A Formal Model of Fuzzy Ontology with Property Hierarchy and Object Membership

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Abstract. In this paper, we propose a formal model of fuzzy ontology with property hierarchy by combining theories in cognitive psychology and fuzzy set theory. A formal mechanism used to determine object memberships in concepts is also proposed. In this mechanism, object membership is based on the defining properties of concepts and properties which objects possess. We show that our model is more reasonable in calculating object memberships and more powerful in concept representation than previous models by an example.

1 Introduction

With the development of the Semantic Web, ontologies play an important role in knowledge representation. Ontologies provide a way to describe and structure the information on the web. An ontology is generally defined as an 'explicit specification of conceptualization' and can be used to provide semantics to resources on the Semantic Web [1].

Traditional ontologies represent concepts as crisp sets of objects [2]. Objects are considered either to belong to or not to belong to a concept. However, there are many vague concepts in reality. These vague concepts have no clear boundaries. For example, 'hot water', 'red car' and so on. To extend the representation ability of ontologies to handle fuzzy concepts, some fuzzy ontologies are proposed based on fuzzy DLs (description logics) [3] [4] [5]. These fuzzy ontologies provide ways to represent the fuzziness of knowledge. However, object memberships are given by users manually or obtained by fuzzy functions defined by users in these fuzzy ontologies. While concepts, objects and properties are building blocks of ontologies, to our best knowledge, there lacks of a formal mechanism to determine memberships of objects in concepts automatically based on the defining properties of concepts and properties which objects possess. Thus, machine cannot obtain object memberships automatically while given defining properties of concepts and objects in ontologies. While properties are generally used in describing concepts and objects in ontology, we consider that it is desirable to formalize object membership in ontology based on properties of concepts and objects.

Au Yeung and Leung [6] consider that methods used by human beings in classification and categorization are useful in modeling a domain by ontology, while

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there is no such a consideration in previous ontology models. They propose a conceptual model of fuzzy ontology which is based on the theories in cognitive psychology. Nevertheless, their model can only represent the conjunction concepts (concepts defined by conjunction of properties). Furthermore, the Au Yeung-Leung model only can handle the concepts defined by independent properties. It requires to assume all properties in the ontology are independent (i.e., there is no relation between properties), and it lacks building blocks to handle the cases with dependent properties. Thus, we cannot infer some implicit knowledge based on the dependence of properties. For example, we cannot infer the property 'is a man' based on property 'is a tall man' because there is no relation between the two properties in the Au Yeung-Leung model.

To overcome the limitations of previous models of ontology, based on theories in cognitive psychology [7] [8], works in [9] and fuzzy set theory [10], we propose a novel formal model of fuzzy ontology with property hierarchy and object membership. Our model extends the expression and reasoning capability of ontologies in handling fuzzy concepts. It can handle the cases with dependent properties in ontology based on a property hierarchy, and represent conjunction concepts, disjunction concepts (concepts defined by disjunction of properties) and combination concepts (concepts defined by conjunction and disjunction of properties). Our model provides a more reasonable formal mechanism to determine object memberships in concepts than previous models. A main feature of this mechanism is that object membership is measured by the defining properties of concepts and properties which objects possess, which is based on the classical view in cognitive psychology.

The structure of this paper is as follows. Section 2 introduces the background and related work. We give a motivating example and state the limitations of the existing models in section 3. In section 4 we propose a novel formal model of fuzzy ontology with property hierarchy. A formal mechanism to determine the object memberships in concepts based on the defining properties of concepts and properties which objects possess is presented in section 5. We illustrate the use of our model by an example in section 6. Section 7 concludes the paper.

2 Background and Related Work

2.1 Classical View of Concept Representation in Cognitive Psychology

In cognitive psychology, how concepts are represented in the human memory is an important concern. It is generally accepted that concepts are characterized by properties [11]. One important model of concept representation based on properties is classical view. The classical view [7] [8] of concepts posits that each concept is defined by a set of properties which are individually necessary and collectively sufficient. Properties are atomic units which are the basic building blocks of concepts. Concepts are organized in a hierarchy and the defining properties of a more specific concept includes all the defining properties of its super-concepts. In classical view, there are clear-cut boundaries between members and non-members of the category. As a result, the classical view cannot handle the vague concepts.

2.2 Formal Models of Fuzzy Ontology

Currently, most ontologies are based on DLs (description logics) [12] and concepts are represented as crisp sets of objects (e.g., ontologies written in OWL DL) [1]. These ontologies cannot represent the fuzzy concepts. Several fuzzy DLs are proposed to handle the fuzzy concepts by combining fuzzy set theory [10] and description logics. For example, Straccia proposes a fuzzy $\mathcal{ALC}$ in [3] and a fuzzy $\mathcal{SHOIN}(\mathcal{D})$ in [4]. Stoilos et al. present a fuzzy $\mathcal{SHIN}$ in [5]. These fuzzy DLs vary in possessing different expressive power, complexity and reasoning capabilities. Some fuzzy ontologies are constructed based on fuzzy DLs or fuzzy logic [13] [14]. Besides, some works apply fuzzy ontologies for some applications. For instance, Cross and Voss [15] explore the potential that fuzzy mathematics and ontologies have for improving performance in multilingual document exploitation. These works can represent membership degrees of different objects in concepts. Nevertheless, in these models, object memberships are given by users manually or obtained by fuzzy functions defined by users. These works lack a formal mechanism to obtain the membership degrees of objects in concepts automatically based on the defining properties of concepts and properties which objects possess. Besides, there is no consideration of how people representing concepts in their mind.

Recently, Au Yeung and Leung [6] propose a formal model for fuzzy ontology by borrowing the idea of classical view. They have formalized the membership degrees of objects (they name the membership degree of objects as *likeliness*) in concepts by constructing several vectors in ontologies. They consider that a concept r can be defined by a single characteristic vector $\overrightarrow{c}_r$ of r which consists all the necessary properties of r. They assume relation among all properties is conjunction and all properties are independent. The value of each element in a characteristic vector is the minimal requirement of a corresponding property. An object a can be represented by a property vector $\overrightarrow{p}_a$, and each element in $\overrightarrow{p}_a$ corresponds to the degree to which the object possesses a property. The likeliness of an object in a concept is the degree to which the object satisfies the minimal requirements of defining properties of the concept.

3 Limitations of Previous Models

We use a motivating example to illustrate the limitations of previous models.

Example 1. Suppose an online-shop will select the top one hundred special customers to give them some discount. The concept 'special-customer' is a fuzzy concept and is defined as the union of two kinds of customers. One kind of special customer is defined by three properties A, B and C (properties of concepts 'special-customer' and 'customer' are given in table 1), i.e., this kind of special customers requires a customer must have bought at least five items (goods) belonging to 'expensive item' and possess average degree of all items that the customer has bought belonging to 'expensive item' as higher as possible. The other kind of special customers is defined by properties A, D, and E, i.e., it requires a customer must have bought at least one hundred items (not necessary expensive items) and there are at least one item that the customer has bought belonging to 'expensive item'. In this example, 'special-customer' is the sub-concept of 'customer' and 'expensive item' is the sub-concept of 'item'.

Table 1. Properties of concepts *'special-customer'* and *'customer'* in the motivating example

A	has customerID	B	buy at least five expensive items
C	possess average degree of all bought items belonging to expensive items	D	buy at least 100 items
E	buy at least one expensive items	F	buy at least one item

We suppose that the definition of the concept 'customer' denoted by C and that of the concept 'special-customer' denoted by *SC* are as following:

$$C : [A]_1 \ and \ [F]_1, \ \ SC : ([A]_1 \ and \ [B]_1 \ and \ [C]_{0.6}) \ OR \ ([A]_1 \ and \ [D]_1 \ and \ [E]_{0.5})$$

where the subscript of each property is the minimal requirement of the property. Objects (e.g., all customers) satisfying all minimal requirements of defining properties of a concept (e.g., 'special-customer') belong to the concept to a degree 1. We want to calculate object memberships for there customers O_1, O_2 and O_3 in concept 'special-customer' and concept 'customer'. Table 2 are items bought by the three customers.

Table 2. Items bought by O_1, O_2 and O_3

O_1		O_2		O_3	
bought item	price	bought item	price	bought item	price
Furniture00002	1550	Book10032	120	Clothes02006	180
Eproduct00307	2500	Book20039	20	Clothes08001	80
...	...	...	...	...	...
Book07005	200	EletronicProduct70032	175	Book03102	140

For fuzzy ontologies based on fuzzy DLs or fuzzy logic (e.g., ontologies in [15]), they provide a model to represent the fuzziness of concepts, and object memberships in concepts are given by users previously or obtained by membership functions defined by users. However, there is no direct or principle of how to give object memberships or to define membership functions, so there may exist arbitrary assignments of object memberships or arbitrary definitions of membership functions. Moreover, while concepts, objects and properties are building blocks of these fuzzy ontologies, they lack a formal mechanism to give membership degrees to objects in concepts automatically based on the defining properties of concepts and properties which objects possess. Thus, for these fuzzy ontologies, machines cannot calculate the object memberships of O_1, O_2 and O_3 in concepts SC and C based on defining properties of the two concepts and properties the three objects possessing automatically.

If using the Au Yeung-Leung model which provides a formal mechanism for calculating object membership based on properties, we can obtain characteristic vectors for SC and C, property vectors of O_1, O_2 and O_3 as following:

$$SC : [A]_1, [B]_1, [C]_{0.6}, [D]_1, [E]_{0.5}, [F]_1; \quad C : [A]_1, [F]_1$$

$$O_1 : [A]_1, [B]_1, [C]_{0.8}, [D]_{0.2}, [E]_1, [F]_1; \quad O_2 : [A]_1, [B]_{0.2}, [C]_{0.1}, [D]_1, [E]_{0.8}, [F]_1$$

$$O_3 : [A]_1, [B]_1, [C]_{0.5}, [D]_{0.5}, [E]_1$$

The subscript of each property is the degree to which the object possessing the property. We calculate the customers' memberships of O_1, O_2 and O_3 in SC and C according to the axioms and equations in the Au Yeung-Leung model and get the results[1] as following: O_1 belongs to SC to a degree 0.2 and to C to a degree 1, O_2 belongs to SC to a degree 0.1 and to C to a degree1, O_3 belongs to both SC and to C to a degree 0. Such results are not reasonable. It is obvious that O_1 satisfies the minimal requirements of the first kind of special customers, while O_2 satisfies the minimal requirements of the second kind of special customers. Thus, O_1 and O_2 should belong to SC to a degree 1. For object O_3, it should be a member of C to a degree 1. The reason is that people can infer O_3 definitely has bought at least one items because O_3 has bought at least five expensive items. Thus it satisfies the minimal requirement of all properties of 'customer'.[2]

Thus, one limitation of the Au Yeung-Leung model is that a concept is represented by a set of properties and the relations among those properties are conjunction. Such a representation cannot represent disjunction concepts and combination concept, and may lead to unreasonable results. For example, concept 'special-customer' is a union of two kinds of customers. Another limitation is that all properties in the Au Yeung-Leung model are assumed to be independent while some of them should be dependent in reality. We cannot infer some properties based on their dependent properties in the Au Yeung-Leung model. For example, property 'buy at least five expensive items' definitely implies property 'buy at least one item'. Besides, there is no formal definition of property and no formal mechanism to obtain the degree to which an object possesses a property in the Au Yeung-Leung model. All degrees of an object possessing properties are given by user.

4 A Novel Formal Model of Fuzzy Ontology with Property Hierarchy

To overcome the limitations of previous models, we propose a novel formal model of fuzzy ontology by combining the classical view and fuzzy set theory. In our model, a concept is defined by properties, and some properties can be dependent within a property hierarchy specifying the subsumption relationships between properties. Membership degree of an object in a concept depends on the comparison of properties of the object and that of the concept.

4.1 A Conceptual Model of Fuzzy Ontology

We consider a fuzzy ontology O in a particular domain Δ as follows:

$$O_\Delta = (C, R, P, I)$$

where C is a set of fuzzy concepts, R is a set of fuzzy roles which are the relations between two objects, P is a set of fuzzy properties of concepts, and I is a set of objects.[3]

[1] Due to lack of space, we omit the details of calculation here.

[2] Because 'item' is the super-concept of 'expensive-item'.

[3] In the rest of this paper, all concepts, roles and properties are referred to fuzzy concepts, fuzzy roles and fuzzy properties respectively unless otherwise specified.

Fuzzy Concept. A fuzzy concept is a fuzzy set of objects. Objects are considered as members of a concept to some degrees. Such a degree is given by a fuzzy function.

Definition 1. *A* ***fuzzy concept*** *C is defined as following:*

$$C = \{a_1^{v1}, a_2^{v2}, ..., a_n^{vn}\}$$

where a_i is an object, vi is the membership degree of object i in concept C.

We say a_i is a member of C or a_i belongs to C to a degree vi. The degree of object a belongs to a fuzzy concept C is given by a fuzzy membership function:

$$\mu_C : A \rightarrow [0, 1]$$

where A is the set of objects. If there are objects whose membership degree in a concept C is greater than zero, and we name those objects as *members of concept C*.

According to classical view, concepts are organized as in a hierarchy. In our model, a *fuzzy concept hierarchy H_C* is a partial order on the set of all fuzzy concepts in the domain defining the subsumption relationship between fuzzy concepts.

Definition 2. *For two concepts X and Y, $X = \{a_1^{w_1}, a_2^{w_2}, ..., a_n^{w_n}\}$ and $Y = \{a_1^{y_1}, a_2^{y_2}, ..., a_n^{y_n}\}$, a_i is an object, w_i is the membership degree of a_i in fuzzy concept X and y_i is the membership degree of a_i in fuzzy concept Y. If $\forall a_i^{w_i} \in X, a_i^{y_i} \in Y, y_i >= w_i$ then X* ***is subsumed by*** *Y (or Y* ***subsumes*** *X) which is denoted as $X \subseteq Y$.*

Fuzzy Role. There may be some binary relations between objects in a domain, and we define them as follows.

Definition 3. *A* ***fuzzy role*** *R is a fuzzy set of binary relations between two objects in the domain. It is interpreted as a set of pairs of objects from the domain denoted by*

$$R = \{< a_1, b_1 >^{w_1}, < a_2, b_2 >^{w_2}, ..., < a_n, b_n >^{w_n}\}$$

where a_i and b_i are two objects, w_i is a real value between zero and one which representing the degree of strength of the relation between the two objects.

For example, we have a statement 'Bob extremely likes football'. There is a relation 'likes' between Bob and football, and the degree w_i of the strength of this relation is very high (extremely).

The degree of strength of the relation between two objects is given by a fuzzy membership function:

$$\mu_R : A \times B \rightarrow [0, 1]$$

where A and B are sets of objects. The set of objects A is named the *domain* of the role while the set of objects B is named the *range* of the role. If there are object pairs $< a_i, b_i >$ whose membership degree in a role R is greater than zero, and we name those object pairs as *members of fuzzy role R*.

In our model, roles are also organized in a hierarchy. A role hierarchy is a partial order on the set of all fuzzy roles in the domain defining the subsumption relationship between roles.

Definition 4. *For two fuzzy roles* S *and* Q*,* $S = \{< a_1, b_1 >^{w_1}, < a_2, b_2 >^{w_2}, ..., < a_n, b_n >^{w_n}\}$ *and* $Q = \{< c_1, d_1 >^{y_1}, < c_2, d_2 >^{y_2}, ..., < c_n, d_n >^{y_n}\}$*, if* $\forall < a_i, b_i >^{w_i} \in S, < a_i, b_i >^{y_i} \in Q, y_i >= w_i$ *then we say* S ***is subsumed by*** Q *(or* Q ***subsumes*** S*) denoted as* $S \subseteq Q$*.* w_i *is the degree of strength of* $< a_i, b_i >$ *in fuzzy role* S *and* y_i *is the degree of strength of* $< a_i, b_i >$ *in fuzzy role* Q*.*

Fuzzy Property. In our model, an object may have several roles with other objects. These roles with different ranges and the same domain (the same object) are considered as properties of the object.

Definition 5. *A* ***fuzzy property*** P *is defined as following:*

$$P = R.C$$

where R *is a fuzzy role,* C *is a fuzzy concept which is the range of the fuzzy role* R*.*

Concept C is a restriction on the range of the role R in property P, and it requires that all objects in the range of role R should be a member of concept C (i.e., $\mu_C(b_i) > 0$). P is interpreted as a fuzzy set of pairs of fuzzy role and fuzzy objects $(< a_i, b_i >, b_i)^{v_i}$. $< a_i, b_i >$ is a member of the fuzzy role R and b_i is a member of fuzzy concept C, and v_i is the degree of the object a_i possessing the property P.

The degree of objects possesses a property $P = R.C$ is given by a function:

$$\mu_P : R \times C \longrightarrow [0, 1]$$

where R is the set of fuzzy roles, C is the set of fuzzy concepts. If an object a has a fuzzy role (relation) $< a, b >$ with object b, $\mu_R(a, b) > 0$ and $\mu_C(b) > 0$, then we say a possesses a *property member* $(< a, b >, b)$ of property $P = R.C$ to a degree $\mu_P(< a, b >, b)$ where $1 \geq \mu_P(< a, b >, b) > 0$. Object a may possess more than one property members of P. All property members of a property belong to the property to a degree greater than zero. There are some axioms for function μ_P to observe.

Axiom 1. For an object a, a fuzzy property $P = R.C$, if $\mu_R(a, c) = 0$ or $\mu_C(c) = 0$ then $\mu_P(< a, c >, c) = 0$.

Axiom 2. For an object a, a fuzzy property $P = R.C$, if $\mu_R(a, c) = 1$ and $\mu_C(c) = 1$, then $\mu_P(< a, c >, c) = 1$.

Axiom 3. For an object a, a fuzzy property $P = R.C$, if $\mu_R(a, c) \geq \mu_R(a, d)$ and $\mu_C(c) \geq \mu_C(d)$, then $\mu_P(< a, c >, c) \geq \mu_P(< a, d >, d)$.

Axiom 4. For two objects a and b, a fuzzy property $P = R.C$, if $\mu_R(a, c) \geq \mu_R(b, d)$ and $\mu_C(c) \geq \mu_C(d)$, then $\mu_P(< a, c >, c) \geq \mu_P(< b, d >, d)$.

Axiom 5. For an object a, two fuzzy properties $P_1 = R.C$ and $P_2 = S.D$, if $\mu_R(a, e) \geq \mu_S(a, e)$, and $\mu_C(e) \geq \mu_D(e)$, then $\mu_{P_1}(< a, e >, e) \geq \mu_{P_2}(< a, e >, e)$.

Axioms 1 and 2 specify the boundary cases of calculating the degree of objects possessing properties. If $\mu_P(< a, c >, c) = 0$, it means $(< a, c >, c)$ is not a property member of P. If $\mu_P(< a, c >, c) = 1$, it means $(< a, c >, c)$ is definitely a member of

P. Axioms 3, 4 and 5 specify the influence of the membership degree of role and that of the range concept on the property memberships.[4]

There is a special kind of property named *fuzzy instance property*. For a property, it consists of some property members. If there is only one property member in the property, the property is so called a fuzzy instance property.

Analogously, a property hierarchy H_P is a partial order on the set of all properties in the domain defining the subsumption relationship between fuzzy properties.

Definition 6. *For two fuzzy properties* P_1 *and* P_2*,*

$$P_1 = \{(< a, c >, c)^{v_{1i}} | < a, c >^{w_{1i}} \in S, c^{y_{1i}} \in C\}$$

and

$$P_2 = \{(< a, c >, c)^{v_{2i}} | < a, c >^{w_{2i}} \in Q, c^{y_{2i}} \in D\}$$

,if $\forall(< a, c >, c), (< a, c >, c)^{v_{1i}} \in P_1, (< a, c >, c)^{v_{2i}} \in P_2, v_{1i} \leq v_{2i}$*, then* P_1 *is said to* ***be subsumed by*** P_2 *(or* P_2 ***subsumes*** P_1*), denoted by* $P_1 \subseteq P_2$*.*

Two theorems are obtained based on axioms and definitions introduced above.[5]

Theorem 1. *For two properties* P_1 *and* P_2*, if* $P_1 = S.C, P_2 = Q.D, S \subseteq Q$*, and* $C \subseteq D$*, then* $P_1 \subseteq P_2$*.*

Theorem 2. *For an object* a *and two properties* P_1 *and* P_2*, suppose* a *possesses* P_1 *to a degree* $v^a_{P_1}$ *and* P_2 *to a degree* $v^a_{P_2}$*. If* $P_1 \subseteq P_2$*, then* $v^a_{P_1} \leq v^a_{P_2}$*.*

For the example in section 3, we assume a customer O_c has a property 'buy.expensiveItem' and there is one property member 'buy.Eproduct00307' of 'buy.expensiveItem' ('Eproduct00307' is an item and 'buy.Eproduct00307' is also an instance property of O_c). According to theorem 1 and 2, we know that 'buy.expensiveItem' is a sub-property of 'buy.Item' ('expensiveItem' is a sub-concept of 'Item') and we can infer that O_c also possesses the property 'buy.Item' to a degree no less than that of 'buy.expensiveItem'.

Object Representation by Fuzzy Instance Properties. For the reason that an object a has several fuzzy relations (roles) with other objects, each specific member of a role and the object which is a member of the role's range concept can form an fuzzy instance property. Thus object a possesses a set of fuzzy instance properties and each of these properties has only one property member.

We consider an object in an ontology is represented by a set of fuzzy instance properties named *object property vector*. The relation among the fuzzy instance properties in the object property vector is conjunction.

$$\overrightarrow{P}_a = (p_{a,1}^{v_{a,1}}, p_{a,2}^{v_{a,2}}, ..., p_{a,n}^{v_{a,n}}), 1 \leq i \leq n$$

where $p_{a,i}$ is a fuzzy instance property a possessing, $v_{a,i}$ is the degree to which a possesses property $p_{a,i}$. For the reason that all properties in the object property vector are instance properties, thus $\forall i, v_{a,i} = 1$.

[4] For the interest of space, we omit all the verification of axioms in this paper.

[5] For the reason of space, we omit all proofs of theorems in this paper.

For the example in section 3, we assume a customer O_c has a customer id '20071202' and has bought two items 'Furniture00002' and 'Eproduct00307'. O_c is represented as

$$\overrightarrow{O}_c = (hasId.2001202 : 1, buy.Furniture00002 : 1, buy.Eproduct00307 : 1)$$

4.2 Two Kinds of Measurements of Objects Possessing Properties

In our model, the measure of the degree to which a possesses p_x is based on the property members of p_x which a possesses. There are two kinds of measurements on the set of property members which a possesses for a specific property p_x, which are named *quantitative measure* and *qualitative measure* for a possessing p_x.

N-property. The quantitative measure for a possessing p_x is a number restriction on property members of p_x which object a possessing. There are a set of quantifiers for modeling number restrictions on properties. We present six quantifiers used frequently here, which are $[\exists]$, $[\forall]$, $[\geq_n]$, $[\leq_n]$, $[>_n]$, $[<_n]$ and n is an integer. We name a property with a quantifier as an *N-property*, e.g., $[\exists]p_x$, $[\forall]p_x$ and so on.[6]

The degrees to which an object a possessing N-Properties presented above are given by fuzzy functions defined as following respectively:

$$\mu_{[\exists]P}(a, P) = max(\mu_{P_1^a}, ..., \mu_{P_m^a}), 1 \leq i \leq m \tag{1}$$

where $\mu_{P_i^a} = \mu_P(\mu_R(a, c_i), \mu_C(c_i))$ and c_i are objects in the domain.

$$\mu_{[\forall]P}(a, P) = min(\mu_{P_1^a}, ..., \mu_{P_m^a}), 1 \leq i \leq m \tag{2}$$

where $\mu_{P_i^a} = max(1 - \mu_R(a, c_i), \mu_C(c_i))$ and c_i are objects in the domain.

$$\mu_{[\geq_n]P}(a, P) = sup_{c_1,...,c_n \in \Delta^I}(min(\mu_{P_{c1}^a}, ..., \mu_{P_{cn}^a})) \tag{3}$$

where $\mu_{P_{ci}^a} = \mu_P(\mu_R(a, c_i), \mu_C(c_i))$ and c_i are objects in the domain.

Furthermore, $\mu_{[>_n]P} = \mu_{[\geq_{n+1}]P}$, $\mu_{[<_n]P} = 1 - \mu_{[\geq_n]P}$, $\mu_{[\leq_n]P} = 1 - \mu_{[>_n]P}$, i.e., $[\leq_n]P = \neg([>_n]P)$, $[<_n]P = \neg([\geq_n]P)$.

For example, if a customer O_c has bought a set of items (e.g., 'Eproduct00307', 'Book07005' and so on). We can use the fuzzy functions defined above to calculate the degree of O_c possessing these N-properties. For instance, we can obtain that O_c possesses the property '[$\exists$]buy.Item' to a degree 1 according to equation 1. It means that O_c definitely buyers at least one item.

L-property. A qualitative measure of object a possessing a property P is a qualitative aggregation on the set of property members of P which object a possessing. We call a property with an aggregation function on property members as an *L-property*, which is in the form of $[\$]P$. $[\$]$ is a qualification aggregation on all property members, and we call it as a *qualifier*.

[6] We use the form of $[quantifier]P$ as syntax of N-property in order to distinguish from some concepts which are with quantifiers and without [] in DLs, e.g., $\exists R.C$ is a concept in DLs.

There are several possible aggregation functions to aggregate all the property members [16]. One of the aggregation used frequently for qualitative measure is an average function for membership degrees of property members which objects possess in P and we present it here as following:

$$\mu_{[\$]P}(a, P) = \frac{\sum_{i=1}^{n} w_i^a}{n} \tag{4}$$

where w_i^a is the membership degree of property member p_i of P object a possessing.

For example, suppose a customer O_c buys two items 'Eproduct00307' and 'Furniture00002' only. Both 'Eproduct00307' and 'Furniture00002' belong to 'expensiveItem' to a degree 1. Then we can obtain that O_c possesses '[$]buy.expensiveItem' to a degree 1 according to equation 4. It means that O_c definitely buys expensive items.

Difference between Properties, L-properties and N-properties. L-Properties and N-properties are used to measure the degree an object possessing properties qualitatively and quantitatively, respectively. An L-property is a qualitative measurement of an object possessing a property based on aggregating all property members the object possessing for the property, while an N-property is a quantitative measurement of an object possessing a property based on a number restriction on all property members the object possessing for the property. To our best knowledge, there is no a formalization of qualitative measurement for the degree of an object possessing a property. These two measurements are frequently used measurements from two perspectives of people.

4.3 Concepts Represented by N-Properties and L-Properties

We combine the classical view and fuzzy set theory so that our model can handle the vague concepts. In our model, all members of a concept should possess all defining properties of the concept to some degrees. For the reason that N-properties and L-properties are quantitative measures and qualitative measures of properties an object possessing respectively, thus a concept can be defined by a set of N-properties and L-properties. Besides, there is a minimal requirement for each defining property of concepts. If an object possesses all defining properties of a concept to higher degrees, then it means that the object satisfies the minimal requirements of defining properties to higher degrees. Thus the object is given a higher membership degree in the concept.

Based on classical view and fuzzy set theory, we generalize the representation of a concept C as following:

$$\overrightarrow{C} = (\overrightarrow{S}_1, \overrightarrow{S}_2, ..., \overrightarrow{S}_m), 1 \leq i \leq m$$

and

$$\overrightarrow{S}_i = (p_{i,1}^{w_{i,1}}, p_{i,2}^{w_{i,2}}, ..., p_{i,n_i}^{w_{i,n_i}}), 1 \leq j \leq n_i$$

where n_i is the number of properties in $\overrightarrow{S}_i$. A $\overrightarrow{S}_i$ is named a *characteristic vector* of C which consists of a set of defining properties. The relation between characteristic vectors is union, and the relation between defining properties in a $\overrightarrow{S}_i$ is conjunction. $p_{i,j}$ is a defining property in a $\overrightarrow{S}_i$ and it can be either N-properties or L-properties. $w_{i,j}$ is considered as a minimal requirement of property $p_{i,j}$ and $w_{i,j} \in (0, 1]$.

5 Fuzzy Membership of Objects in Concepts

In our model, membership degree of an object a in concept C depends on the comparison of object property vector of a and characteristic vectors of C. If an object a possesses all the defining properties in one of characteristic vectors $\overrightarrow{S}_i$ of C to a degree greater than zero, then a is a member of C to some degree.[7] Besides, while object a possesses all the defining properties of any $\overrightarrow{S}_i$ of C to degrees which are greater than or equal to the minimal requirements of all defining properties of the specific $\overrightarrow{S}_i$ in C, the membership of a in concept C is equal to one. For the reason that concepts are represented by N-properties and L-properties while objects are represented by fuzzy instance properties, and properties in our model may be not independent, we need to do property alignment (aligning fuzzy instance properties of objects to defining properties of concepts) before measuring the membership of objects in concepts based on properties comparison.

5.1 Measuring Degrees of Objects Possessing Defining Properties of Concepts

For the reason that a concept is represented by a set of disjoint characteristic vectors, we need to align the property vector of object a to each characteristic vectors. We define a function for the alignment between object property vectors and characteristic vectors.

$$alignO : P_a \times S_x \rightarrow S_x^a$$

where P_a is the set of object property vectors, S_x is set of characteristic vectors and S_x^a is the set of aligned property vectors. The function $alignO(\overrightarrow{p}_a, \overrightarrow{S}_x)$ is used to align object property vector $\overrightarrow{p}_a$ to characteristic vector $\overrightarrow{S}_x$, the result of $alignO(\overrightarrow{p}_a, \overrightarrow{S}_x)$ is an aligned property vector $\overrightarrow{S}_x^a$ as following:

$$\overrightarrow{s}_x^a = (p_{x,1}^{w_{x,1}^a}, p_{x,2}^{w_{x,2}^a}, ..., p_{x,n}^{w_{x,n}^a}), 1 \leq j \leq n$$

where n is the number of properties of $\overrightarrow{S}_x$ and $w_{x,j}^a$ is the degree of object a possessing property $p_{x,j}$ in characteristic vector $\overrightarrow{S}_x$. In our model, we can obtain the degree of object a possessing each defining property $p_{x,j}$ ($p_{x,j}$ can be N-properties or L-properties) by the fuzzy membership function $\mu_{p_{x,j}}(\overrightarrow{p}_a, p_{x,j})$. The reason is that object a is represented by a vector of instance properties (i.e., a vector of property members) and measuring the degree of object a possessing an N-property or L-property is based on all property members of a possessing. Thus we can obtain $w_{x,j}^a = \mu_{p_{x,j}}(\overrightarrow{p}_a, p_{x,j})$ for each property $p_{x,j}$ where $\mu_{p_{x,y}}(\overrightarrow{p}_a, p_{x,j})$ is one of the membership functions of N-properties or L-properties defined in section 4.2 (e.g., equation 3 and 4).

5.2 Calculation of Object Fuzzy Memberships in Concepts

For a concept C and object a, we can align $\overrightarrow{p}_a$ to each characteristic vector $\overrightarrow{S}_x$ of C and get its aligned property vector $\overrightarrow{S}_x^a$. The degree of a property vector $\overrightarrow{p}_a$ satisfying

[7] If object a possesses all the defining properties of $\overrightarrow{S}_i$ of C to higher degrees, then its membership degree in C is higher.

the minimal requirements of a characteristic vector $\overrightarrow{S}_x$ is calculated by a comparison function of vectors.

$$\varphi : S^a_x \times S_x \rightarrow [0, 1]$$

where S^a_x is the set of aligned property vectors and S_x is the set of characteristic vectors. There are some axioms for $\varphi(\overrightarrow{S}^a_x, \overrightarrow{S}_x)$ to observe.

Axiom 6. For a characteristic vector $\overrightarrow{S}_x$ of a concept and its aligned property vector $\overrightarrow{S}^a_x$, if for some properties $p_{x,i}$ in $\overrightarrow{S}^a_x$, we have $w^a_{x,i} = 0$, then $\varphi(\overrightarrow{S}^a_x, \overrightarrow{S}_x) = 0$.

Axiom 7. For a characteristic vector $\overrightarrow{S}_x$ of a concept and its aligned property vector $\overrightarrow{S}^a_x$, if for each properties $p_{x,i}$ in $\overrightarrow{S}^a_x$, we have $w^a_{x,i} \geq w_{x,i}$, then $\varphi(\overrightarrow{S}^a_x, \overrightarrow{S}_x) = 1$.

Axiom 8. For an object property vector $\overrightarrow{p}_a$, two characteristic vectors $\overrightarrow{S}_{x1}$ and $\overrightarrow{S}_{x2}$ of a concept, $\overrightarrow{S}^a_{x1}$ is the aligned property vector of $\overrightarrow{p}_a$ for $\overrightarrow{S}_{x1}$ and $\overrightarrow{S}^a_{x2}$ is the aligned property vector of $\overrightarrow{p}_a$ for $\overrightarrow{S}_{x2}$, if $w_{x1,i} \leq w_{x2,i}$ for some properties $p_{x,i}$, and $w_{x1,j} = w_{x2,j}$ for others properties $p_{x,j}$ where $i \neq j$, then $\varphi(\overrightarrow{S}^a_{x1}, \overrightarrow{S}_{x1}) \geq \varphi(\overrightarrow{S}^a_{x2}, \overrightarrow{S}_{x2})$.

Axiom 9. For a characteristic vector $\overrightarrow{S}_x$ of a concept, two aligned property vectors $\overrightarrow{S}^a_x$ and $\overrightarrow{S}^b_x$ for object a and b respectively, if $w^a_{x,i} \geq w^b_{x,i}$ for some properties $p_{x,i}$ and $w^a_{x,j} = w^b_{x,j}$ for others properties $p_{x,j}$ where $i \neq j$, then $\varphi(\overrightarrow{S}^a_x, \overrightarrow{S}_x) \geq \varphi(\overrightarrow{S}^b_x, \overrightarrow{S}_x)$.

Axioms 6 and 7 specify the boundary cases of objects satisfying the minimal requirements of properties of concepts. Axioms 8 and 9 concern how the degree of an object property vector satisfying the minimal requirement of a characteristic vector is varied.

Here, we present a possible function which satisfies axioms 6, 7, 8 and 9.

$$\varphi(\overrightarrow{S}^a_x, \overrightarrow{S}_x) = min(\tau_1, \tau_2, ..., \tau_n) \tag{5}$$

where

$$\tau_i = \begin{cases} \frac{w^a_{x,i}}{w_{x,i}} & w^a_{x,i} < w_{x,i} \\ 1 & w^a_{x,i} \geq w_{x,i} \end{cases} \tag{6}$$

where $w^a_{x,i}$ is the degree to which a possessing property $p_{x,i}$ and $w_{x,i}$ is the minimal requirement of property $p_{x,i}$ in $\overrightarrow{S}_x$.

Besides, we consider the fuzzy membership of an object a in fuzzy concept C depends on the following equation:

$$\mu_C(a) = max(\varphi(\overrightarrow{S}^a_1, \overrightarrow{S}_1), \varphi(\overrightarrow{S}^a_2, \overrightarrow{S}_2), ..., \varphi(\overrightarrow{S}^a_n, \overrightarrow{S}_n)) \tag{7}$$

One object may satisfy all the property minimal requirements of more than one characteristic vectors. We choose the maximal value of $\varphi(\overrightarrow{S}^a_i, \overrightarrow{S}_i)$ as the the membership of a in C because that the relation among $\overrightarrow{S}_i$ is disjunction. This is in line with fuzzy set theory.

6 An Illustrating Example

Let's revisit the example discussed in section 3. The concept 'special-customer' denoted by *SC* and the concept 'customer' denoted by *C* are defined as following using our model (Properties of SC and C formalized in our model are shown in table 3.):

Table 3. Properties of SC and C formalized in our model

A'	[∃]hasId.customerID	B'	$[\geq_5]$buy.expensiveItem	C'	[$]buy.expensiveItem
D'	$[\geq_{100}]$buy.Item	E'	$[\geq_1]$buy.expensiveItem	F'	[∃]buy.Item

$$\overrightarrow{C} = (A' : 1, F' : 1), \quad \overrightarrow{SC} = \begin{pmatrix} \overrightarrow{SC_1} = (A' : 1, B' : 1, C' : 0.6) \\ \overrightarrow{SC_2} = (A' : 1, D' : 1, E' : 0.5) \end{pmatrix}$$

For O_1, O_2 and O_3 in section 3, they are represented by fuzzy instance properties and items bought by the three customers are showed in table 2 in section 3. We align the property vectors of them to characteristic vectors of SC as following.

$$\overrightarrow{O_1} = (A' : 1, B' : 1, C' : 0.8) \cup (A' : 1, D' : 0.2, E' : 1)$$

$$\overrightarrow{O_2} = (A' : 1, B' : 0.2, C' : 0.1) \cup (A' : 1, D' : 1, E' : 0.8)$$

$$\overrightarrow{O_3} = (A' : 1, B' : 1, C' : 0.5) \cup (A' : 1, D' : 0.5, E' : 1)$$

The degrees of each object possessing defining properties (e.g., '[∃]buy.expensiveItem') is calculated based on all property members (e.g., 'buy.Furniture00002') possessed by the object for the corresponding property (e.g., 'buy.expensiveItem') using equations 1, 2, 3 and 4 in section 4.2.[8] For example, according to table 2, object O_1 has property members for property '[∃]buy.Item' such as O_1 possessing 'buy.Furniture00002', 'buy.Eproduct00307' and 'buy.Book07005', and these property members are belonged to '[∃]buy.Item' to degree 1. Then the degree of object O_1 possessing the property '[∃]buy.Item' is calculated using equation 1 as following:

$$\mu_{[\exists]buy.Item}(O_1, [\exists]buy.Item) = \max(1, 1, ...1) = 1$$

Then we can get the following result for SC by axioms 6, 7, 8, 9 and equations 5, 6, 7 introduced in section 5:

$$\mu_{SC}(O_1) = 1, \mu_{SC}(O_2) = 1, \mu_{SC}(O_3) = 0.83$$

Analogously, we can get the result for C as following:

$$\mu_C(O_1) = 1, \mu_C(O_2) = 1, \mu_C(O_3) = 1$$

Such results are more reasonable than that in previous models. For the reason that O_1 satisfies all minimal requirements of properties in $\overrightarrow{SC_1}$ while O_2 satisfies that in $\overrightarrow{SC_2}$ and O_3 satisfies a part of that in $\overrightarrow{SC_1}$, we obtain $\mu_{SC}(O_1) = 1, \mu_{SC}(O_2) = 1, \mu_{SC}(o_3) = 0.83$. Further more, according to theorem 1, we can obtain that 'buy.expensiveItem' is a sub-property of 'buy.Item'. Thus we can obtain $\mu_C(O_1) = 1, \mu_C(O_2) = 1, \mu_C(O_3) = 1$ without knowing the degree of each object possessing property F'.

[8] For the interest of space, we omit all the fuzzy functions of concepts and the calculation details here.

7 Conclusion

In this paper, we propose a novel formal model of fuzzy ontology with property hierarchy and object membership by combining the classical view and fuzzy set theory, and show that our model is more reasonable and powerful than previous models. Our model can handle the cases of representing concepts by dependent properties in ontology and represent all kinds of concepts (including conjunction concepts, disjunction concepts and combination concepts). Besides, our model also provides a formal mechanism to determine object memberships in concepts automatically based on the defining properties of concepts and properties which objects possess.

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