreted as a tendency for the disc to "thin" so much as to produce a small hole very quickly. From a mathematical point of view, the deficiencies in the Tresca solution can be avoided by applying a non-associated flow rule, e.g., combining the Tresca criterion with a Levy-Mises flow rule, or by applying the Huber-von Mises criterion and its associated flow rule (Rees, 1999). The latter must resort to a numerical iteration method because of the nonlinear expression of the Huber-von Mises criterion.

The limit of angular velocity of a rotating disc based on the Tresca criterion always gives the lowest estimation. The effect of different yield criterion on the limit of angular velocity of a rotating disc with variable thickness has not been well studied because the solution based on the Huber-von Mises criterion can only be obtained with numerical iteration (Rees, 1999).

1.5 Shakedown Analysis of Structures

The concept and method of shakedown analysis were first brought up in the 1930s and widely explored in the 1950s. The most significant milestones in shakedown theory of elasto-plastic structures are the pioneering works by Bleich (1932), Melan (1936), and Koiter (1953; 1960). They brought up two crucial shakedown theorems, namely the static shakedown theorem (also called the Melan's theorem, the first shakedown theorem, or the lower bound shakedown theorem), and the dynamic shakedown theorem (also called the Koiter's theorem, the second shakedown theorem or the upper bound shakedown theorem). The later developments of shakedown analysis can be categorized into the static and the dynamic shakedown analysis methods. The shakedown theory has constituted a well-established branch of plasticity theory. The bound to shakedown loads was discussed by Zouain and Silveira (2001).

In recent years, the shakedown analysis of an elasto-plastic structure has gradually attracted attention in engineering due to the requirements of modern technologies such as nuclear power plants, the chemical industry, aeronautical and astronautical technologies. The shakedown theory has been applied successfully in a number of engineering problems such as the construction of nuclear reactors, highways and railways, and employed as one of the tools for structural design and safety assessment in some design standards, rules, and regulations (König, 1987; Polizzotto, 1993; Feng et al., 1993; 1994; Maier, 2001). Shakedown analysis of the shape-memory-alloy structures is presented by Feng and Sun (2007). A new unified solution of the shakedown of cylinder and rotating disc for non-SD materials and SD materials is given by Xu and Yu (2005). Xu and Yu (2005) also give a shakedown analysis of a thick-walled spherical shell of material with different strengths in tension and compression.

1.6 Plastic Limit Analysis Based on the Unified Strength Theory

There have been significant developments in plastic limit analyses of plates and rotationally symmetrical solids based on the unified strength theory in recent years. This book mainly updates these developments which have considerable potential in extending the derived solutions to various other structural forms.

The yield loci of the unified strength theory cover all the traditional convex yield criteria. The Tresca yield criterion, the Huber-von Mises yield criterion, the twin-shear yield criterion and a series of other new linear yield criteria are special cases or approximations of the unified strength theory. It provides a new approach to the study of the load-carrying capacities of structures in a unified manner.

Ma et al. (1993; 1994; 1995a; 1995b; 1995c) derived a unified plastic limit solution to a circular plate under uniform and partially uniform load. Ma et al. (1998) gave a unified solution to simply supported circular plates and clamped circular plates in terms of the Yu's unified yield criterion. Applications of the unified yield criterion to unified plastic limit analysis of circular plates under arbitrary load were reported by Ma et al. (1998; 1999; 2001). The unified solutions of the limit speed of disc and cylinder using the unified yield criterion were derived by Ma et al. (1994; 1995a; 1995b; 1995c) for non-SD materials. The solutions in terms of maximum principal stress criterion, the Tresca yield criterion, the Huber-von Mises criterion, and the twin-shear yield criterion are all special cases or close approximations to the solutions using the unified yield criterion.

The unified solutions of the plastic limit and shakedown load to plate, cylinder, and limit speed of rotating disc and cylinder for SD materials have been reported by Wang and Fan (1998), Li and Yu (2001), Wei and Yu (2002), Wang and Yu (2002; 2005), Xu and Yu (2004; 2005). It will be described in this book, too.

A new orthotropic yield criterion, which is an extension of the unified yield criterion, has been suggested by Ma et al. (2002). The new orthotropic yield criterion is applicable to the plastic limit analysis of orthotropic plates.

The results derived from the present study show the influence of different yield criteria, which is helpful in achieving optimized design of structures and in validating the numerical models in plastic analysis.

1.7 Summary

Various single yield criteria are used for the limit analysis, shakedown analysis and dynamic plastic analysis of structures. The solution is a single result adapted for one kind of material. Owing to the development of the unified strength theory, a series of unified solutions of limit analysis, shakedown analysis and dynamic plastic analysis of structures were presented during the last decade.

The unified strength theory is an accumulation of serial yield criteria adapted for non-SD materials and SD materials. The serial criteria cover all areas between the lower bound (single-shear criterion, Tresca-Mohr-Coulomb (1864; 1900)) and upper bound (twin-shear criteria, Yu (1961; 1983; 1985)). It is well known that all the yield criteria of the unified strength theory are piecewise linear with the attendant simplification of the analytical solution of the structure. The application of the unified strength theory gives not only a single solution, but also a series of solutions. The serial solution for a structure is referred to the unified solution, which can be adopted for more materials and structures.

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