Shallow Models for Non-iterative Modal Logics

Lutz Schröder[1,⋆] and Dirk Pattinson[2,⋆⋆]

[1] DFKI-Lab Bremen and Department of Computer Science, Universität Bremen
[2] Department of Computing, Imperial College London

Abstract. Modal logics see a wide variety of applications in artificial intelligence, e.g. in reasoning about knowledge, belief, uncertainty, agency, defaults, and relevance. From the perspective of applications, the attractivity of modal logics stems from a combination of expressive power and comparatively low computational complexity. Compared to the classical treatment of modal logics with relational semantics, the use of modal logics in AI has two characteristic traits: Firstly, a large and growing variety of logics is used, adapted to the concrete situation at hand, and secondly, these logics are often non-normal. Here, we present a shallow model construction that witnesses PSPACE bounds for a broad class of mostly non-normal modal logics. Our approach is uniform and generic: we present general criteria that uniformly apply to and are easily checked in large numbers of examples. Thus, we not only re-prove known complexity bounds for a wide variety of structurally different logics and obtain previously unknown PSPACE-bounds, e.g. for Elgesem's logic of agency, but also lay the foundations upon which the complexity of newly emerging logics can be determined.

Special purpose modal logics abound in applied logic, and in particular in artificial intelligence, where new logics emerge at a steady rate. They often combine expressiveness and decidability, and indeed many modal logics are decidable in PSPACE, i.e. not dramatically worse than propositional logic. While lower PSPACE bounds can typically be obtained directly from seminal results of Ladner [12] by embedding a PSPACE-hard logic such as K or KD, upper bounds are often non-trivial to establish. In this respect, non-normal logics have received much attention in recent research, which has lead e.g. to PSPACE upper bounds for graded modal logic [20] (correcting a previously published incorrect algorithm and refuting a previous EXPTIME hardness conjecture), Presburger modal logic [4], coalition logic [17], and various conditional logics [14].

The methods used to obtain these results can be broadly grouped into two classes. Syntactic approaches presuppose a complete tableaux or sequent system and establish that proof search can be performed efficiently. Semantics-driven approaches, on the other hand, directly construct shallow tree models. Both approaches are intimately connected in the case of normal modal logics with relational semantics: counter models can usually be derived directly from search trees [10]. However, the situation is quite different in the non-normal case, where the structure of models often goes far beyond mere graphs. We have previously shown [19] that the *syntactic* approach uniformly

⋆ This work forms part of the DFG project *Generic Algorithms and Complexity Bounds in Coalgebraic Modal Logic* (SCHR 1118/5-1).

⋆⋆ Partially supported by EPSRC grant EP/F031173/1.

A. Dengel et al. (Eds.): KI 2008, LNAI 5243, pp. 324–331, 2008.

generalises to a large class of modal logics. Here, we present a generic *semantic* set of methods to establish uniform PSPACE bounds using a direct shallow model construction which in particular does not need to rely on an axiomatisation of the logic at hand. Apart from the fact that both methods use substantially different techniques, they apply to different classes of examples. Examples not easily amenable to the syntactic approach, because either no axiomatisation has been given or known axiomatisations are hard to harness, include probabilistic modal logic [6] and Presburger modal logic [4].

We emphasise that our methods go far beyond establishing the complexity of a particular logic: they employ the semantic framework of *coalgebraic modal logic* [16] to obtain results that are parametric in the underlying semantics of particular logics. In this paradigm, the role of models is played by coalgebras, which associate a structured collection of successor states to every state of the model: a coalgebra with state set C is a function $C \to TC$ where the parametrised datatype (technically: functor) T represents the structuring of successors, e.g. relational, probabilistic [6], game-oriented [17], or non-monotonically conditional [1]. Our approach is now best described as investigating *coherence conditions* between the syntax and the semantics, parametric in both, that guarantee the announced complexity bounds. While these methods have so far been limited to logics of *rank 1*, given by axioms whose modal nesting depth is uniformly equal to one, the present results apply to *non-iterative logics* [13], i.e. logics axiomatised without nested modalities (rank-1 logics additionally exclude top-level propositional variables). This increase in generality, achieved by working with *copointed functors* in the semantics, substantially extends the scope of the coalgebraic method, in particular where relevant to AI. E.g. all conditional logics covered in [14], Elgesem's logic of agency [5], and the graded version Tn of T [7] are non-iterative logics.

Our main technical tool is to cut model constructions for modal logics down to the level of *one-step logics* which semantically do not involve state transitions, and then amalgamate the corresponding *one-step models* into shallow models for the full modal logic. This requires the logic at hand to support a small model property for its one-step fragment, the *one-step polysize model property (OSPMP)*, which is much easier to establish than a shallow model property for the logic itself (e.g. to reprove Ladner's PSPACE upper bound for K, one just observes that to construct a set that intersects n given sets, one needs at most n elements). As a by-product of our construction, we obtain NP-bounds for bounded rank fragments, generalizing corresponding results for the logics K and T from [8]. We illustrate our method by concise new proofs of known PSPACE upper bounds for various conditional logics, probabilistic modal logic, and Presburger modal logic. As a new result, we prove e.g. that Elgesem's logic of agency is in PSPACE. Despite the emphasis we place on examples, we stress that the main intention of this work is to provide a *standard method* that goes beyond mere informal recipes, being based on formal theorems with easily verified and well-structured application conditions. A full version of this work is available as e-print arXiv:0802.0116.

1 Preliminaries: Coalgebraic Modal Logic

We give a self-contained introduction to the syntax and coalgebraic semantics of modal logics. A *(modal) similarity type* Λ is a set of modal operators with associated finite arity. The similarity type Λ determines two languages: firstly, the *modal logic* of Λ, whose set $\mathcal{F}(\Lambda)$ of *Λ-formulas* $\psi, \dots$ is defined by the grammar

$$\psi ::= \bot \mid \psi_1 \wedge \psi_2 \mid \neg\psi \mid L(\psi_1, \dots, \psi_n) \qquad (L \in \Lambda \; n\text{-ary}).$$

Propositional atoms are treated as nullary modalities and therefore do not explicitly appear in the syntax. Secondly, the signature Λ determines the *one-step logic* of Λ, whose formlas may be represented as *one-step pairs* (ϕ, ψ) where ϕ is a propositional formula over a set V of variables and ψ is a propositional combination of atoms $L(a_1, \dots, a_n)$, with $a_1, \dots, a_n \in V$ and $L \in \Lambda$ n-ary. Equivalently, one may use *one-step formulas*, i.e. propositional combinations of formulas $L(\phi_1, \dots, \phi_n)$, where the ϕ_i are propositional formulas over V [18]; i.e. the modal logic of Λ is distinguished from the one-step logic in that it admits nested modalities. The *rank* $\mathrm{rank}(\phi)$ of $\phi \in \mathcal{F}(\Lambda)$ is the maximal nesting depth of modalities in ϕ. The *bounded-rank fragments* of $\mathcal{F}(\Lambda)$ are the sets $\mathcal{F}_n(\Lambda) = \{\phi \in \mathcal{F}(\Lambda) \mid \mathrm{rank}(\phi) \leq n\}$.

The semantics of both the one-step logic and the modal logic of Λ are parametrized coalgebraically by the choice of a *set functor*, i.e. an operation $T : \mathsf{Set} \to \mathsf{Set}$ taking sets to sets (w.l.o.g. preserving the subset relation) and maps $f : X \to Y$ to maps $Tf : TX \to TY$, preserving identities and composition. The standard setup of coalgebraic modal logic using all coalgebras for a set functor covers only *rank-1 logics*, i.e. logics axiomatised by one-step formulas [18] (a typical example is the K-axiom $\Box(a \to b) \to \Box a \to \Box b$). Here, we improve on this by considering the class of coalgebras for a given *copointed* set functor (in a slightly restricted sense), which enables us to cover the more general class of *non-iterative logics*, axiomatised by arbitrary formulas without nested modalities (such as the T-axiom $\Box a \to a$).

Definition 1. A *copointed functor* S with *signature functor* $S_0 : \mathsf{Set} \to \mathsf{Set}$ is a subfunctor of $S_0 \times Id$ (where $(S_0 \times Id)X = S_0 X \times X$). We say that S is *trivially copointed* if $S = S_0 \times Id$. An *S-coalgebra* $A = (X, \xi)$ consists of a set X of *states* and a *transition* function $\xi : X \to S_0 X$ such that $(\xi(x), x) \in SX$ for all x.

We view coalgebras as generalised transition systems: the transition function maps a state to a structured collection of successors, with the structure prescribed by the signature functor, which thus encapsulates the branching type of the transition systems employed. Copointed functors additionally impose local frame conditions that relate a state to the collection of its successors. We refer to elements of sets SX as *successor structures*. Generalising earlier work, *coalgebraic modal logic* [16] abstractly captures the interpretation of modal operators using predicate liftings:

Definition 2. An *n-ary predicate lifting* ($n \in \mathbb{N}$) for S_0 is a family $(\lambda_X : \mathcal{P}(X)^n \to \mathcal{P}(S_0 X))_{X \in \mathsf{Set}}$ of maps satisfying *naturality*, i.e. $\lambda_X(f^{-1}[A_1], \dots, f^{-1}[A_n]) = (S_0 f)^{-1}[\lambda_Y(A_1, \dots, A_n)]$ for all $f : X \to Y$, $A_1, \dots, A_n \in \mathcal{P}(Y)$.

A coalgebraic semantics for Λ, i.e. a *coalgebraic modal logic* $\mathcal{L}$, consists of a copointed functor S with signature functor S_0 and an assignment of an n-ary predicate lifting $[\![L]\!]$ for S_0 to every n-ary modal operator $L \in \Lambda$. When S is trivially copointed, we will mention only S_0. *We fix $\mathcal{L}$, Λ, S, S_0 throughout.* The semantics of the modal language $\mathcal{F}(\Lambda)$ is defined inductively as a satisfaction relation $\models_C$ between states x of S-coalgebras $C = (X, \xi)$ and Λ-formulas. The clause for an n-ary modal operator L is

$$x \models_C L(\phi_1, \dots, \phi_n) \;\Leftrightarrow\; \xi(x) \in [\![L]\!]_X([\![\phi_1]\!], \dots, [\![\phi_n]\!])$$

where $[\![\phi]\!] = \{x \in X \mid x \models_C \phi\}$. Our interest is in the *satisfiability problem of* $\mathcal{L}$, which asks whether for a given formula ϕ, there exist an S-coalgebra C and a state x in C such that $x \models_C \phi$.

In contrast, the semantics of the one-step logic is defined over single successor structures, in particular does not involve a notion of state transition:

Definition 3. A *one-step model* (X, τ, t, x) over V consists of a set X, a $\mathcal{P}(X)$-valuation τ for V, $t \in S_0X$, and $x \in X$ such that $(t, x) \in SX$. We omit the mention of x if S is trivially copointed. For a one-step pair (ϕ, ψ), τ induces interpretations $[\![\phi]\!]\tau \subseteq X$ and, using the given predicate liftings, $[\![\psi]\!]\tau \subseteq S_0X$. We say that (X, τ, t, x) is a *one-step model of* (ϕ, ψ) if $[\![\phi]\!]\tau = X$ and $t \in [\![\psi]\!]\tau$.

Example 4. 1. *The modal logics K and T* [1] have a single unary modal operator $\Box$. The standard Kripke semantics of K is modelled coalgebraically over the powerset functor $\mathcal{P}$, whose coalgabras are just Kripke frames, by $[\![\Box]\!]_X(A) = \{B \in \mathcal{P}X \mid B \subseteq A\}$. The logic T (i.e. K extended with the non-iterative axiom $\Box a \to a$) is modelled by moving to the copointed functor R given by $RX = \{(A, x) \in \mathcal{P}X \times X \mid x \in A\}$. R-coalgebras are reflexive Kripke frames.

2. *Conditional logics* have a single binary infix modal operator $\Rightarrow$, read as a non-monotonic conditional (default, relevance, ...). The standard semantics of the conditional logic CK is modelled coalgebraically over the functor Cf given by $Cf(X) = \mathcal{P}(X) \to \mathcal{P}(X)$, with $\to$ denoting function space, by

$$[\![\Rightarrow]\!]_X(A, B) = \{f : \mathcal{P}(X) \to \mathcal{P}(X) \mid f(A) \subseteq B\}.$$

Cf-coalgebras are *conditional frames* [1]. The conditional logic $CK + ID$ extends CK with the rank-1 axiom $a \Rightarrow a$. Its semantics is modelled by passing to the subfunctor Cf_{ID} of Cf defined by $Cf_{ID}(X) = \{f \in Cf(X) \mid \forall A \in \mathcal{P}(X).\, f(A) \subseteq A\}$. Similarly, the logic $CK + MP$ extends CK with the non-iterative axiom $(a \Rightarrow b) \to (a \to b)$. (This axiom is undesirable in default logics, but often considered in relevance logics.) Semantically, this amounts to passing to the copointed functor Cf_{MP} defined by

$$Cf_{MP}(X) = \{(f, x) \in Cf(X) \times X \mid \forall A \in \mathcal{P}(X).\, x \in A \Rightarrow x \in f(A)\}.$$

3. *Modal logics of quantitative uncertainty:* The similarity type of *likelihood* has polyadic modal operators expressing linear inequalities between likelihoods $l(\phi)$, variously interpreted as probabilities [6], upper probabilities, Dempster-Shafer degrees of belief, or Dubois-Prade degrees of possibility [9]. Extensions generalise likelihood to *expectations* [9] for linear combinations of formulas. These logics are captured coalgebraically by suitable distribution functors. E.g. the case of probabilities is modelled by the probability distribution functor D_ω, where $D_\omega X$ is the set of finitely supported probability distributions on X, with D_ω-coalgebras corresponding to Markov chains; and the case of upper probabilities is modelled by the sets-of-distributions functor $\mathcal{P} \circ D_\omega$. In the literature, one-step logics of quantitative uncertainty are often introduced independently and only later extended to full modal logics [6].

4. *Graded and Presburger modal logic:* Graded modal logic [7] has operators $\Diamond_k$ read 'in more than k successor states, it holds that ...'. Variants of these operators have found their way into modern description logics as *qualified number restrictions*. More generally, Presburger modal logic [4] has n-ary modal operators $\sum_{i=1}^{n} a_i\#(\_) \sim b$, where b and the a_i are integers and $\sim \in \{<, >, =\} \cup \{\equiv_k \mid k \in \mathbb{N}\}$, with $\equiv_k$ read as equality modulo k. The original Kripke semantics is equivalent to a coalgebraic semantics over the finite multiset functor $\mathcal{B}$, which maps a set X to the set of maps $B : X \to \mathbb{N}$ with finite support, understood as multisets containing $x \in X$ with multiplicity $B(x)$. $\mathcal{B}$-coalgebras are graphs with $\mathbb{N}$-weighted edges, over which the given modal operators are interpreted as suggested by the notation, adding up multiplicities [3].

5. *Agency:* Logics of agency, concerned with agents bringing about states of affairs, play a role in planning and task assignment in multi-agent systems [2,11]. A standard approach due to Elgesem [5], intended as an abstraction of pure agency to be used as a building block in more complex logics, has modalities E and C, read 'the agent brings about' and 'the agent is capable of realising', respectively. Their semantics is defined over a certain restricted class of conditional frames $(X, f : X \to (\mathcal{P}(X) \to \mathcal{P}(X)))$ (see above) by $x \models E\phi$ iff $x \in f(x)([\![\phi]\!])$ and $x \models C\phi$ iff $f(x)([\![\phi]\!]) \neq \emptyset$. Most of the information in such models is disregarded: one only needs to know whether $f(x)(A)$ is non-empty and contains x. We may thus equivalently use the following coalgebraic semantics: put $3 = \{\bot, *, \top\}$, ordered $\bot < * < \top$ (to code the cases $f(x)(A) = \emptyset$, $x \notin f(x)(A) \neq \emptyset$, and $x \in f(x)(A)$), and take as signature functor the 3-*valued neighbourhood functor* N_3 given by $N_3(X) = (\mathcal{P}(X) \to 3)$. The restrictions on conditional frames imposed by Elgesem translate into using the copointed functor $\mathcal{A}$ over N_3 where $(f, x) \in \mathcal{A}(X)$ iff for all $A, B \subseteq X$, $f(\emptyset) = \bot$, $f(X) = \bot$ ('the agent cannot bring about logical truths'), $f(A) \wedge f(B) \leq f(A \cap B)$, and $f(A) = \top \Rightarrow x \in A$ ('what the agent brings about is actually the case'). The operators E, C are interpreted by $[\![E]\!]_X A = \{f \in N_3(X) \mid f(A) = \top\}$ and $[\![C]\!]_X A = \{f \in N_3(X) \mid f(A) \neq \bot\}$. Notably, the logic is non-monotone, i.e. Ea does not imply $E(a \vee b)$, which makes typical approaches to proving PSPACE bounds hard to apply.

2 A Generic Shallow Model Construction

We now turn to the announced construction of polynomially branching shallow models for modal logics whose one-step logic has a small model property; this construction leads to a PSPACE decision procedure. For finite X, we assume given a representation of elements $(t, x) \in SX$ as strings of size $\mathrm{size}(t, x)$ over some finite alphabet, where crucially do not require that all elements of SX are representable.

Definition 5. The logic $\mathcal{L}$ has the *one-step polysize model property (OSPMP)* if there exist polynomials p and q such that, whenever a one-step pair (ϕ, ψ) over V has a one-step model (X, τ, t, x), then it has a one-step model (Y, κ, s, y) such that $|Y| \leq p(|\psi|)$, (s, y) is representable with $\mathrm{size}(s, y) \leq q(|\psi|)$, and $y \in \kappa(a)$ iff $x \in \tau(a)$ for all $a \in V$.

It is crucial that the polynomial bound depends only on ψ, as the proof of Thm. 7 below uses one-step pairs (ϕ, ψ) with exponential-size ϕ. In terms of one-step formulas, this amounts to discounting the potentially exponential-sized inner propositional layer.

Definition 6. A *supporting Kripke frame* of an S-coalgebra (X, ξ) is a Kripke frame (X, R) such that for each $x \in X$, $\xi(x) \in S_0\{y \mid xRy\} \subseteq S_0X$.

Theorem 7 (Shallow model property). *The OSPMP implies the* polynomially branching shallow model property*: There exist polynomials p, q such that every satisfiable Λ-formula ψ is satisfiable in an S-coalgebra (X, ξ) which has a supporting Kripke frame (X, R) such that removing all loops xRx from (X, R) yields a tree of depth at most* $\mathrm{rank}(\psi)$ *and branching degree at most $p(|\psi|)$, and $(\xi(x), x) \in S\{y \mid xRy\}$ is representable with* $\mathrm{size}(\xi(x), x) \leq q(|\psi|)$.

This theorem leads to a nondeterministic decision procedure for satisfiability that recursively traverses shallow models in a depth-first fashion, guessing at each level a polynomial-size successor structure. Checking whether such a structure satisfies the local requirements imposed by the input formula is encapsulated as follows:

Definition 8. The *one-step model checking problem* of $\mathcal{L}$ is to check, given a string s, a finite set X, $A_1, \dots, A_n \subseteq X$, and $L \in \Lambda$ n-ary, whether s represents some $(t, x) \in SX$ and whether $t \in [\![L]\!]_X(A_1, \dots, A_n)$.

Theorem 9. *Let $\mathcal{L}$ have the OSPMP.*

1. *If the one-step model checking problem of $\mathcal{L}$ is in PSPACE, then the satisfiability problem of $\mathcal{L}$ is in PSPACE.*
2. *If the one-step model checking problem of $\mathcal{L}$ is in P, then the restriction of the satisfiability problem of $\mathcal{L}$ to the bounded-rank fragment $\mathcal{F}_n(\Lambda)$ is in NP for every $n \in \mathbb{N}$.*

Theorem 9.2, which generalises known results for K and T [8], follows from the fact that Thm. 7 implies a polynomial-size model property for bounded-rank fragments.

In cases where the OSPMP fails, one can often use a relaxed criterion, the *one-step pointwise polysize model property (OSPPMP)*, provided that S_0 is *pointwise κ-bounded*, i.e. $|S_0X| \leq \kappa^{|X|}$, for some cardinal κ; we then assume $S_0X \subseteq \kappa^X$. E.g. the functors $\mathcal{P}$ and D_ω, but not *Cf* and $\mathcal{P} \circ D_\omega$, are pointwise bounded. Assuming a (partial) representation of elements of κ, we put $\mathrm{maxsize}(t) = \max_{x \in X} \mathrm{size}\, t(x)$ for $t \in S_0X \subseteq \kappa^X$. The OSPPMP essentially requires the existence of one-step models (X, τ, t, x) such that $\mathrm{maxsize}(t)$ is polynomially bounded. Then we have

Theorem 10. *If $\mathcal{L}$ has the OSPPMP and one-step model checking of (X, τ, t, x) is decidable on a non-deterministic Turing machine with input tape that uses space polynomial in* $\mathrm{maxsize}(t)$ *and accesses each input symbol at most once, then the satisfiability problem of $\mathcal{F}(\Lambda)$ is in PSPACE.*

The crucial point in the proof is that if input symbols are read at most once, they can be guessed without having to be stored.

Example 11. In all example applications, tractability of one-step model checking is straightforward, so that we concentrate on small one-step models.

1. *Modal logics K and T:* To verify the OSPMP for K, let (X, τ, A) be a one-step model of a one-step pair (ϕ, ψ) over V; w.l.o.g. ψ is a conjunctive clause over atoms

$\Box a$, where $a \in V$. For $\neg\Box a$ in ψ, there exists $x_a \in A$ such that $x_a \notin \tau(a)$. Taking Y to be the set of these x_a, we obtain a polynomial-size one-step model (Y, τ_Y, Y) of (ϕ, ψ), where $\tau_Y(a) = \tau(a) \cap Y$ for all a. The construction for T is the same, except that the point x of the original one-step model (X, τ, A, x) is retained in Y. By Thm. 9, this reproves Ladner's PSPACE upper bounds for K and T [12], as well as Halpern's NP upper bounds for bounded-rank fragments [8].

2. *Conditional logic:* To avoid exponential blowup, we represent elements of $\mathit{Cf}(X) = \mathcal{P}(X) \to \mathcal{P}(X)$ as *partial* maps, extended to total maps using default value $\emptyset$. The OSPMP for CK is proved as follows: given a one-step model (X, τ, f) of a one-step pair (ϕ, ψ), where ψ is w.l.o.g. a conjunctive clause, retain values of f only at the sets $\tau(a_i)$ and cut down to a polynomial-size set Y containing elements y_{ij} of the symmetric difference of $\tau(a_i)$ and $\tau(a_j)$ whenever $\tau(a_i) \neq \tau(a_j)$ and elements $z_i \in f(\tau(a_i)) \setminus \tau(b_i)$ whenever ψ contains $\neg(a_i \Rightarrow b_i)$. The proofs for $\mathit{CK}+\mathit{ID}$ and $\mathit{CK}+\mathit{MP}$ are the same, up to changing the default value for $f(A)$ in the representation of $(f, x) \in \mathit{Cf}_{\mathit{MP}}(X)$ to $A \cap \{x\}$. Thus, we reprove that CK, $\mathit{CK}+\mathit{ID}$, and $\mathit{CK}+\mathit{MP}$ are in PSPACE [14] (hence PSPACE-complete, as they contain known PSPACE-hard sublogics) and obtain a new NP upper bound for their bounded-rank fragments.

3. *Modal logics of quantitative uncertainty:* Polynomial size model properties for one-step logics and complexity estimates for one-step model checking have been proved for various logics of quantitative uncertainty [6,9]. The polynomial bounds are stated in the cited work as depending on the size of an entire one-step formula ψ; however, inspection of the given proofs shows that the bounds are in fact independent of the inner propositional layer, and hence actually establish the OSPMP, with ensuing (tight) upper complexity bounds as in Thm 9. A proof of the PSPACE upper bound for the modal logic of probability is sketched in [6]; the NP upper bound for bounded-rank fragments is new. In the remaining cases, also the PSPACE upper bounds are new, if only because just the one-step versions of these logics appear in the literature so far.

4. *Elgesem's logic of agency:* For X finite, we let a *partial* map $f_0 : \mathcal{P}(X) \rightharpoonup 3$ represent the element $f \in N_3(X) = \mathcal{P}(X) \to 3$ that maps $B \subseteq X$ to the maximum of $\bigwedge_{i=1}^{n} f_0(A_i)$, taken over all sets $A_1, \dots, A_n \subseteq X$ such that $\bigcap A_i = B$ and $f_0(A_i)$ is defined for all i. In the proof of the OSPMP, a one-step model $(X, \tau, f : \mathcal{P}(X) \to 3, x)$ of a one-step pair over V is reduced to polynomial size by restricting f to the $\tau(a)$, $a \in V$, and cutting down to a set Y containing: the point x; an element $y_{ab} \in \tau(a) \setminus \tau(b)$ whenever $\tau(a) \not\subseteq \tau(b)$; an element $z_a \in \bigcap\{\tau(b) \mid b \in V, \tau(a) \subseteq \tau(b), f(\tau(b)) > f(\tau(a))\} \setminus \tau(a)$ for each $a \in V$; and an element $w_0 \in \bigcap\{\tau(b) \mid f(\tau(b)) > \bot\}$, where w_0 and the z_a exist by the definition of $\mathcal{A}$. Thus, *the modal logic of agency is in PSPACE*, and its bounded-rank fragments are in NP. Both results (and even decidability) seem to be new. We conjecture that the PSPACE upper bound is tight.

5. *Presburger Modal Logic:* The functor $\mathcal{B}$ is pointwise ω-bounded. It follows easily from estimates on solution sizes of integer linear equalities [15] that Presburger modal logic has the OSPPMP, and hence is in PSPACE [4]. One easily incorporates non-iterative frame conditions such as reflexivity (modelled by the copointed functor $SX = \{(B, x) \in \mathcal{B}X \times X \mid B(x) > 0\}$) or e.g. the condition that at least half of all transitions from a given state are loops (modelled by the copointed functor $SX = \{(B, x) \in \mathcal{B}X \times X \mid B(x) \geq B(X - \{x\}\}$). In particular, this implies that

graded modal logic over reflexive frames (i.e. the logic Tn of [7]) *is in PSPACE*, to our knowledge a new result. This extends straightforwardly to show that the concept satisfiability problem in description logics with role hierarchies, reflexive roles, and qualified number restrictions is in PSPACE over the empty TBox.

Acknowledgements. The authors wish to thank Joseph Halpern for useful discussions. Erwin R. Catesbeiana has contributed his opinion on consistent formulas.

References

1. Chellas, B.: Modal Logic. Cambridge Univ. Press, Cambridge (1980)
2. Cholvy, L., Garion, C., Saurel, C.: Ability in a multi-agent context: A model in the situation calculus. In: Toni, F., Torroni, P. (eds.) CLIMA 2005. LNCS (LNAI), vol. 3900, pp. 23–36. Springer, Heidelberg (2006)
3. D'Agostino, G., Visser, A.: Finality regained: A coalgebraic study of Scott-sets and multisets. Arch. Math. Logic 41, 267–298 (2002)
4. Demri, S., Lugiez, D.: Presburger modal logic is only PSPACE -complete. In: Furbach, U., Shankar, N. (eds.) IJCAR 2006. LNCS (LNAI), vol. 4130, pp. 541–556. Springer, Heidelberg (2006)
5. Elgesem, D.: The modal logic of agency. Nordic J. Philos. Logic 2, 1–46 (1997)
6. Fagin, R., Halpern, J.Y.: Reasoning about knowledge and probability. J. ACM 41, 340–367 (1994)
7. Fine, K.: In so many possible worlds. Notre Dame J. Formal Logic 13, 516–520 (1972)
8. Halpern, J.: The effect of bounding the number of primitive propositions and the depth of nesting on the complexity of modal logic. Artificial Intelligence 75, 361–372 (1995)
9. Halpern, J., Pucella, R.: Reasoning about expectation. In: Uncertainty in Artificial Intelligence, UAI 2002, pp. 207–215. Morgan Kaufmann, San Francisco (2002)
10. Halpern, J.Y., Moses, Y.O.: A guide to completeness and complexity for modal logics of knowledge and belief. Artificial Intelligence 54, 319–379 (1992)
11. Jones, A., Parent, X.: Conventional signalling acts and conversation. In: Dignum, F.P.M. (ed.) ACL 2003. LNCS (LNAI), vol. 2922, pp. 1–17. Springer, Heidelberg (2004)
12. Ladner, R.: The computational complexity of provability in systems of modal propositional logic. SIAM J. Comput. 6, 467–480 (1977)
13. Lewis, D.: Intensional logics without iterative axioms. J. Philos. Logic 3, 457–466 (1975)
14. Olivetti, N., Pozzato, G.L., Schwind, C.: A sequent calculus and a theorem prover for standard conditional logics. ACM Trans. Comput. Logic 8 (2007)
15. Papadimitriou, C.: On the complexity of integer programming. J. ACM 28, 765–768 (1981)
16. Pattinson, D.: Coalgebraic modal logic: Soundness, completeness and decidability of local consequence. Theoret. Comput. Sci. 309, 177–193 (2003)
17. Pauly, M.: A modal logic for coalitional power in games. J. Log. Comput. 12, 149–166 (2002)
18. Schröder, L.: A finite model construction for coalgebraic modal logic. J. Log. Algebr. Prog. 73, 97–110 (2007)
19. Schröder, L., Pattinson, D.: PSPACE reasoning for rank-1 modal logics. In: Logic in Computer Science, LICS 2006, pp. 231–240. IEEE, Los Alamitos (2006)
20. Tobies, S.: PSPACE reasoning for graded modal logics. J. Log. Comput. 11, 85–106 (2001)