

# 10. Connectivity and Competition in Airline Networks

## A Study of Lufthansa's Network

Aura Reggiani<sup>1</sup>, Peter Nijkamp<sup>2</sup> and Alessandro Cento<sup>3</sup>

<sup>1</sup>Department of Economics, University of Bologna, Italy, [aura.reggiani@unibo.it](mailto:aura.reggiani@unibo.it)

<sup>2</sup>Department of Spatial Economics, VU University Amsterdam, The Netherlands, [pnijkamp@feweb.vu.nl](mailto:pnijkamp@feweb.vu.nl)

<sup>3</sup>KLM Royal Dutch Airlines, Milan, Italy, [alessandro.cento@klm.com](mailto:alessandro.cento@klm.com)

### Abstract

Air transport networks have exhibited a trend towards complex dynamics in recent years. Using Lufthansa's networks as an example, this paper aims to illustrate the relevance of various network indicators – such as connectivity and concentration – for the empirical analysis of airline network configurations. The results highlight the actual strategic choices made by Lufthansa for its own network, as well in combination with its partners in Star Alliance.

### Towards Connected and Competitive Airline Networks

The airline industry has moved from a patchwork of individual and protected companies to a liberalized system of globally interconnected corporate organizations (see Martin & Voltes-Dorta, 2008 and Nijkamp, 2008). The aviation sector has traditionally been a publicly controlled industry, with a high degree of government intervention, for both strategic and economic reasons. Already in 1919, the Paris Convention stipulated that states have sovereign rights in the airspace above their territory. Consequently, a series of bilateral agreements was established between countries that the airlines wished to fly over. The Chicago Convention (1944) made a distinction between various forms of freedom for using the airspace, ranging from the 1st freedom (the right to fly over the territory of a contracting state without landing) to the 8th freedom (the right to transport passengers and cargo within another state between the airports in that state). The airline sector ultimately became an overregulated – and thus inefficiently operating – industrial sector in the post-war period all.

The US Airline Deregulation Act (1978) set the tone for a clear market orientation of the aviation sector in the USA, where US-based airlines were allowed to autonomously determine their routes, destinations, frequencies and airfares on their domestic flights, while new firms that were fit, willing and able to properly perform air transportation were free to enter the market. The resulting competition led to a rise in efficiency and innovative strategies in the airline industry and resulted in lower airfares, the entry of many new companies, and a significant increase in demand.

The airline deregulation in Europe has taken a much slower pace, due to the heterogeneity among European countries, the diversity of air traffic control systems and nationalistic motives for promoting a national carrier. Since the year 1988, Europe has gradually introduced a series of steps (so-called packages) to ensure a full deregulation of the European airline sector by the end of the last century, based on an integrated airline market characterized by fair competition and sound economic growth.

The next step in this deregulation process has been the Open Skies Agreement between the USA and Europe, which has opened up many more opportunities for carriers on both sides of the Atlantic to increase their financial viability and their market shares in a free competition across the Atlantic.

The changes in regulatory regimes in the European airline sector have prompted various new actions and strategies of European carriers in the past decade, such as mergers, take-overs and alliances. But the fierce competition has also led to bankruptcy of several existing carriers (such as Swissair and Sabena). More competition in a free market in Europe has largely had the same effects as in the USA, except for the fact that flag carriers still kept a large share of the market. But there are striking similarities in developments, in particular:

- A trend towards the development of hub-and-spokes networks of the existing major airlines in Europe (though less pronounced than in the USA, because of the greater diversity in Europe);
- The trend towards advanced computer reservation systems and electronic booking systems, in order to reduce transaction costs;
- The emerge of a wide variety of – often less transparent – airfare systems, which can even fluctuate daily, depending on demand and capacity (yield management systems);
- The growth in loyalty programmes in order to create bonds with various groups of frequent-flyer passengers;
- The development of various forms of airline alliances, not only within Europe, but also worldwide (such as Sky Team and Star Alliance), allowing also for efficient forms of code-sharing among participating companies as well;
- The emergence of low cost carriers which have taken a significant market share in the European aviation industry, next to charter companies, based on an aggressive pricing policy.

The above mentioned trends are largely similar to those in the USA, but there are a few marked differences:

- Europe is still strongly influenced by nationally oriented carriers (although flag carriers are rapidly losing their influence);
- Most European flights are international, but cover only relatively small distances, so that a competition with the railway system (especially the fast trains) is also emerging;
- The European air traffic control system is still made up of a patchwork of various systems, and this hampers an efficient management of the air control system in Europe;
- The charter market in Europe is well-developed, and has become a serious competitor to the scheduled airline sector (in contrast to the USA);
- Airports in Europe are often still largely in the hands of national or regional governments or authorities, and, as a consequence, their operation often does not meet the highest efficiency standards.

It is clear that the European airline sector has witnessed rapid changes and challenges in recent years, in particular (1) disruptions caused by external conditions (for example, September 11 2001, the Iraq war, the SARS virus), (2) the emergence of low cost carriers (LCCs) with a rapidly rising market share, and (3) the need to comply with environmental standards. Nevertheless, there has been a general trend towards more competition, more passengers, more mergers, more entries of new firms, a decline in airfares, and more variability in forces in most markets.

In Europe, we currently observe – as a result of the deregulation packages – three airline business models: (1) full-service carriers (offering a variety of services and network linkages); (2) LCCs (offering a limited number of services on specific segments of the network (for example, regional airports) at low prices; (3) charter companies (offering various services to specific holiday destinations). The changing scene in competition in response to the deregulation has prompted a variety of network strategies (ranging from hub-and-spoke systems to point-to-point systems) and yield management practices (for example, through market segmentation, product differentiation, booking classes, price setting and distribution channels). Various alliances have also occurred, but less mergers, to strike a balance between scale advantages and national identity/visibility.

Among the above recent developments, it should be noted that one of the most striking facts in Europe has been the rapid emergence of LCCs (for example, Ryanair, easy Jet). Despite the relatively low fares, most LCCs manage to be profitable and to conquer a significant part of the (rising) passenger demand. In most cases, they offer elementary services and fly uniform – but often modern – aircraft. A major challenge for the near future will be the question whether – and to which extent – LCCs will be able to benefit from the Open Skies Agreement on transatlantic routes.

In conclusion, deregulation policy has had a deep impact on the airline industry in Europe, in terms of airfares, number of passengers, market coverage and product

variability. A new major question will now be how the sector will respond to tighter environmental policy constraints (for example, noise, CO<sub>2</sub> emission). This will be decisive for the future of the aviation industry in Europe.

The above described for field has had far-reaching implications for the network strategies of airline companies. In the present paper we will investigate the structure and evolution of the airline network of Lufthansa, both individually and in association with its international partners (for example, Star Alliance). The paper is organized as follows. After this introduction on airline networks from an organizational and policy viewpoint, section "Network Analysis" will illustrate the principal elements of network analysis useful to characterise our case study, that is, four Lufthansa networks, by focussing on the critical indicators concerning the network topology, viz. concentration and connectivity. These indicators will then be applied to the four Lufthansa's network configurations under analysis, and subsequently employed in a final experiment (carried out by means of multicriteria analysis) aiming to classify these four network configurations according to the above indicators/criteria (section "Application to Airline Networks: the case of Lufthansa"). The final section "Retrospect and Prospect" will offer some concluding logical reflections, in the light of future policy and research strategies.

## Network Analysis

Boolean algebra in combination with digital information form the constituents of network analysis, as exemplified for instance by traditional graph theory. Network analysis has become an established tool in, for example, operations research, telecommunication systems analysis and transportation science, while in more recent years it has also become an important analytical tool in industrial organization, sociology, social psychology, and economics and business administration (Barthélemy, 2003; Gorman & Kulkarni, 2004; Gorman, 2005; Schintler, Gorman, Reggiani, Patuelli, & Nijkamp, 2005; Schintler, Gorman, Reggiani, Patuelli, Gillespie, et al., 2005; Reggiani & Nijkamp, 2006; Patuelli, 2007). Air transport is a prominent example of modern network constellations and will be addressed in this paper from a connectivity perspective. Air transport shows indeed clear network features, which impact on the way single airline carriers operate (Button & Stough, 2000). The abundant scientific literature on airline networks has addressed this topic in terms of theoretical modelling and empirical measurements on different typologies of airline network configurations.

In this context, interesting research has emerged that mainly addressed the issue of describing and classifying networks by means of geographical concentration indices of traffic or flight frequency (Caves, Christensen, & Tretheway, 1984; Toh & Higgins, 1985; McShan, 1986; Reynolds-Feighan, 1994, 1998, 2001; Bowen, 2002; Lijesen, 2004; Cento, 2006). These measures, such as the Gini concentration index or the Theil index, provide a proper measure of frequency or traffic concentration of the main airports in a simple, well-organized network. However, if a

real-world network structure is complex, including multi-hub or mixed point-to-point and hub-spokes connections, the concentration indices may record high values for all types of structure, but fail to clearly discriminate between different network shapes (Alderighi, Cento, Nijkamp, & Rietveld, 2007). There is a need for a more appropriate measurement of connectivity structures in complex networks.

Starting from the above considerations and research challenges, the present paper aims to investigate the scientific potential and applicability of a series of network connectivity/concentration indices, in order to properly typify and map out complex airline network configurations. The application of an analysis will address Lufthansa's network, both European and World-Wide, while making a distinction between Lufthansa as an individual firm and Lufthansa in combination with Star Alliance.

Modelling complex networks is also a great challenge: on the one side, the topology of the network is governing the complex connectivity dynamics (see, for instance, Barabási & Oltvai, 2004); on the other side, the functional-economic relationships in such networks might also depend on the type of connectivity structure. The understanding of these two interlinked network aspects may be instrumental for capturing and analysing airline network patterns.

In the last decades network theory has gained scientific interest and sophisticated network models have been used in different fields, including economics and geography (Waters, 2006). This trend faced also quite some difficulty, because existing models were not able to clearly describe the network properties of many real-world systems, whose complexity could not fully be understood (Barabási & Albert, 1999).

Spatial-economics systems – including air transport networks – are complex, because agents interact, obtaining significant benefits by means of a joint activity (Boschma, 2005). This interacting process may become a permanent feature thus leading to a new meso- or macro structure, for example, to the creation of clusters.

Air transport systems have over the past years been experiencing such clustering processes. An example is provided by airlines' alliances.<sup>1</sup> The main reason why airline carriers cooperate or aggregate stems from cost reductions they can thus obtain. Being a member of an alliance impacts on the carriers' strategy for a long time and also influences the network configuration they adopt. It is worth noteworthy that alliances play also an important role in determining market dynamics; in 2005, the three main alliances in air transport accounted for 80 per cent of the total capacity offer.<sup>2</sup> Therefore, we need to develop airline network models that can adequately take into account clustering and merger processes.

A further important trend many real networks show is the so-called 'Small-World (SW) effect'. This term indicates that the diameter<sup>3</sup> of a network is so short that it takes only a few movements along links in order to move between any two nodes of a network (Reggiani & Vinciguerra, 2007). In air transport systems, we

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<sup>1</sup> The processes underlying the creation of an alliance can be clearly depicted by considering the integration of Lufthansa and Swiss, described in the Lufthansa Annual Report (2005); available on the website [http://konzern.lufthansa.com/en/html/ueber\\_uns/swiss/index.html](http://konzern.lufthansa.com/en/html/ueber_uns/swiss/index.html).

<sup>2</sup> See <http://www.tourismfuturesintl.com/special%20reports/alliances.html>.

<sup>3</sup> The concept of diameter is defined in Table 10.1.

can point out the SW effect by taking into consideration and comparing the network configuration of single carriers or of alliances; such systems exhibit a clear SW effect when it takes only a small number of flights to link the two most distant airports in the network.

Alongside the SW effect, the SW network model has been developed in order to take into account both the SW effect and the related clustering processes (Watts & Strogatz, 1998). The main features of this model are a short diameter and a high clustering coefficient.

A further elaboration of the SW model is the so called Scale-Free (SF) network introduced by Barabási and Albert (1999) in order to incorporate two mechanisms upon which many real networks have proven to be based: *growth* and *preferential attachment*. The former points to the dynamic character of networks, which grow by the addition of new nodes and new vertices; the latter explains how new nodes enter the network, namely by connecting themselves to the nodes having the highest number of links.

An important feature of SF networks is represented by their vertex degree distribution<sup>4</sup>  $P(k)$  which is proportional to  $k^{-\gamma}$  (with  $k$  being the number of links), that is, to a power law. The value of the degree exponent  $\gamma$  depends on the attributes of the single systems and is crucial to detect the exact network topology, in particular the existence of the hubs (highly connected nodes). As Barabási and Oltvai (2004) highlight, a SF network embeds the proper hub-and-spoke model only when  $\gamma = 2$ , while for  $2 < \gamma \leq 3$  a hierarchy of hubs emerge. For  $\gamma > 3$ , the hub features are absent and the SF network behaves like a random one.

In air transport systems, we can point out SW networks by considering fullservice carriers. Without national or political impediments in a free market, these carriers typically organize their network into a hub-and-spoke system, where one or a few central airports called 'hubs' have a high number of links to the other airports called 'spokes'. Passengers travelling from a place of origin to a place of destination have to stop typically in one or a few hubs to change aircraft. Hubs are organised in order to allow flight connectivity by coordinating the scheduled timetable of the arriving and departing flights. Investigating the airline strategy in designing hub connectivity and timetable coordination has been the aim of several empirical network studies. Some examples of theoretical and empirical investigation of hub connectivity can be found in the works of Bootsma (1997), Dennis (1998), Rietveld and Brons (2001), Veldhuis and Kroes (2002), and Burghouwt and de Wit (2003). As a consequence, the hub has to manage normally a high volume of traffic at the same time, due to their central connecting role in the network.

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<sup>4</sup>  $P(k)$  is the probability that a chosen node has exactly  $k$  links (Barabási and Oltvai 2004). See also Equation (10.1).

In contrast to SF networks, we have to highlight also random networks (Erdős & Rényi, 1959), which display homogeneous, sparse patterns, without cluster characters. Their vertex degree distribution follows a Poisson distribution.<sup>5</sup>

In air transport, random networks are useful to map point-to-point connections, as it is the case for low-cost airlines (Cento, 2006). In the ideal point-to-point network all airports are connected to each other, so that passengers can fly from one airport to any other directly without stopping in any hub to change aircrafts. These networks have a low diameter, as a consequence of the high number of direct links between airports. Reggiani and Vinciguerra (2007: 148) point out that a random network can be seen as ‘a homogeneous system which gives accessibility to the majority of the nodes in the same way’. Furthermore, as it is evident by looking at the plot of the exponential function, the probability to find highly connected nodes is equal to 0. Therefore, no clear hubs exist, and the network configuration appears to be random because no single airport displays a dominant role in a connected network.

The vertex degree distribution is one of the key tools we may use to point out the network configuration (Reggiani & Vinciguerra, 2007), since this function determines the way nodes are connected. It can be defined as the probability  $P(k)$  of finding nodes with  $k$  links. In general, we can state that:

$$P(k) = N(k) / N, \quad (10.1)$$

where  $N(k)$  is the number of nodes with  $k$  links and  $N$  is the number of nodes of the network.

With regard to the network topologies developed in the framework of graph theory, complex systems tend to show two main degree distributions: the *Poisson* distribution (Erdős & Rényi, 1959) and the *power-law* function (Barabási & Bonabeau, 2003). The former is defined as:

$$P(k) \sim e^{-\langle k \rangle} \frac{\langle k \rangle^k}{k!}, \quad (10.2)$$

and describes networks – so-called random networks – where the majority of nodes have approximately the same number of links, close to the average  $\langle k \rangle$  (Barabási & Albert, 1999). Equation (10.2) is a distinctive feature of point-to-point networks, such as those adopted by low-cost airlines; this network topology is typical of equilibrated economic-geographical areas, where a high number of direct links can be profitably operated.

The power-law function is defined as:

$$P(k) \sim k^{-\gamma} \quad (10.3)$$

and characterizes networks having a small number of nodes with a very high degree while the majority of nodes have a few links. Equation (10.3) has impor-

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<sup>5</sup> For a review of random models, SW models and SF models, see Albert and Barabási (2002) and Jeong (2003).

tant economic implications: it characterizes SF networks, where the term SF refers to the fact that ‘*the power-law distribution does not change its form no matter what scale is used to observe it*’ (Reggiani & Vinciguerra, 2007: 150), and that, in these networks, distances are irrelevant. Therefore, we expect to find SF networks in ‘global networks’, such as the Internet and air transport, and in general in those networks where relevant economic aggregation clusters (preferential attachments) attract flows from distant nodes.

Networks can be analyzed from the perspective of their geometry and their concentration. Various relevant indices are included in Tables 10.1 and 10.2, respectively.

**Table 10.1** Network’s topology indices

Index or measurement	Description	Formulation	Variables	Source
Degree	The degree of a node is given by the number of its links	$k(v)$	$k(v)$ is the number of links of node $v$	Barabási and Oltvai (2004)
Closeness	It indicates a node’s proximity to the other nodes	$C(v) = \frac{1}{\sum_{t \in V} d_{vt}}$	$d_{vt}$ is the shortest path (geodesic distance) between nodes $v$ and $t$ ; $n$ is the number of nodes in the network	Newman (2003)
Betweenness	It indicates a node’s ability to stand between the others, and therefore, to control the flows among them	$B(v) = \sum_{s \neq t \in V} \frac{\sigma_{st}(v)}{\sigma_{st}}$	$\sigma_{st}(v)$ and $\sigma_{st}$ are, respectively, the number of geodesic distances between $s$ and $t$ that pass through node $v$ , and the overall number of geodesic distances between nodes $s$ and $t$	Freeman (1977)
Diameter	It measures the maximum value of the geodesic distances between all nodes	$D = \max_{s, t \in V, s \neq t} d_{st}$	$d_{st}$ is the geodesic distance between nodes $s$ and $t$	Boccaletti, Latora, Moreno, Chavez, and Hwang (2006)
Clustering coefficient	It measures the cliquishness of a node	$Cl(v) = \frac{l_v}{\max l_v}$	$l_v$ and $\max l_v$ are, respectively, the number of existing and maximum possible links between the nodes directly connected to node $v$ (its neighbours)	Watts and Strogatz (1998)



**Table 10.2** Network’s concentration indices

Indicator	Formula	Use	Variables used	Sources
Gini concentration index	$G = \frac{\sum_{i=1}^n \sum_{j=1}^n  x_i - x_j }{2n^2 \mu}$	It is a measure of geographical concentration	$x_i, x_j$ are the number of weekly flights from airports $i$ and $j$ , ranked in increasing order; $n$ is the number of airports in the network; $\mu$ is $\sum_i x_i / n$	Cento (2006)
Freeman centrality index	$F_B = \frac{\sum_i [F_B(x^*) - F_B(x_i)]}{n^3 - 4n^2 + 5n - 2}$	It is a measure of similarity to a perfect star network	$F_B(x_i) = \sum \sum b_{jk}(x_i)$ is the $j < k < k$ betweenness centrality of node $x_i$ ; $F_B(x^*)$ is the highest betweenness centrality value of the distribution	Cento (2006)
Entropy function	$E = -\sum_{ij} p_{ij} \ln p_{ij}$	It measures the degree of spatial organization and variety in a system	$p_{ij}$ is the probability of a link between nodes $i$ and $j$	Nijkamp and Reggiani (1992); Frenken and Nuvolari (2004)

All the indicators in Tables 10.1 and 10.2 will be utilized in the empirical analysis concerning the exploration of the Lufthansa network’s topology and concentration (See the following section).

## Application to Airline Networks: the Case of Lufthansa

### Introduction

We will now address the geographical analysis of Lufthansa’s aviation network in the year 2006. The airline network measurement is essential for exploring the airline behaviour and its implications for the supply, the traffic demand, the airports’ infrastructure and aviation planning. The airline network can be subdivided into domestic, international or intercontinental configurations depending on whether the airports connected are located within a country, a continent or in different continents. Furthermore, an airline network can be interconnected or interlined to partner’s networks within the alliance concerned. This classification is based on geographical, air transport-political and economic characteristics, such as airlines’ degree of freedom from the Chicago Convention (see Cento, 2006) market liberalization, or costs and traffic demand. Therefore, the overall network configuration is the result of the integrated optimisation of the domestic, international, and

intercontinental parts of the total network. These sub-network configurations may range from fully-connected or point-to-point to hub-and-spokes configurations to alliances (fully-contracted) or to a mix of these configurations. Within this conceptual framework, we will position our analysis of four sub-networks of Lufthansa. As summarized in Table 10.3, we coin networks A1 and A2, referring respectively to the flights operated by Lufthansa in Europe and in the whole world, while networks B1 and B2 take into consideration – respectively at a European and at a global level – the flights operated by all the carriers which are members of Star Alliance (to which Lufthansa belongs).<sup>6</sup>

**Table 10.3** Lufthansa's network constellation (2006)

Network	Area under consideration	Carrier or alliance operating the flight	Nodes	Total number of links
A1	Europe	Lufthansa	111	522
A2	World	Lufthansa	188	692
B1	Europe	Star Alliance	111	3,230
B2	World	Star Alliance	188	6,084

The variable under analysis is represented by the number of direct connections of each airport in the summer season of the year 2006, measured on a weekly basis. In all four cases we only consider those airports where Lufthansa operates with its fleet and not by partner's airlines. When we consider A1 and A2 networks, we clearly see that the majority of Lufthansa's flights are operated at a continental level. On the contrary, nearly half of Star Alliance's flights are operated outside Europe. This finding is not surprising, if we consider that the carriers making up Star Alliance are mainly from non-European countries.

### *Network Geometry*

In order to examine the nodes' location, we have computed the three centrality measures (degree, closeness and betweenness) described in Table 10.1. Concerning the investigation of the nodes' relations, we have examined the diameter and the clustering coefficient of the network (see again Table 10.1).

<sup>6</sup> The Star Alliance member carriers are currently: Air Canada; Air New Zealand; ANA; Asiana Airlines; Austrian; bmi; LOT Polish Airlines; Lufthansa; Scandinavian Airlines; Singapore Airlines; South African Airlines; Spanair; Swiss; TAP Portugal; THAI; United Airlines; US Airways; VARIG (the list was retrieved from [www.staralliance.com](http://www.staralliance.com)).

The degree of a node (Table 10.1) can be seen as a measure of centrality if we assume – in the framework of our analysis – that the best connected airports have a greater power over the whole network, as they can control a considerable amount of all flights. In all networks we find that the airports of Frankfurt and Munich have always the highest degree (see Table 10.8 in Appendix A).

A further analysis of nodes' centrality focuses on their 'ease-of-access' to the other nodes.<sup>7</sup> In order to investigate this concept we have computed the closeness centrality<sup>8</sup> (Table 10.1). The values of this index for the networks under consideration (listed in Table 10.9 in Appendix A) show that the highest values usually correspond to the best connected nodes; therefore, closeness centrality is able to map out – in the framework of our study – the most important airports in terms of connectivity. A similar trend can be observed by considering betweenness centrality (Table 10.1; the values for networks A1, A2, B1 and B2 are listed in Table 10.10 in Appendix A). This finding is not surprising, since hubs – in the framework of the hub-and-spoke model – are chosen from those airports falling among the highest possible number of pairs of other airports (O'Kelly & Miller, 1994; Button & Stough, 2000).

The networks' topology can also be explored by examining how the various nodes relate and link, since this last attribute impacts the configuration of the whole structure. For this purpose we have computed the clustering coefficient (defined in Table 10.1; the ten highest values for the nodes of the four networks of our experiments are listed in Table 10.11 in Appendix A). The values indicate a significant difference between the networks A1 and A2 and the networks B1 and B2; in the former case the airports of Frankfurt and Munich dominate the chart; in the latter case, other airports appear to emerge, thus showing that flights are spread more equally on the whole network.

In addition, we will also consider the diameter of the above networks in order to investigate how the links' patterns influence the ability to move inside the network. Both A1 and A2 have a diameter of 4, while B1 and B2 have a diameter of 2. This can be justified only if there is no significant difference in the geographical configuration between A1 and A2, approximately a hub-and-spoke, while B1 and B2 can be a mixture of hub-and-spoke and point-to-point networks. In other words, the integration of Lufthansa network in the Star Alliance reduces the travel distance, as the passengers can benefit from more connections and thus shorter paths to travel between the origin and the destination. This has important implications in the context of our study, because it entails that Lufthansa's networks shrink, when we consider the flights of all Star Alliance members.

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<sup>7</sup> It can be assumed that access to the network is easier when nodes are closer (Freeman 1979).

<sup>8</sup> We compute the closeness centrality, as well as the subsequent betweenness centrality, using the Pajek software (<http://vlado.fmf.uni-lj.si/pub/networks/pajek/>).

## *Network Concentration*

The study of the networks' degree of concentration – which is carried out in the present subsection – is crucial in order to detect the exact network topology, because the hub-and-spoke model is highly concentrated, while point-to-point networks do not show this feature.

First, Table 10.4 presents the normalized Gini index (see Table 10.1) for the four networks under consideration. Both Star Alliance networks are less concentrated than the Lufthansa counterparts, meaning that when we enlarge the measurement to a broader network including intercontinental destinations and partners' networks, the configuration will probably evolve into a mix of multi hub-and-spoke and point-to-point structures. In particular, network A2 appears to be the most concentrated.

The information provided by the Gini index refers to the degree of concentration existing in a network, without any evidence on how this concentration impacts on the network topology. For this last purpose the Freeman centrality index (Table 10.1) has been computed. Its normalized values are represented in Table 10.4. This index assumes the value 1 for a hub-and-spoke network, and the value 0 for a point-to-point network (Cento, 2006).

**Table 10.4** Concentration indices

Network	Gini index	Freeman index	Entropy
A1	0.762	0.504	5.954
A2	0.813	0.757	6.194
B1	0.524	0.059	7.790
B2	0.699	0.056	8.389

According to the Freeman index, again networks A1 and A2 turn out to be the most concentrated ones. In particular, A2 network seems to be again the closest to the hub-and-spoke model; we may suppose that this network is characterized by a strong hierarchy among nodes.

Finally, concerning the last concentration index, that is, entropy (Table 10.1), Table 10.4 shows the related values for the networks A1, A2, B1 and B2. The results show that the entropy values are higher when we consider those flights operated by Lufthansa's partners (networks B1 and B2). A likely explanation for this increase is given by the process of construction of these networks, obtained by the addition of flights to the nodes of A1 and A2, respectively. Both B1 and B2 are therefore the 'sum' of the networks implemented by the different carriers that are members of Star Alliance, and hence they are not the result of a specific strategy, as is the case for A1 and A2. Clearly, the above values indicate that A1 and A2 networks are more concentrated and less dispersed than the B1 and B2 networks; more specifically, A1 appears to be the most concentrated network.

In conclusion, from the above three indicators, networks A1 and A2 appear to be the most concentrated. However, among these two networks, A2 seems the most concentrated with respect to two indicators (Gini and Freeman), while A1 seems the most concentrated with respect to the entropy index.

## Network Configuration

### Degree Distribution of the Lufthansa Networks

The vertex degree distribution function is important in order to detect the most plausible network configuration. In this section, we will explore whether the variable ‘number of weekly connections’ is rank-distributed – over A1, A2, B1 and B2 – according to either an exponential or a power function. The  $R^2$  values and the  $b$  coefficients of the two interpolating functions (exponential and power) concerning the four ranked distributions (in log terms) are listed in Table 10.5. The plots of both functions for the four networks under consideration are displayed in Appendix B (Figs. 10.1 and 10.2).

**Table 10.5** Exponential and power fitting of rank distributions

	Network → A1		A2		B1		B2	
	$R^2$	$b$	$R^2$	$b$	$R^2$	$b$	$R^2$	$b$
Distribution function ↓								
Power	0.95	0.99	0.93	0.82	0.75	0.67	0.70	0.65
Exponential	0.75	0.03	0.67	0.01	0.66	0.02	0.48	0.01

Both Table 10.5 and Figs. 10.1 and 10.2 (in Appendix B) highlight that our data sets better fit a power function, as the higher  $R^2$  values indicate. It is worth noting that the  $b$  coefficient of the power function for the networks A1, A2, B1 and B2 is respectively equal to 0.99, 0.82, 0.67 and 0.65. If we carry out a transformation<sup>9</sup> of these coefficients, we observe that the A1 network displays a power-law exponent equal to 2, thus indicating a stronger tendency to a hub-and-spoke system according to Barabási and Oltvai (2004), while the other three networks A2, B1 and B2 display a power-law exponent between 2 and 3, thus indicating a tendency to a hierarchy of hub/agglomeration patterns.

A further issue concerns the fitting of the exponential function. Also in this case we obtain high  $R^2$  values, although inferior to the ones emerging in the power

<sup>9</sup> Adamic (2000) shows that the power-law exponent  $\gamma$  (emerging from the nodes’ probability distribution (Equation (3))) is related to the power function coefficient  $b$  (emerging from the distribution relating the degree of the nodes to their rank (rank size rule) (see Figs. B1 and B2 in Annex B) as follows:  $\gamma = 1 + (1/b)$ .

case; however, the coefficient of the exponential function is always very low, ranging from 0.01 to 0.03 (Table 10.5). Therefore, if we look at the  $R^2$  indicators, all networks under consideration appear to be in a 'border-line' situation (that is, an ambiguity between a power and exponential fitting). Nevertheless, if we look at the coefficient values, the four networks seem to show a tendency toward an agglomeration structure of SF type, expressed by a clear power-law vertex degree distribution, with the degree exponent  $\gamma$  equal to 2 (network A1), or varying between 2 and 3 (networks A2, B1, B2).

A further consideration concerns the plots of networks B1 and B2 (Fig. 102 in Appendix B). We can clearly see that both identify a power function with a cut-off. Thus, if we eliminate – in both networks B1 and B2 – those nodes which have less than 10 links, we slightly improve the fitting of their power function, obtaining for networks B1 and B2 respectively  $R^2$  values of 0.84 and 0.75, but still lower than the  $R^2$  values regarding A1 and A2.

In conclusion, from our estimation results, the networks A1, A2 appear to show the strongest characteristics of concentration and preferential attachment. In particular, network A1 appears to be the closest to the hub-and-spoke model, from the perspective of Barabási and Oltvai's approach. Given these preliminary results, it is worth to examine these configurations, jointly with some indicators of network concentration and topology previously implemented. Consequently, a multidimensional method, such as Multicriteria Analysis (MCA), taking into account – by means of an integrative approach – all adopted indicators and related results, was next carried out and utilized for further analysis.

### **Classification of the Lufthansa Networks by means of Multicriteria Analysis**

A multidimensional assessment approach, such as MCA, will now be applied<sup>10</sup> to the four Lufthansa networks in order to identify the 'best' system, according to the network indicators previously calculated.

Consequently, the alternatives are the four networks A1, A2, B1, B2 under consideration, while the criteria have been grouped according to three macro-criteria: network concentration, topology and connectivity (Table 10.6). It should be noted that, concerning the geometric criteria, we have considered the diameter and the clustering coefficient, since these two indices provide the network geometry's features. In particular, concerning the latter, the average clustering coefficient has been adopted (Barabási & Oltvai, 2004).

The first group of macro-criteria is related to the networks' concentration. It should be noted that in our MCA procedure, the entropy indicator needs to be transformed positively because the real values of the entropy function increase when networks are more heterogeneous, that is, less concentrated. The second group of macro-criteria refers to the networks' physical measurement. Here, the diameter needs to be converted in utility, because its value is higher when

<sup>10</sup> Here the Regime method and software has been used (Hinloopen & Nijkamp, 1990).

networks are less centralized. The third group of macro-criteria is related to connectivity. This property is investigated through the interpolation of the ranked degree distributions, where – in the power function – the highest exponent of 0.99 implies a value of the exponent degree<sup>11</sup> – in the associated power-law distribution – close to 2 (perfect hub-and-spoke). The *R*<sup>2</sup> and the coefficient of the exponential function need to be converted to utility, since both values indicate random and homogeneous patterns.

**Table 10.6** Alternatives and criteria

Alternatives	A1 (Lufthansa, Europe) A2 (Lufthansa, World) B1 (Star Alliance, Europe) B2 (Star Alliance, World)
‘Concentration’ criteria	Gini index Freeman index Entropy
‘Topology’ criteria	Diameter Average Clustering Coefficient
‘Connectivity’ criteria	<i>R</i> <sup>2</sup> of the fitted power function (ranked degree distribution) Coefficient of the power function <i>R</i> <sup>2</sup> of the fitted exponential function (ranked degree distribution) Coefficient of the exponential function

We have carried out five scenarios by considering: (a) all the criteria mentioned above; (b) each macro-criteria separately; (c) concentration and topology criteria together. In each scenario an equal weight, that is, unknown priority, has been given to the single criteria. The results are listed in Table 10.7.

**Table 10.7** Findings of multi-criteria analyses

Criteria considered	All criteria combined	Concentration criteria	Topology criteria	Connectivity criteria	Concentration and topology criteria
Hierarchy of the alternatives	A1	A2	B1	A1	A1
	A2	A1	B2	B1	B1
	B2	B2	A1	A2	A2
	B1	B1	A2	B2	B2

These findings point out that network A1 prevails, however with two exceptions. The former is represented by network A2, which is the top-scorer when we consider the criteria related to the networks’ concentration/geography: this finding comes from the higher centralization and concentration degree of network A2, as

<sup>11</sup> See Footnote 9.

demonstrated by the Freeman and Gini indices. The latter exception is represented by network B1, which prevails when we consider the criteria related to the physical measurement of networks.

It turns out that the Lufthansa network A1 is the most connected one; we can conjecture that A1 is close to a hub-and-spoke system, according to the values expressed by its exponent degree in the power-law distribution (see Table 10.5). This result confirms the dual-hubs network strategy advocated by the German carrier (Lufthansa, 2005). Frankfurt and Munich act as central hubs, where all intercontinental flights depart and arrive in conjunction with the European and domestic flights. This timetable coordination is designed to allow passengers to transfer from one flight to another for different national and international destinations.

## Retrospect and Prospect

Network analysis turns out to be a powerful tool for analyzing the structure and evolution of transportation systems. Airline networks are fascinating examples of emerging complex and interacting structures, which may evolve in a competitive environment under liberalized market conditions. They may exhibit different configurations, especially if a given carrier has developed a flanking network framework together with partner airlines.

The present paper has investigated the network structure of four networks of Lufthansa by considering several indicators concerning the concentration, topology and connectivity (degree distribution) functions characteristics of this carrier. An integrated multidimensional approach, in particular multicriteria analysis has been adopted, in order to take into account all information obtained by the above indices, and thus extrapolate the most 'appropriate' network, according to these indicators.

The related results point out that all the four Lufthansa networks can be properly mapped into the SF model of the Barabási type. In particular, network A1 can be formally identified as a hub-and-spoke structure. In general, we can conjecture a 'tendency' towards a hubs' hierarchy or hub-and-spoke configuration in Lufthansa's European network (network A1), as also witnessed by the emergence of various nodes (Frankfurt, Munich and Dusseldorf) which are organized as hubs in the framework of Lufthansa's activities. All in all the four networks exhibit a hierarchical structure mainly dominated by German airports.

The results obtained thus far highlight various characteristic features of complex aviation networks, but need to be complemented with additional investigations, in particular, on the structure and driving forces of the demand side (types of customers, in particular). Furthermore, the market is decisive in a liberalized airline system, and hence also price responses of customers as well as competitive responses of main competitors would need to be studied in the future.



From a methodological viewpoint a refined weighted network analysis – taking into account the strength of each connecting link – might offer better insights into the topological structure of the airline network at hand (see, for example, Barrat, Barthélemy, Pastor-Satorras, & Vespignani, 2004).

Another, and perhaps more interesting type of new research on network topologies might be to identify the existence of ‘structural holes’, which refers to the strategic importance of a relationship of nonredundancy between two contacts or nodes (see Burt, 1992). Such analyses are particularly important to map out the individual gains or losses of being connected to other parts of a complex network. It is thus clear that modern network analysis offers a wealth of new and important research challenges to the scientific community.

## Acknowledgments

The present chapter is a revised version of a previous paper “Network Measures in Civil Air Transport: A Case Study of Lufthansa”, published in *‘Networks, Topology and Dynamics’*, Springer Series Lecture Notes in Economics and Mathematical Systems, 2008, Vol. 613 (A.K. Naimzada, S. Stefani, A. Torriero, eds). The authors wish to thank Sara Signoretti (University of Bologna) for her assistance concerning the empirical application, as well as Roberto Patuelli (Institute for Economic Research, University of Lugano) for his comments on the present chapter, as well as for his cooperation in the editing process.

## References

- Adamic, L. A. (2000). Zipf, power-laws, and Pareto – A ranking tutorial. <http://www.hpl.hp.com>. Accessed 16 April 2007.
- Albert, R., & Barabási, A. -L. (2002). Statistical mechanics of complex networks. *Reviews of Modern Physics*, 74, 47–97.
- Alderighi, M., Cento, A., Nijkamp, P., & Rietveld, P. (2007). Assessment of new hub-and-spoke and point-to-point airline network configurations. *Transportation Reviews*, 27, 529–549.
- Barabási, L. A., & Albert, R. (1999). Emerging of scaling in random networks. *Science*, 286, 509–512.
- Barabasi, L. A., & Bonabeau, E. (2003). Scale-free networks. *Scientific American*, 288, 60–69.
- Barabási, L. A., & Oltvai, Z. N. (2004). Network’s biology: understanding the cell’s functional organization. *Nature Reviews Genetics*, 5, 101–113.
- Barrat, A., Barthélemy, M., Pastor-Satorras, R., & Vespignani, A. (2004). The architecture of complex weighted networks. *Proceedings of the National Academy of Science of the United States of America*, 8 March. <http://www.pnas.org/cgi/doi/10.1073/pnas.0400087101>.
- Barthélemy, M. (2003). Crossover from scale-free to spatial networks. *Europhysics Letters*, 63, 915–921.
- Boccaletti, S., Latora, V., Moreno, Y., Chavez, M., & Hwang, D. -U. (2006). Complex networks: structure and dynamics. *Physics Reports*, 424, 175–308.
- Boschma, R. A. (2005). Proximity and innovation. A critical assessment. *Regional Studies*, 39, 61–74.
- Bootsma, P. D. (1997). *Airline flight schedule development; analysis and design tools for European hinterland hubs*. Utrecht: University of Twente.

- Bowen, J. (2002). Network change, deregulation, and access in the global airline industry. *Economic Geography* 78, 425–439.
- Burghouwt, G., & de Wit, J. (2003). *The temporal configuration of European airline networks*. University of Montreal, Publication AJD-74.
- Burt, R. S. (1992). *Structural holes*. Cambridge, MA: Harvard University Press.
- Button, K., & Stough, R. (2000). *Air transport networks: Theory and policy implications*. Cheltenham: Edward Elgar.
- Caves, D. W., Christensen, L. R., & Tretheway, M. W. (1984). Economics of density versus economies of scale: why trunks and local service airline costs differ. *The Rand Journal of Economics* 15, 471–489.
- Cento, A. (2006). Challenge to the airline industry: emergence of carriers and low cost carriers. PhD Thesis, VU University Amsterdam.
- Dennis, N. P. S. (1998). Competition between hub airports in Europe and a methodology for forecasting connecting traffic. *8th World Conference on Transport Research*, Antwerp.
- Erdős, P., & Rényi, A. (1959). On random graphs I. *Publications Mathématiques*, 6, 290–297.
- Freeman, L. C. (1977). A set of measures of centrality based on betweenness. *Sociometry*, 40, 35–41.
- Freeman, L. C. (1979). Centrality in social networks: a conceptual clarification. *Social Networks*, 1, 215–239.
- Frenken, K., & Nuvolari, A. (2004). The early development of the steam engine: an evolutionary interpretation using complexity theory. *Industrial and Corporate Change*, 13, 419–450.
- Gorman, S. P. (2005). *Networks, security and complexity*. Cheltenham, Edward Elgar.
- Gorman, S.P., & Kulkarni, R. (2004). Spatial small worlds. *Environment and Planning B* 31, 273–296.
- Hinloopen, E., & Nijkamp, P. (1990). Qualitative multiple criteria choice analysis: the dominant regime method. *Quality and Quantity*, 24, 37–56.
- Jeong, H. (2003). Complex scale-free networks. *Physics A*, 321, 226–237.
- Lijesen, M.G. (2004). Adjusting the Herfindahl index for close substitutes: an application to pricing in civil aviation. *Transportation Research E* 40, 123–134.
- Lufthansa (2005). *Annual report*. [http://konzern.lufthansa.com/en/html/ueber\\_uns/swiss/index.html](http://konzern.lufthansa.com/en/html/ueber_uns/swiss/index.html) (2005)
- Martin, J. C., & Voltes-Dorta, A. (2008). Theoretical evidence of exciting pitfalls in measuring hubbing practices in airline networks. *Networks and Spatial Economics*, 9, 161–182.
- McShan, W. S. (1986). An economic analysis of hub-and-spoke routing strategy in the airline industry. PhD Thesis, Northwestern University.
- Newman, M. E. J. (2003). A measure of betweenness centrality based on random walks. arXiv:condmat/0309045 (2003)
- Nijkamp, P. (2008). Policy developments in the airline industry. In: D. De Jong, B. Kaashoek, & W. J. Zondag (eds.) *Blue Skies or Storm Clouds: Essays on Public Policy and Air Transport*. Scienceguide, The Hague, pp. 20–25.
- Nijkamp, P., & Reggiani, A. (1992). *Interaction, evolution and chaos in space*. Berlin: Springer.
- O' Kelly, E. M., & Miller, H. J. (1994) The hub network design problem: an overview and synthesis. *Journal of Transport Geography*, 2, 31–40.
- Patuelli, R. (2007). *Regional labour markets in Germany: statistical analysis of spatio-temporal disparities and network structures*. PhD Thesis, VU University Amsterdam, September.
- Reggiani, A., & Nijkamp, P. (eds.). (2006). *Spatial dynamics, networks and modelling*. Cheltenham: Edward Elgar.
- Reggiani, A., & Vinciguerra, S. (2007). Network connectivity models: an overview and empirical applications. In: T. Friesz (ed.), *Network Science, Nonlinear Science and Infrastructure Systems*. New York: Springer, 147–165.

- Reynolds-Feighan, A. J. (1994). EC and US air freight markets: network organisation in a deregulated environment. *Transportation Reviews*, 14, 193–217.
- Reynolds-Feighan, A. J. (1998). *The impact of US airline deregulation on airport traffic patterns*. *Geographical Analysis*, 30, 234–253.
- Reynolds-Feighan, A. J. (2001). Traffic distribution in low-cost and full service carrier network in the US air transport market. *Journal of Air Transport Management*, 7, 265–275.
- Rietveld, P., & Brons, M. (2001). Quality of hub-and-spoke networks; the effects of timetable coordination on waiting time and rescheduling time. *Journal of Air Transport Management*, 7, 241–249.
- Schintler, L. A., Gorman, S. P., Reggiani, A., Patuelli, R., & Nijkamp, P. (2005). Small-world phenomena in communication networks: a cross-Atlantic comparison. In A. Reggiani, L. A. Schintler (eds.), *Methods and models in transport and telecommunications: cross Atlantic perspectives*. Berlin: Springer, 201–220.
- Schintler, L. A., Gorman, S. P., Reggiani, A., Patuelli, R., Gillespie, A., Nijkamp, P., & Rutherford, J. (2005). Complex network phenomena in telecommunication systems. *Networks and Spatial Economics*, 5, 351–370.
- Toh, R. S., & Higgins, R. G. (1985). The impact of hub-and-spoke network centralization and route monopoly on domestic airline profitability. *Transportation Journal*, 24, 16–27.
- Veldhuis, J., & Kroes, E. (2002). Dynamics in relative network performance of the main European hub airports. *European Transport Conference*, Cambridge.
- Waters, N. (2006). Network and nodal indices. Measures of complexity and redundancy: a review. In: A. Reggiani, P. Nijkamp (eds.), *Spatial dynamics, Networks and Modelling*. Cheltenham: Edward Elgar, 16–33.
- Watts, D. J., & Strogatz, S. H. (1998) Collective dynamics of small world networks. *Nature*, 393, 440–442.

## Appendix A Top-Ten Airports

In this Appendix, we will present the top ten scores of the airports – according to the main topological indices illustrated in Table 10.1 – belonging to the four airline networks A1, A2, B1 and B2 (see Tables 10.8–10.12).

**Table 10.8** Top-ten scores of airports according to the degree index (corresponding values in brackets)

A1	A2	B1	B2
MUC (82)	FRA (138)	FRA (106)	FRA (183)
FRA (81)	MUC (100)	MUC (105)	MUC (179)
DUS (39)	DUS (41)	BRE (97)	HAM (172)
HAM (24)	HAM (24)	HAM (97)	DUS (171)
STR (18)	STR (18)	BSL (94)	STR (168)
TXL (10)	TXL (10)	DUS (94)	LEJ (166)
CDG (8)	CDG (8)	LEJ (92)	ZRH (165)
NUE (8)	NUE (8)	NUE (92)	TXL (164)
BRU (7)	BRU (7)	STR (92)	NUE (163)
LHR (6)	MXP (6)	CGN (89)	BRE (162)

**Table 10.9** Top-ten scores of airports according to the closeness index (corresponding values in brackets)

A1	A2	B1	B2
MUC (0.78)	FRA (0.79)	FRA (0.96)	BRE (1)
FRA (0.76)	MUC (0.64)	MUC (0.95)	DUS (1)
DUS (0.60)	DUS (0.53)	HAM (0.89)	ZRH (1)
HAM (0.55)	HAM (0.51)	DUS (0.87)	FRA (0.98)
STR (0.54)	STR (0.50)	NUE (0.86)	MUC (0.95)
TXL (0.51)	CDG (0.49)	STR (0.86)	HAM (0.93)
CDG (0.51)	NUE (0.49)	LEJ (0.85)	STR (0.91)
NUE (0.51)	BRU (0.48)	CGN (0.84)	LEJ (0.89)
LHR (0.51)	LHR (0.48)	TXL (0.84)	NUE (0.89)
MXP (0.51)	MXP (0.48)	ZRH (0.84)	FMO (0.85)
	VIE (0.48)		

**Table 10.10** Top-ten scores of airports according to the betweenness index (corresponding values in brackets)

A1	A2	B1	B2
MUC (0.51)	FRA (0.76)	MUC (0.06)	MUC (0.06)
FRA (0.50)	MUC (0.03)	FRA (0.06)	FRA (0.06)
DUS (0.06)	DUS (0.03)	DUS (0.05)	DUS (0.06)
KUF (0.05)	BKK (0.02)	HAM (0.05)	BRE (0.05)
HAM (0.03)	KUF (0.02)	STR (0.05)	CGN (0.05)
GOJ (0.02)	HAM (0.01)	BRE (0.04)	HAM (0.05)
STR (0.01)	CAI (0.01)	HAJ (0.04)	NUE (0.05)
CDG ( $4.5e^{-4}$ )	CAN (0.01)	NUE (0.04)	STR (0.05)
CGN ( $9.5e^{-5}$ )	GOJ (0.01)	TXL (0.04)	ZRH (0.05)
BRU ( $1.9e^{-5}$ )	GRU (0.01)	CGN (0.04)	CGN (0.05)
	JED (0.01)		DRS (0.05)
	KRT (0.01)		LEJ (0.05)
	LOS (0.01)		
	PHC (0.01)		

**Table 10.11** Top-ten scores of airports according to the clustering coefficient (corresponding values in brackets)

A1	A2	B1	B2
MUC (0.82)	FRA (0.75)	FRA (0.96)	BRE (1)
FRA (0.80)	MUC (0.48)	MUC (0.89)	DUS (1)
DUS (0.24)	DUS (0.11)	LEJ (0.77)	ZRH (1)
HAM (0.10)	HAM (0.04)	ZRH (0.67)	FRA (0.96)
STR (0.06)	STR (0.02)	BSL (0.66)	MUC (0.88)
CDG (0.01)	TXL (6e-3)	STR (0.57)	LEJ (0.84)
TXL (0.01)	CDG (5e-6)	DUS (0.55)	BSL (0.81)
NUE (9e-3)	NUE (4e-3)	HAM (0.55)	GVA (0.67)
BRU (6e-3)	BRU (2e-3)	GVA (0.48)	HAM (0.63)
MXP (4e-4)	ZRH (2e-3)	TXL (0.47)	STR (0.60)
VIE (4e-4)			

**Table 10.12.** Nomenclature of airports under study

BKK	Bangkok	JED	Jedda
BRE	Bremen	KRT	Khartoum
BRU	Bruxelles	KUF	Samara
BSL	Basel	LEJ	Leipzig
CDG	Paris Charles de Gaulle	LHR	London-Heathrow
CGN	Koln	LOS	Laos
DRS	Dresden	MUC	Munich
DUS	Dusseldorf	MXP	Milano-Malpensa
FMO	Munster	NUE	Nuremberg
FRA	Frankfurt	PHC	Port Harcour
GOJ	Novgorod	STR	Stuttgart
GRU	Sao Paulo	TXL	Berlin-Tegel
GVA	Geneva	VIE	Wien
HAM	Hamburg	ZRH	Zurich

## Appendix B Rank Distributions

In this appendix, we will present the rank distribution fitting for the networks A1, A2, B1 and B2, with reference to the following variables:  $y$ -axis = number of weekly connections;  $x$ -axis = airport (node) rank. The related fitting has been carried out by considering both an exponential and a power interpolation (see Table 10.5 for the synthesis of the results) (see Figs. 10.1 and 10.2).

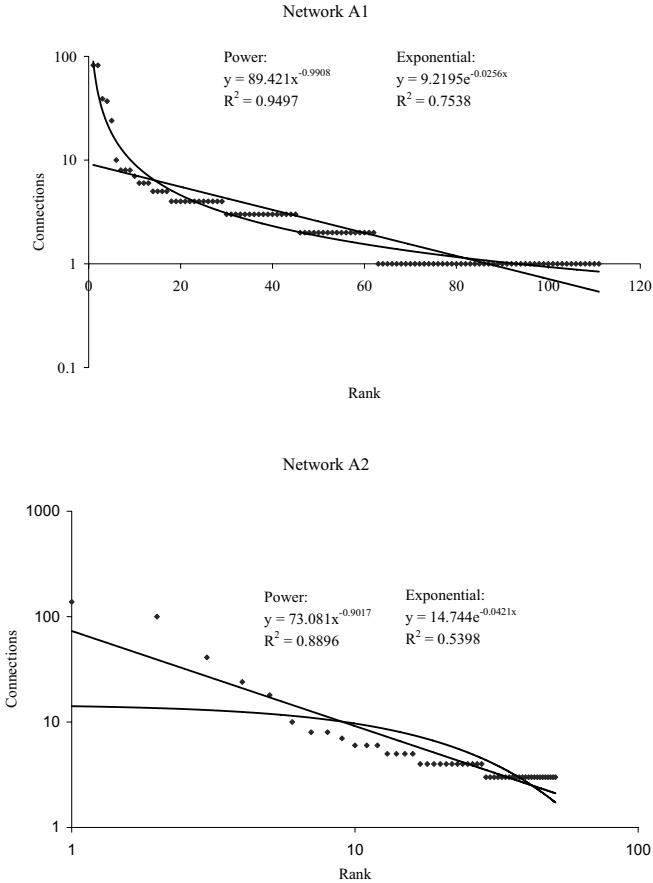
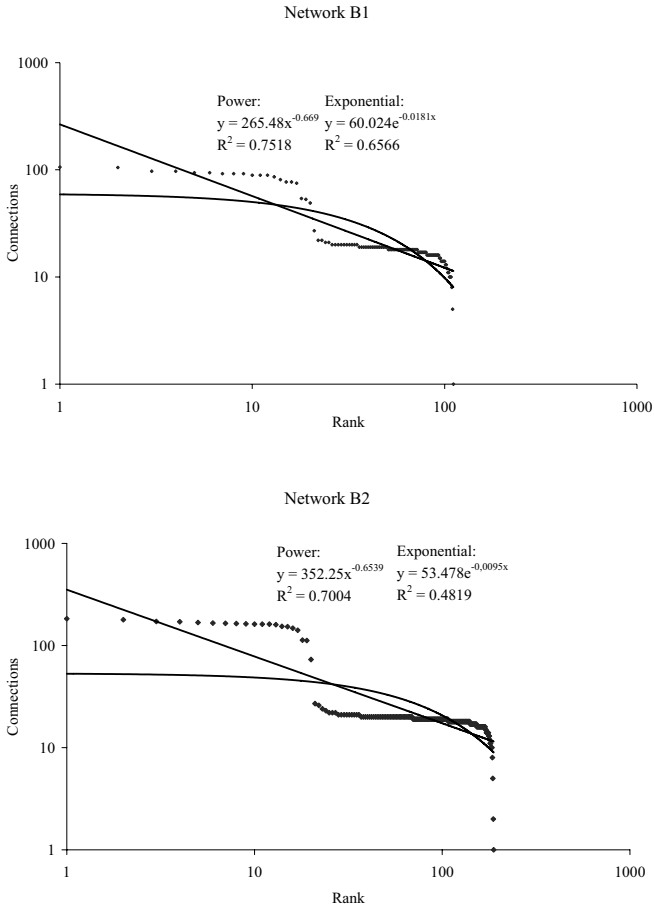


Fig. 10.1 Rank distribution fitting for networks A1 and A2



**Fig. 10.2** Rank distribution fitting for networks B1 and B2