Complementarity in Bistable Perception

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1 Introduction

The idea of complementarity already appears in William James' (1890a, p. 206) *Principles of Psychology* in the chapter on "the relations of minds to other things". Later, in 1927, Niels Bohr introduced complementarity as a fundamental concept in quantum mechanics. It refers to properties (observables) that a system cannot have simultaneously, and which cannot be simultaneously measured with arbitrarily high accuracy. Yet, in the context of classical physics they would both be needed for an exhaustive description of the system.

In contrast to the concept of a "complement" in mathematics, which refers to the negation of a proposition,[4] complementarity refers to properties that are not simply negations of each other. A nice example is mentioned by James (1890b, p. 284): "The true opposites of belief ... are doubt and inquiry, not disbelief." Disbelief would be the complement of belief in the Boolean sense, while doubt and inquiry are concepts that are complementary to belief. Another pertinent example for complementarity may be "learning" and "knowing" in data processing systems. In addition to James and Bohr, Wolfgang Pauli was one of those scientists who always thought that the idea of complementarity is significant far beyond the objectively measurable realms of physics.

In quantum mechanics, complementarity is mostly used in the context of observables such as "momentum" and "position" which are, technically speaking, non-commuting observables. Although complementarity soon became an important ingredient in the so-called Copenhagen interpretation of quantum theory, there exists no rigorous and unique mathematical definition of complementarity which all scientists agree upon. There are many definitions which all

[4] For instance, the complement of a set in Boolean set theory consist of those elements which are not elements of that set.

emphasize the non-commutativity of the corresponding mathematical objects but which differ with respect to more restrictive conditions.

Several years ago, Atmanspacher *et al.* (2002, 2006) formulated a mathematical framework for dealing with observations and measurements, which generalizes the framework used in quantum (and classical) mechanics so that applications in psychology and cognitive science become possible. The Necker-Zeno model for bistable perception, which will be explained in this contribution, marks a first success in this direction.

Note that it is not the aim of the generalized quantum theory to explain mental or cognitive phenomena in terms of quantum *physics*. The idea is rather to use elements of the *mathematical* framework of quantum theory (in particular those elements which appear when an observation of a system changes its state) and apply them to non-quantum (i.e. classical) physical systems, and eventually even to non-physical systems.

Concerning terminology, we will sometimes use the terms “classical” and “non-classical” in the following sense: The behavior of a system is called classical if an observation of the system has no (or, at least, negligible) influence on the state of the observed system. In this limit observations commute and we obtain a behavior that is typical for systems in classical physics. In those cases, however, for which an observation of a system has an unavoidable effect on the state of the observed system, we may encounter non-commutative observations and thus non-classical behavior. This is the domain for which the generalized quantum theory is intended. There may be different levels of non-classical behavior, from simple examples for non-commutative observables up to non-classical behavior manfesting itself in the violation of Bell’s inequalities.

Bistable perception is a particularly suited scenario for applying the generalized formalism of quantum theory. In a first approximation, one has to distinguish only two different mental states corresponding to the two different representations of an ambiguous stimulus (such as, e.g., the Necker cube). Simple assumptions about the state dynamics between representations lead to the Necker-Zeno model proposed by Atmanspacher *et al.* (2004) and refined by Atmanspacher *et al.* (2008). This model not only accounts for the feature that switches of the representation cannot be avoided. It also predicts a quantitative relation between three different cognitive time scales: the time scale at which the sequence of perceived stimuli becomes undecidable, the time scale at which perceptions of stimuli become consciously accessible, and the time scale at which mental states in bistable perception switch.

This contribution reviews the basic ideas of the generalized quantum theory and its application to the bistable perception of ambiguous stimuli, the so-called Necker-Zeno model. In the following Sec. 2 we will describe the phenomenon of bistable perception and some relevant experimental data. Before we then introduce the Necker-Zeno model in Sec. 4, we will briefly sketch the main ideas of the generalized quantum theory according to Atmanspacher *et al.* (2002, 2006). Finally, in Sec. 5, we will speculate about a new idea to use so-called temporal Bell inequalities (see, e.g., Leggett and Garg, 1985; Mahler, 1994) as a test for non-classical behavior (in the sense indicated above) in mental systems.

2 Bistable Perception

The bistable perception of ambiguous stimuli such as the Necker cube is a well-known phenomenon in cognitive science (Kruse and Stadler, 1995; Long and Toppino, 2004). It refers to the effect that the mental state of subjects perceiving an ambiguous stimulus, e.g. an image which can be interpreted in two (or more) different ways, switches spontaneously between the two (or more) possible perceptions, often perspectivally different. The time between two successive shifts, i.e. the inverse *reversal rate*, will be called *dwell time.* Key predictions of the Necker-Zeno model refer to the functional dependence of the dwell time on experimentally controllable parameters.

The perception of ambiguous stimuli shares many features with another scenario called *binocular rivalry*, where two different unambiguous stimuli are offered each to one eye of the observer (Blake and Logothetis 2001). However, there are also important differences, in particular with respect to the issue of voluntary control over the reversal rate (Meng and Tong, 2004). In this contribution we we will not address binocular rivalry and restrict ourselves to the perception of ambiguous stimuli.

A simple and often used example for an ambiguous stimulus leading to bistable perception is the Necker cube (Fig. 1, left), a projection of the edges of a three-dimensional cube onto a plane. There are two ways to give this drawing a three-dimensional interpretation: either the front side is lower left or it is upper right (Fig. 1, right).

In experimental studies of Necker-cube perception, subjects are asked to direct their view onto a fixation cross in the center of the image and report, e.g. by pressing a button, whenever they perceive a "switch" of the perspective. Fig. 2 shows a typical switching behavior between states 1 and 2 as a function of time with dwell times in the range between 1 and 5 seconds. Typical dwell time distributions (over many trials) are similar to gamma distributions (Brascamp *et al.*, 2005; Atmanspacher *et al.*, 2008) with a mean dwell time T.

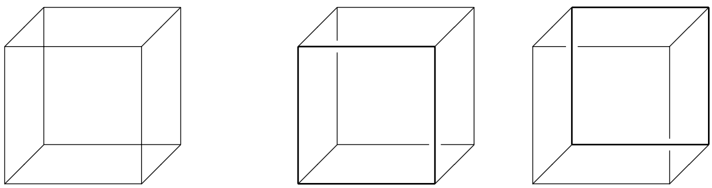

Fig. 1. The Necker cube (left) and the two ways how it can be interpreted (right).

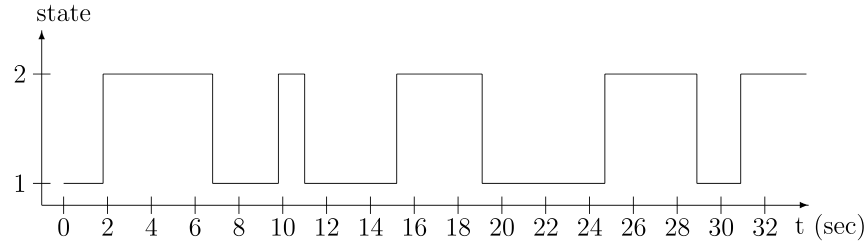

Fig. 2. Schematic representation of the bistable switching between states 1 and 2 as a function of time t.

It should be noted that inter-individual variations of dwell times can exceed intra-individual variations, sometimes by far. Different from usually found values of $T \approx 3$ seconds, Carter *et al.* (2005) reported that particular types of meditation can lead to dwell times that are increased up to several hundreds or even thousands of seconds.

The Necker-Zeno model explains why the mental state of subjects cannot be kept in one of the two representations of the Necker cube for an arbitrarily long time (see Sec. 4.2). On this basis, it provides a relation between the mean dwell time $T = \langle t \rangle$ and other cognitive time scales. A recently refined version of the model also gives correct predictions of the shape of the dwell time distribution and the cumulative dwell time probability (see Sec. 4.3).

3 Complementarity in Generalized Quantum Theory

The Necker-Zeno model was developed in the context of a generalized quantum theory (Atmanspacher *et al.*, 2002, 2006) and its application in cognitive science. In this section, we give a brief summary of the framework of generalized quantum theory with particular emphasis on possible formalizations of the concept of complementarity.

The developement of quantum theory in the 1920s and 1930s made it obvious that the assumption of a non-intervening or non-invasive measurement is unsuitable for systems with only a few elementary degrees of freedom. The concept of "observation" has to include the experimental fact that any observation may have an intrinsic and unavoidable influence on the state of the observed system and its associated observables. While in classical physics observables are mathematically represented as *functions* on the space of states (the phase space of a system), in quantum physics observables are represented as *operators* acting on the space of states.

Despite the significant differences between classical theory and quantum theory it turned out that both theories fit into one general algebraic framework – observables form a C*-algebra. In this framework the key distinction between classical and quantum physics is the distinction between the commutativity (classical physics) and the non-commutativity (quantum physics) of the observables. In both cases, a state is a positive, normalized, linear functional on the algebra of observables, associating to each observable its expectation value in that state.

Instead of referring to the algebra of all observables, one sometimes uses the structure of the set of propositions – observables with only 0 or 1 as a possible outcome of a measurement. Classical propositions form what is called a distributive lattice (corresponding to commutative observables), while quantum propositions form a non-distributive lattice (corresponding to non-commutative observables).

Even though the algebraic framework is general enough to comprise both classical and quantum physics, it contains some quite restrictive postulates. For instance, it is assumed that for any two observables A and B, also the sum $A+B$ is defined to be an observable, even though there exists no operational rule to derive the experimental protocol for the measurement of $A+B$ from the protocols for measuring A and B separately.

The generalized quantum theory provides a scheme for a mathematical representation of observables which is applicable to any system "which has enough internal structure to be a possible object of a meaningful study" (Atmanspacher *et al.*, 2002) In this respect, systems of interest in psychology and cognitive science are a particular challenge.

The complete and detailed axiomatic set-up of the generalized quantum theory has been published elsewhere (Atmanspacher *et al.*, 2002, 2006). Here it is sufficient to sketch the main ingredients. The basic elements of the theory are a set of states $\{z\}$ and a set of observables $\{A\}$. Observables act on the set of states as mappings, i.e., they can change the states. The main axioms are:[5]

- Observables have a *spectrum*, which is the set of all possible results of a measurement or an observation. The nature of the possible results of measurements remains unspecified. In particular, it is not required that the results can be expressed in terms of real numbers, or that results can be added or multiplied.
- Observables can be multiplied, which is related to the fact that their measurement can be performed in sequential order, i.e. in temporal succession. It should be noted, however, that not even in quantum mechanics the temporal succession of two observations is represented by the product of the corresponding observables (expressed by the operator or matrix product of linear mappings). The relation is more subtle: In quantum theory we assume that for each measurement of an observable A with duration t_1 there exists a time evolution operator $U_A(t)$ which describes the time evolution of the system (including the measuring device) during the measurement

[5] Other axioms, like the existence of an identity observable, a zero observable, and the existence of a zero state, are relevant for the developement of the mathematical structure but unimportant for a conceptual discussion.

process. The later measurement of a second observable B with duration t_2 is represented by a different time evolution operator $U_B(t)$. The time evolution corresponding to the temporal succession of both processes is represented by $U_B(t_2)U_A(t_1)$. These two operators do not commute if the corresponding operators A and B do not commute. The multiplication of observables in generalized quantum theory refers to the operators $U_A(t)$, $U_B(t)$ rather than to the operators A, B representing the observables in quantum mechanics.

In this framework, a number of significant features of quantum mechanics are missing: There is not necessarily a Hilbert space of states, there is no *a priori* probability interpretation, there is no unitary Schrödinger evolution describing the time evolution of states, etc. However, despite the small set of axioms, the conditions required for the observables yield several options to define complementarity:

1. A most general definition of complementarity refers to the commutativity of observables: two observables A and B are said to be complementary if the corresponding mathematical representations of these observables do not commute, i.e., if the results of temporally successive measurements of these observables depend on their temporal order.
   This definition of complementarity is quite weak: It may happen that two observables do not commute on a few exceptional states but commute on the majority of states.
2. More restrictively, we may call two observables A and B complementary if they do not commute on any state z which is not the zero state:
$$ABz \neq BAz \quad \text{for all} \quad z \neq 0 \quad .$$
   In the framework of conventional quantum mechanics this implies that there are no states for which the two observables A and B assume definite values simultaneously. This corrolary is not necessarily equivalent with the definition according to generalized quantum theory and may, therefore, be considered as an alternative definition of complementarity.
3. A more restricted version of complementarity would require (apart from non-commutativity) that the observables A and B generate (by multiplication and any other additional operation which may be defined in special cases) the complete set of observables.
   For instance, this definition is satisfied if we think of position Q and momentum P for systems with only one degree of freedom in conventional quantum mechanics. It is not satisfied if there are several degrees of freedom, related to several particles and/or several dimensions and/or internal degrees of freedom. One can also give meaning to this definition of complementarity by generalizing the concept of complementarity from two observables A and B to two sets of observables $\{A_i\}$ and $\{B_i\}$, and requiring that both $\{A_i\}$ and $\{B_i\}$ commute among themselves and that A_i and B_i do not commute pairwise.

4. Finally, there are even more restrictive definitions of complementarity in quantum mechanics. The strongest definition of complementarity requires that the dispersion-free states[6] related to two observables A and B have a "maximal distance". While dispersion-free states can be defined in the context of generalized quantum theory as well, the concept of a "distance of states" needs more structure than provided by the framework of generalized quantum theory.

4 The Necker-Zeno Model

The Necker-Zeno model for bistable perception was first proposed by Atmanspacher *et al.* (2004). It is based on the same idea as the quantum Zeno effect introduced by Misra and Sudarshan (1977). Therefore we shall first describe the quantum Zeno effect in the form used for quantum systems proper. This does not mean that we want to hold a genuine quantum effect responsible for bistable perception. However, it will be shown that parts of the mathematical framework used for describing the quantum Zeno effect coincide (in the sense of generalized quantum theory) with the mathematical framework that is applicable to describe bistable perception.

4.1 The Quantum Zeno Effect

In a simple two-state model, the quantum Zeno effect can be described by the following ingredients:

1. Observations are represented by the operator
$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} .$$
Immediately after an observation the system will be in one of the two eigenstates
$$\psi_1 = |+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \text{or} \qquad \psi_2 = |-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} .$$
2. Without loss of generality, we may assume that the time evolution of the unperturbed system is generated by a Hamilton operator
$$H = g\sigma_1 = g\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} ,$$
where g is some coupling constant related to the velocity at which the states change. The corresponding unitary operator of time evolution is given by

[6] A state z is called dispersion-free with respect to an observable A if the possible results of measurements of A in systems prepared in state z are precisely identical.

$$U(t) \;=\; e^{iHt} \;=\; \begin{pmatrix} \cos(gt) & i\sin(gt) \\ i\sin(gt) & \cos(gt) \end{pmatrix} \quad .$$

Now, the unperturbed time evolution of the system (without observation) can be compared with the case where repeated observations are performed at time intervals ΔT. The probability that the unperturbed system in, say, state $|+\rangle$ at time $t = 0$ is still found in state $|+\rangle$ at time t is given by:

$$w(t) \;=\; |\langle + | U(t) | + \rangle|^2 \;=\; \cos^2(gt) \quad . \tag{1}$$

The time scale $t_0 = 1/g$ characterizes the "relaxation" or "decay" of the unperturbed system into an observation eigenstate.

Considering repeated observations after time intervals ΔT, the joint probability that the system is in state $|+\rangle$ at $t = 0$ and at all subsequent observations until time $t = N \cdot \Delta T$ is given by:

$$w_{\Delta T}(t) \;=\; [\cos(g\,\Delta T)]^{2N} \;=\; \exp\left(2N \ln[\cos(g\,\Delta T)]\right) \quad . \tag{2}$$

For $g\,\Delta T \ll 1$ or $\Delta T \ll t_0$ we may expand the cosine and the logarithm and obtain:

$$w_{\Delta T}(t) \;\approx\; \exp\left(-g^2 \Delta T^2 \cdot N\right) \;=\; \exp -(t/T) \quad . \tag{3}$$

The decay time t_0 in the unperturbed case is now replaced by a slower time scale T for the decay of the system:

$$T \;=\; (g^2 \Delta T)^{-1} \;=\; t_0^2/(\Delta T) \quad . \tag{4}$$

For $\Delta T \to 0$ we find $T \to \infty$, i.e., for continuous observations the mean time for a change of the system from state $|+\rangle$ to state $|-\rangle$ tends to infinity: The system becomes frozen in $|+\rangle$ under the influence of the observations. Figure 3 illustrates the three time scales of the quantum Zeno effect.

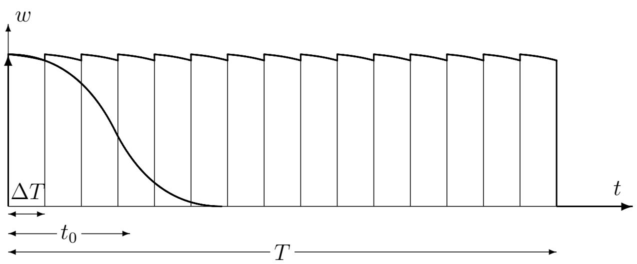

Fig. 3. Time scales of the quantum Zeno effect: ΔT is the time interval between successive observations of the system, t_0 is the time scale for the decay of the unperturbed (unobserved) system, and T is the mean decay time if the system is observed at time intervals ΔT.

4.2 The Necker-Zeno Model for Bistable Perception

The quantum Zeno effect can be related to the perception of ambiguous stimuli if the following correspondences are assumed:

1. The two states of the quantum Zeno effect correspond to the two possible representations of the ambiguous Necker cube (Fig. 1).
2. Without updates due to successive observations the mental representation "decays" with a probability which for small times t is given by

$$w(t) \;\approx\; 1 - g^2 t^2 \;+\; \mathrm{O}(t^4) \quad . \tag{5}$$

   This expression coincides in lowest non-trivial order with the corresponding probability in the quantum Zeno effect (Eq. 1). The higher-order terms are not needed in the derivation of the quantum Zeno effect, and the oscillatory behavior for large t is not to be expected for the dwell time in bistable perception anyhow.
3. The observation of the stimulus provides "updates" at time intervals ΔT. After each update the mental state corresponds to one of the two possible three-dimensional representations of the Necker cube.

In the resulting Necker-Zeno model, two types of processes can be considered as complementary: (i) the bistable perception dynamics (formalized by σ_1) tends towards a decay of the actualized mental state, while (ii) the successive updates (formalized by σ_3) stabilize this state in one of the two representations . Let us emphasize again that we do not require the decay or update dynamics to be a genuine quantum process. Nevertheless, generalized quantum theory allows us to speak of complementarity in a well-defined manner.

The calculations for the Necker-Zeno model are the same as for the quantum Zeno effect, but the interpretation of the time scales is different. The probability that the mental representation has not changed after a time $t = N \cdot \Delta T$ due to N successive updates of the mental state separated by ΔT is given by

$$w_{\Delta T}(t) \;=\; \exp(-t/T) \quad , \tag{6}$$

with a mean dwell time T that satisfies:

$$T \cdot \Delta T \;=\; t_0^2 \quad . \tag{7}$$

We associate the following cognitive time scales with the parameters of the model (for more details see Atmanspacher *et al.*, 2004):

- ΔT is an internal update time for the mental state during observation of the Necker cube. We interpret this quantity as the interval between two successive stimuli that is necessary for a correct assignment of the sequence of their presentation. This so-called order threshold (Pöppel, 1997) is of the order of $25 - 70\,\mathrm{ms}$. With smaller time intervals, the stimuli can still be distinguished but their sequence cannot be correctly determined.

- t_0 is the period of oscillations between the two states under the assumption that no updating observations take place. It is plausible to assume that the decay out of one state and the relaxation into the other occur on the same time scale. It can be related to the so-called P300 component in event-related potentials and is, thus, assumed to be of the order of 300 ms.
- T is the average time between successive switches of the mental state when the Necker cube is observed. It is usually characterized as roughly $T \approx 3\,\mathrm{s}$ and has large inter-individual differences (Brascamp *et al.*, 2005).

Relation (7) is clearly satisfied for these three cognitive time scales. Moreover, the predictive power of the model has been convincingly demonstrated with empirical results obtained under discontinuous stimulus presentation if it is possible to vary one of the time scales (t_0) as an independent variable and measure another one (T) as a function of t_0. Assuming that ΔT remains constant, Eq. (7) predicts a quadratic dependence for $T = T(t_0)$.

Under certain conditions, the time scale t_0 can be approximated by the off-time in discontinuous presentation, so it is indeed possible to test the model with experimental data. A comparison of observations by Kornmeier *et al.* (2007) with the predictions of the Necker-Zeno model is shown in Figure 4. The plotted symbols show observed values of T as a function of off-times. The solid curve represents a one-parameter fit of the Necker-Zeno prediction $T = t_0^2/\Delta T$ where $\Delta T \approx 70\,\mathrm{ms}$ gives the best results.

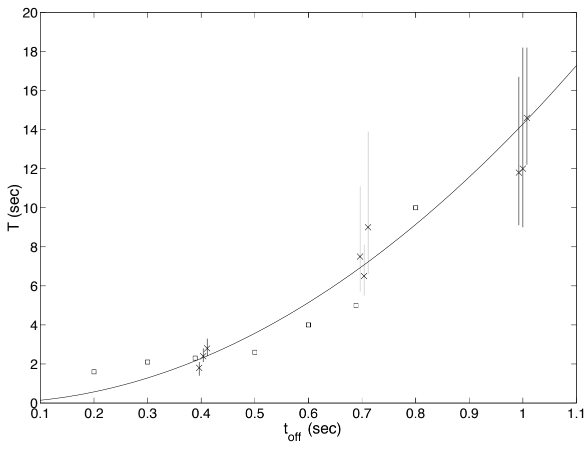

Fig. 4. Mean dwell time T as a function of the off-time t_0 in discontinuous presentation. The solid curve is a one-parameter fit (leading to $\Delta T = 70\,\mathrm{ms}$) from the prediction of the Necker-Zeno model under the assumption that the off-time can be identified with the decay time t_0 of the unperturbed system.

4.3 The Refined Necker-Zeno Model

While the Necker-Zeno model in the form just presented gives good predictions for the relation of the mean dwell time T to the other cognitive time scales ΔT and t_0, the predicted probability distribution of T differs significantly from experimental data. The cumulative probability that a change of the mental state has occurred up to time t is given by:

$$W(t) \;=\; 1 - w_{\Delta T}(t) \;=\; 1 - e^{-t/T} \quad , \quad T = t_0^2/\Delta T \quad . \tag{8}$$

From this we obtain the probability density $P(t)$ that a switch of the mental state occurs at time t:

$$P(t) \;=\; \frac{\mathrm{d}W(t)}{\mathrm{d}t} \;=\; \frac{e^{-t/T}}{T} \quad . \tag{9}$$

This probability density as well as the cumulative probability $W(t)$ are shown in Fig. 5.

A comparison with experimental data published by Brascamp *et al.* (2005) shows good agreement for times $t > 2\,\mathrm{s}$, but it reveals a completely different behavior for small t. The experimental probability density resembles a gamma distribution

$$P(t) \;\propto\; t^b\, e^{-\gamma t} \quad ,$$

with an exponent of $b \approx 5$, while the Necker-Zeno model according to Sec. 4.2 predicts a simple exponential decay. Similarly, the prediction for the cumulative probability from the Necker-Zeno model leads to linear behavior for small t while the experimental data rather exhibit power-law behavior with a large exponent.

Although the Necker-Zeno model in its original version was not intended to be valid for small values of t, it is nevertheless tempting to refine the model such that it provides the experimentally observed probability functions. The mathematical details of this refined version are published elsewhere (Atmanspacher *et al.*, 2008). We sketch the main results briefly.

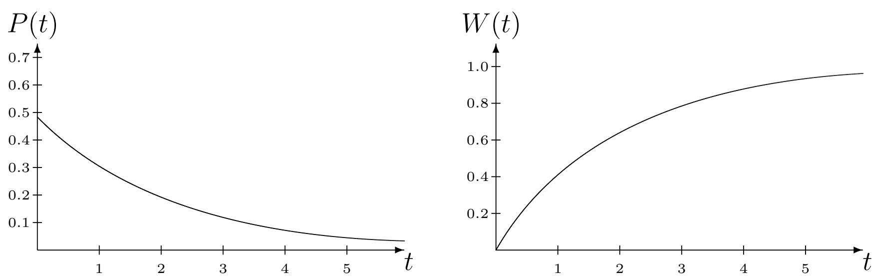

Fig. 5. The probability density $P(t)$ (left) for a switch of the mental state at time t and the cumulative probability $W(t)$ (right) that a switch occurred up to time t.

There are basically two possibilities to refine the original Necker-Zeno model for small values of t:

1. The parameter g, which determines the "decay velocity" of a mental state after the stimulus is turned off, is time dependent: $g \to f(t) \cdot g$.
2. The update intervals ΔT are time dependent: $\Delta T \to f(t) \cdot \Delta T$.

For large values of t the function $f(t)$ in both options approaches the constant value 1, such that the orignial Necker-Zeno model is recovered in this regime. For small values, however, $f(t)$ starts from zero with some power-law as shown in Fig. 6. Both a shorter update time for small t (case 2) as well as a slower decay for small t (case 1) can be interpreted as a form of increased attention. For more details concerning possible interpretations and applications see Atmanspacher *et al.* (2008) and Franck and Atmanspacher (2008).

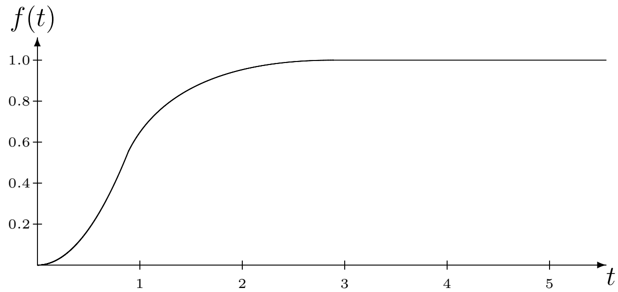

Fig. 6. The function $f(t)$ which determines the small-t behavior of either the parameter g or the update time ΔT in the refined Necker-Zeno model.

Despite the fact that in the probability distribution according to (2) only the product $g \cdot \Delta T$ enters, the physical interpretation of ΔT as a time interval leads to a different behavior of the probability distributions considered as a function of time. If for small values of t the function $f(t)$ starts with t^k, the behavior of $W(t)$ for small t is given by either t^k (for case 2, $\Delta T \to f(t) \cdot \Delta T$) or by t^{2k} (case 1, $g \to f(t) \cdot g$). Hence, the observed large values for the power law in the probabilities for small t are explained more naturally if g rather than ΔT is considered as time-dependent.

5 Temporal Bell Inequalities

In 1964, John Bell derived a set of inequalities which the expectation values of observables have to satisfy in any theory (i) that is local (i.e., any causal dependence respects the constraints given by Einstein's theory of relativity) and (ii) for which the results of a measurement are (at least in principle) already determined before the measurement is actually performed (Bell, 1966).

This second requirement was used by Einstein *et al.* (1935) in their famous EPR-argument as a definition of "elements of reality".

Experimental tests provided clear evidence that quantum mechanics violates Bell's inequalities (Aspect *et al.*, 1982a, 1982b). In particular, this implies that the statistical aspects of quantum mechanics cannot be explained by the introduction of local hidden variables. In general, it is assumed that in quantum mechanics the outcome of a measurement is not predetermined, even in principle, but rather the result of the measuring process itself.

Let Q_1, Q_2, Q_3, Q_4 be four different observables for which the result of a measurement can only assume one of the two values $+1$ or -1. In quantum mechanics, observables of this type are typically realized by measurements of polarizations of photons or by spin orientations of electrons. Let $E_{ij} = \langle Q_i Q_j \rangle$ be the expectation value for the (simultaneous) correlation function of Q_i and Q_j for the same system. One form of Bell's inequalities for this situation would be:

$$-2 \leq E_{12} + E_{23} + E_{34} - E_{41} \leq +2 \quad . \tag{10}$$

This inequality can be violated in quantum mechanics.

Bell's inequalities are expressed in terms of expectation values of two (or more) observables. The violation of Bell's inequalities in quantum mechanics involves the expectation values of (pairwise) non-commuting observables. Such observables cannot be measured simultaneously with arbitrarily high accuracy. In order to test the violation of Bell's inequalities in quantum mechanics, one makes use of particular correlations between two spatially separated systems which, however, are in an entangled state. Only under the assumption that "elements of reality" exist can one interpret the results as simultaneous correlation functions for one of the systems.

Obviously, the requirements for measuring a violation of Bell's inequalities as the key criterion for non-classical behavior are quite high. For a possible application to mental systems, the preparation of entangled states may be a particularly difficult problem. In addition, despite the fact that the quantum Zeno model provides the necessary non-commuting observables, the Necker-Zeno model for bistable perception has only one observable (σ_3) that serves to describe an "observation" of one of the two perspectives of the Necker cube. It is not clear if any of the other observables of the quantum Zeno model makes sense as an additional observable for the Necker-Zeno model. From this point of view it seems almost hopeless to test Bell's inequalities in the context of bistable perception.

But there may be an alternative option. In 1985, Leggett and Garg derived a set of inequalities which involve the expectation values of correlations of *one observable measured at different time instants* (Leggett and Garg, 1985). These so-called *temporal* Bell inequalities can be formulated in generalized quantum theory if the dynamics of a system does not commute with the observable (i.e., if the observable is not a constant of motion).

This is precisely the case for the Necker-Zeno model. Let $K(t_i, t_j) = \langle\sigma_3(t_i)\sigma_3(t_j)\rangle$ be the expectation value for a first "observation" of one of the two perspectives on the Necker cube at time t_j and a second one at time t_i. Then the following inequality should hold, if the mental state follows a "classical trajectory" (like the one shown in Fig. 2) with respect to the representation of the Necker cube:

$$|K(t_1,t_2) + K(t_2,t_3) + K(t_3,t_4) - K(t_1,t_4)| \leq 2 \quad . \tag{11}$$

If, on the other hand, during the periods of non-observation the mental state cannot always be described in terms of one of the two perspectives, this inequality can be violated. This might, for instance, be the case if the mental state is in a kind of superposition with respect to the two perspectives.[7]

For spin models, the largest violation of inequality (10) has been found to occur for measurements of the spin orientation along the angles $0°, 45°, 90°$ and $135°$. Similarly, one would expect the largest violation of temporal Bell inequalities in the framework of the Necker-Zeno model to occur at times t determined by the conditions $gt = 0, \pi/4, \pi/2, 3\pi/4$. For $t_0 = 300\,\mathrm{ms}$ this would correspond to measurements at $t = 0, 236, 471$ and $707\,\mathrm{ms}$. In order to avoid effects of observational updates, these values should be the off-times between brief stimulus presentations and observations.

This option to detect non-classical behavior in mental systems is as thrilling as challenging, but there is an important *caveat* to it. In the derivation of inequality (11) it is assumed that observations made on the same system do not influence each other. This is necessary for the determination of $K(t_i, t_j)$: The first measurement at t_j should have no effect on the result of the second measurement at t_i.[8] Such a requirement of "non-invasive measurements" might be difficult to realize for temporal Bell inequalities. On the other hand, it might be possible to estimate the degree of how much an observation of a mental state at time t_1 influences the observation of a mental state at time t_2. If this influence is smaller than the effect by which temporal Bell inequalities are violated, this could provide a terrific route toward evidence for non-classical behavior in mental systems.

[7] This gives an interesting twist to a question posed by Sudarshan (1983, p. 465): "Can we perceive a quantum system directly?" He speculates about a mode of awareness in which (p. 466) "sensations, feelings, and insights are not neatly categorized into chains of thoughts, nor is there a step-by-step development of a logical-legal argument-to-conclusion. Instead, patterns appear, interweave, coexist; and sequencing is made inoperative. Conclusion, premises, feelings, and insights coexist in a manner defying temporal order."

[8] This requirement corresponds to the locality requirement for the expectation values E_{ij} when the measurements are performed on different (but entangled) parts of a system: A measurement of Q_i at one part should have no influence on the measurement of Q_j at the other part.

6 Summary

The concept of complementarity has been defined in an axiomatic framework generalizing the quantum mechanical axioms for states and observables to systems involving invasive and thus, in general, non-commutative operations. In this framework, a novel approach to understand the bistable perception of ambiguous stimuli has been achieved, where the dynamics of the switch between different representations of a stimulus (e.g., the Necker cube) is complementary to the process of observation of these representations.

The corresponding Necker-Zeno model, referring to mental states and observables as well as their dynamics, is in agreement with experimental data for (1) the dwell time distributions (inverse reversal rates) in bistable perception and (2) the dependence of dwell times on off-times if stimuli are presented discontinuously. Finally, we have speculated about the possibility to formulate temporal Bell inequalities for this scenario. Their violation would imply evidence for fundamentally "non-classical" behavior in mental systems.

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