Psychological Research on Insight Problem Solving

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1 Introduction

"Albert! How did you find the theory of relativity ?" Max Wertheimer, the famous Gestalt psychologist, posed this question to his friend Albert Einstein in an attempt to understand the genesis of Einstein's groundbreaking scientific discovery (Wertheimer, 1959). Together they reconstructed the thinking processes underlying the discovery in several conversations and stumbled upon an ingenious thought experiment that Einstein had come up with. He considered it as the turning point where suddenly many open questions that had bothered his mind for a long time were easily and almost effortlessly resolved.

Imagine you are travelling in the middle of a moving train while two bolts simultaneously strike the front and the back of the train. Imagine further that there is an external observer at the embankment of the railway. Would you perceive the struck of the bolts as being simultaneous ? Would the observer ? Einstein recognized that the moving person and the observer would likely give different answers. This, in turn, led him to the crucial insight that physical measurements depend on particular frames of reference. Einstein's postulate about the "relativity of observations" had an extreme impact on the type of explanations that were conceivable in physics (Gruber, 1995; Knoblich and Öllinger, 2006).

Although many anecdotes describe flashes of insight as coming out of the blue, this was not the case for Einstein (and probably also not for other famous scientists who made important discoveries). Einstein had, of course, a profound knowledge in classical physics and mathematics. Nevertheless, he pondered for months and even years on the problems that led to the discovery of the theory of relativity. However, his expertise, shared with many other physicists at the time, was insufficient to find the right answers. The problem was not a lack of expertise or intellectual power. The problem was that the known theories and findings had to be seen, combined, structured, or integrated in a completely new way. Einstein's thought experiment allowed him to achieve this restructuring.

Of course, not everybody is a genius. Nevertheless, research in psychology has shown that insight is a general phenomenon that can also be observed in the average person. We get stuck with a problem we are actually competent to solve, but it seems unsolvable even if we try very hard, until at some point the solution appears out of the blue. Psychological research calls the processes that lead to such insights restructuring processes. In this contribution we will provide an overview of the cognitive and neural mechanisms enabling the restructuring of problems and the resulting insights. We start with potential definitions of the term "insight" and point out the problems with such definitions. We then sketch the Gestalt psychologists' view of productive thinking that initiated psychological research on insight at the beginning of the 20th century. Next we discuss cognitive psychologists' attempts to understand the "mysterious insight phenomenon" (Bowden *et al.*, 2005) using computational models of thinking. Finally, we give an overview of current perspectives discussed in cognitive science and cognitive neuroscience.

2 Definitions of Insight

So far there is no single definition of insight in psychological research that all researchers accept (Metcalfe and Wiebe, 1987; Weisberg, 1995). However, one can identify three different dimensions that different definitions of insight focus on: a *phenomenological dimension*, a *task dimension*, and a *process dimension.*

On a *phenomenological dimension* insight can be described as a sudden, unexpected, unintended, and surprising moment where a solution pops into someone's mind. The accompanying experience is often called "aha"-experience (Bowden and Jung-Beeman, 2003; Bowden *et al.*, 2005) and is in stark contrast to other types of problem solving where problems are solved stepwise and systematically through an exhausting and laborious process. The following description from Wegner (2002, pp. 81–82) illustrates the involuntary nature of insight:

> "The happiest inconsistency between intention and action occurs when a great idea pops into mind. The 'action' in this case is the occurrence of the idea, and our tendency to say 'Eureka!' or 'Aha!' is our usual acknowledgement that this particular insight was not something we were planning in advance. Although most of us are quite willing to take credit for our good ideas, it is still true that we do not experience them as voluntary."

Wegner's description gets at the core of the paradoxical character of insight problem solving. After several conscious, laborious, and voluntary solution attempts have repeatedly failed, an unintended and unexpected idea leads to the solution of a difficult problem.

Another approach to define insight is to identify particular tasks that provoke sudden solution ideas and to contrast them with another class of problems that are more likely to provoke stepwise solutions. The focus here is on the *task dimension*. Accordingly, researchers have tried to come up with a taxonomy of insight problems, and a variety of studies tried to identify the features that characterize insight problems and distinguish them from non-insight problems. So far there is no agreement about the criteria that clearly differentiate insight problems from non-insight problems (Weisberg and Alba, 1982; Metcalfe, 1986; Metcalfe and Wiebe, 1987; Weisberg, 1992; Weisberg, 1995; Bowden and Jung-Beeman, 2003; Chronicle *et al.*, 2004; Bowden *et al.*, 2005). One criterion we find useful is the ratio between problem difficulty and the size of the problem space (all logically possible problem states). Regular problems are easy when the problem space is small and difficult when the problem space is large. In contrast, insight problems are often very difficult, although the problem space is (very) small (Knoblich *et al.*, 1999; Öllinger *et al.*, 2006). However, from a logical point of view, task-based definitions of insight are problematic, because there is always the danger that the definition becomes circular: Insight problems are problems that require insight, and insight occurs when insight problems are solved (Dominowski and Dallob, 1995).

Therefore, now most researchers use a definition of insight that is linked to particular cognitive models of insight and focus on a *process dimension*. The core assumption here is that solving insight problems involves specific processes that are not involved in stepwise solutions of problems. One guiding assumption that has driven insight research for the past 20 years is that insight involves a number of processes that change the initial problem representation (Ohlsson, 1992; Dominowski and Dallob, 1995; Knoblich *et al.*, 1999; Knoblich *et al.*, 2001; Grant and Spivey, 2003; Jones, 2003; Kershaw and Ohlsson, 2004; Knoblich *et al.*, 2005; Knoblich and Öllinger, 2006; Öllinger *et al.*, 2006). In particular, it is assumed that problem solvers initially establish inadequate problem representations that make the solution of insight problems impossible. All solution attempts repeatedly fail and problem solvers hit an impasse. To overcome such impasses the solvers' problem representation needs to change, and this representational change can be brought about through a number of different processes. The changed problem representation enhances the space of possibilities to solve the problem.

Before we describe in more detail which specific processes for restructuring are discussed in modern research on insight we should address the roots of psychological research on insight problem solving in Gestalt psychology. Not only did the Gestalt psychologists coin the term restructuring that is still central in insight research. They were also the first to systematically study insight in the psychological laboratory.

3 Insight in Gestalt Psychology

Three facts are important for the understanding of the Gestalt psychology view on insight. First, visual perception was of central concern to the Gestalt psychologists and therefore they held the view that thinking is a lot like perception (Wertheimer, 1912; Wertheimer, 1923). Their research on visual perception resulted in the famous Gestalt laws, e.g., laws of proximity, similarity, and closure (Metzger, 1986; Ash, 1998). In the Gestalt view a Gestalt is "something else" (Koffka, 1935) than the mere representation of the given physical facts. This led to the famous *credo* that the whole is more than the sum of its parts. Despite Gestalt psychology's focus on perception it also had a lasting influence on other domains of psychology like social psychology or the psychology of thinking (Wertheimer, 1959).

Second, in the Gestalt psychologists' view thinking is by definition a kind of problem solving, characterized as goal-directed behavior, that clears out existing barriers preventing the solution to problems. The underlying metaphor is that a problem is considered as a disturbed Gestalt that "asks for" being transformed into a good Gestalt ("gute Gestalt"), *the solution*. The idea is that a disturbed Gestalt exerts a kind of driving force that pulls towards the good Gestalt. The mechanism that releases and bundles this force is called *restructuring*.

Third, the Gestalt psychologists thought of themselves as a counter movement to behaviorism. They distinguished between productive thinking (good thinking) and re-productive thinking (as a bad, blind, and mindless trial-and-error strategy) and claimed that even chimpanzees would be able to solve new problems with insight (Köhler, 1921).

As described above, restructuring was the Gestalt psychologists' core concept to address thinking. Figure 1a shows an example for a problem that, according to Wertheimer (1959), requires restructuring and allows us to explain

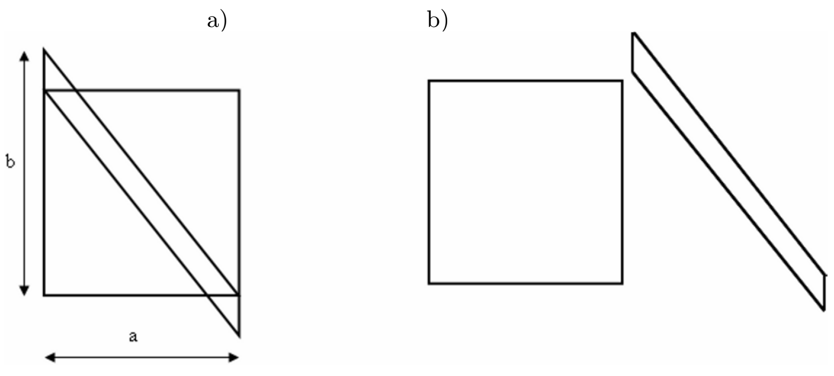

Fig. 1. The square-parallelogram problem: a) the given problem situation; b) the re-productive approach.

the difference between productive and re-productive thinking. The diagram depicts two geometrical figures, a square and a parallelogram superimposed on the square. The task is as follows: Given the lengths a and b of the two sections, what is the sum of the areas of the two figures? If you try to find the solution by yourself you may be rewarded with the feeling of a sudden insight (but it may take some time to find it).

Most people who try to solve this problem come up with more and more complex mathematical equations by systematically varying the two given dimensions a and b. This is what their prior knowledge about the calculation of the areas of squares and parallelograms suggests. The Gestalt psychologists characterize applying one's prior knowledge in this way as re-productive thinking (Figure 1b).

However, there is a more elegant and parsimonious solution that requires productive thinking. It is illustrated at the end of this contribution in Section 10. Here the problem situation is seen from a completely new perspective. The process of restructuring leads to a rearrangement of the problem constituents that results in a much better Gestalt than the original problem (see Figure 5). The lines of the two figures are perceptually re-combined so that two rectangular triangles result. These, in turn, can be seen as a rectangle and the resulting area $a \times b$ can be easily read off. In this problem the disturbed (bad) Gestalt is literally transformed into a good Gestalt through restructuring.

Driven by their opposition against behaviorism the Gestalt psychologists believed that prior knowledge impaired productive thinking rather than supporting it. Therefore, they tried to find ways to foster productive thinking, to find out why productive thinking is often very difficult, and why people tend to "blindly" apply their prior knowledge. Karl Duncker, a disciple of Max Wertheimer and Wolfgang Köhler, investigated *functional fixedness* as a critical component that is in the way of productive thinking. In his famous candle experiment Duncker (1945) asked participants to create a ledge on the wall to rest a candle on. The given material was a candle, a matchbox, and tacks. He found that problem solvers were fixated on the "container" function of the matchbox. As a consequence they had difficulties to perceive other potential functions of the box, e.g., "support for a candle". The correct solution to the problem is to light the candle, to fix the matchbox to the wall using the tacks, to put wax on the matchbox, and to fix the candle on the box – voila!

Duncker performed further variations of this experiment. He found that presenting an empty matchbox increased the solution rate, because now the container function of the matchbox was less emphasized. Duncker made the general claim that realizing the functional-value ("Funktionalwert") of an object is the initial event that triggers successful restructuring (Duncker, 1935). For example, reaching for something that is out of one's reach, e.g., a ball under the bed requires finding a "tool" that reduces the distance to the desirable object. The functional-value of the tool is "reducing distance". Assume that an umbrella is on top of the bed. According to Duncker, two things need to happen in order for a person to use the umbrella to get the ball: First, it has to be recognized that the umbrella is a long object satisfying the required functional-value, and second, the traditional function of the umbrella "shielding against rain" has to be overcome. Further experiments confirmed Duncker's assumption (e.g., Maier, 1931) and the concept of functional fixedness is still used in cognitive psychology today.

Luchins (1942) extended Duncker's finding by demonstrating that the repeated application of the same solution procedure can result in a mental set that keeps problem solvers from finding better solutions to routine problems. Luchins defined a mental set as a state of mind that is blind for alternative and possibly easier solutions. Luchins examined mental set effects using the now famous water jug problems (Luchins, 1942; Luchins and Luchins, 1959; Lovett and Anderson, 1996). For example, given three jugs A, B, and C, with volumes of 21, 127, and 3 units, respectively, the goal might be to fill an amount of 100 units into one of the jugs. The solution to this task is to pour water into B (127), then use the water in B to fill C twice, leaving 121 units in B. The final step is to fill A using the water in B, to leave 100 units in B.

In Luchins' famous experiments participants solved a set of about two to five problems that could all be solved with the same solution procedure, B – 2C – A. Then participants were presented with a test problem that could either be solved with this solution procedure or with a simpler procedure. For example, given the volumes 23, 49, and 3 in jugs A, B, and C, with the goal of attaining 20 units, the procedure B – 2C – A can be used, but a much simpler alternative is A – C (fill A, pour once into C, and 20 units are left in A). Luchins' experiments demonstrated that participants who had used the same solution procedure on multiple problems continued to use the more complicated solution. A control group that only solved the test problems almost always applied the easier procedure. Luchins concluded that the repeated application of the same procedure makes people blind to a better approach. Of course, this was an attack against the behaviorist conviction that practice makes perfect.

To summarize, the Gestalt psychologists distinguished re-productive and productive thinking, and they thought that productive thinking was the key to make humans smart. The process of restructuring was assumed to be the core of productive thinking. This process allows problem solvers to overcome hindrances to productive thinking such as functional fixation and mental set. We will see in the next sections that the Gestalt psychologists' ideas are still very important in current research addressing insight in problem solving.

4 Cognitive Theories of Insight Problem Solving

Since the early 1960s the computer has become the dominating metaphor in research on human problem solving. After Newell and Simon (1972) published a highly influential book that conceptualized problem solving as a search in a problem space, many researchers were fairly optimistic that it would be simply a matter of time until human thinking was entirely understood and could be implemented in computational models. However, although a lot of progress has been made, today researchers are less optimistic. One reason is that computational models work best for well-defined toy problems. The mysterious nature of insight (Bowden *et al.*, 2005) remains still poorly understood. In 1986 Michael Wertheimer (the son of Max Wertheimer) raised serious doubts about whether cognitive psychology could contribute to a deeper understanding of insight problem solving (Wertheimer, 1985, p. 31):

> " ... does modern cognitive psychology do justice to the Gestalt problem of insight ? ... from the perspective of Max Wertheimer's book *Productive Thinking*, the answer is an unequivocal no ... It is not that the modern information-processing approaches are wrong as such; they simply do not speak to the issue of insight. They have bypassed it completely. So the basic Gestalt problem remains as unsolved and as crucial – as it was before cognitive psychology ... came on the scene."

Although some of these doubts persist, cognitive psychologists have tried to come up with information processing models to explain insight and restructuring. Before we address some of these models we will provide an overview of the most important assumptions of problem space theory (Newell and Simon, 1972) and discuss why this theory cannot explain insight.

5 Problem Space Theory

According to problem space theory (PST), problem solving is defined as a search in a problem space. The problem space is a space of logical possibilities that is defined by an initial state (the problem), a goal state (the solution) and the operators that can be applied to a problem. The problem space encompasses all potential states that can be generated by applying the available operators to the problem, usually resulting in an exorbitant number of intermediate states that separate the initial state and the goal state. The size of the problem space depends on the given problem elements and the number of available operators. It is assumed that the problem difficulty covaries with the size of the problem space – the larger the problem space the more difficult the problem.

A classical problem that can be elegantly described and formalized by PST is the famous Tower-of-Hanoi problem (Fig. 2). The problem solver is asked to move the two disks on peg A to peg C by obeying the following rules: a) only one disk may be moved at one time and b) a larger disk may never be placed onto a smaller one. Figure 2 shows the complete problem space for the two-disk problem. If the number of disks is increased the size of the problem space explodes.

Further assumptions of PST concern the search process. Newell and Simon argued that problem solvers do not search by trial and error, randomly trying

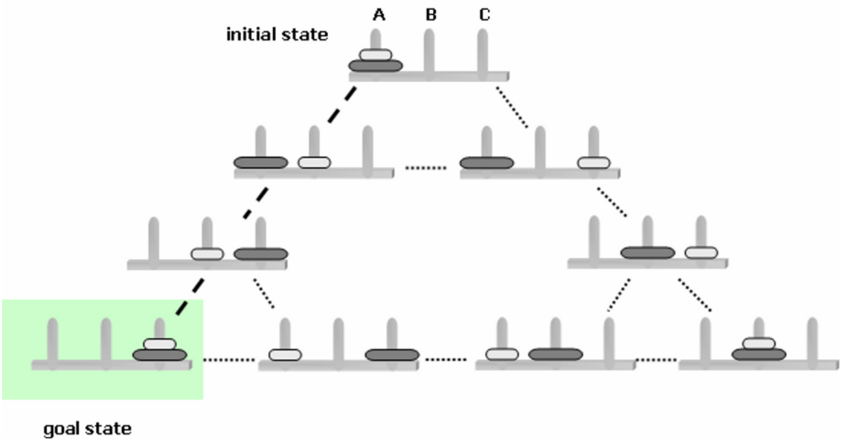

Fig. 2. The easiest version of the Tower-of-Hanoi problem with two disks. The problem space increases exponentially with the number of disks.

moves that come to mind. Instead, problem solvers apply heuristics that constrain the number of possible solution paths in a problem space. Heuristics make problem solving more efficient and more goal directed, although there is no guarantee that this will lead to a solution (Lindsay and Norman, 1981). However, heuristics can be very powerful to help to avoid searching parts of the problem space that are unlikely to contain the solution, and they help to avoid visiting the same state repeatedly. We will discuss two important heuristics, *hill climbing* and *means ends analysis*, in a bit more detail, because some accounts of insight problem solving refer to them.

The rule underlying the hill climbing heuristic is quite straightforward: Always select the move that transforms the current state into one that is as similar as possible to the goal state. This presupposes some sort of distance measure that assesses the similarity between the current state and the goal state. In the Tower-of-Hanoi problem, similarity can be defined as the number of disks that are already on peg C. This example also illustrates the problem of hill climbing – the existence of local maxima. After putting the small disk on peg C (the first move right below the initial state in Fig. 2), the current state is more similar to the goal state than the initial state, but in this constellation the solution becomes impossible, because now the larger disk can not be put on peg C. However, in many cases hill climbing is a fairly effective and parsimonious heuristic (Greeno, 1974; Thomas, 1974).

A more important heuristic is the means ends analysis (MEA). The most important characteristic of MEA is the introduction of sub-goals. MEA comprises three successive steps. In the first step the distance between initial state and goal state is determined. In the second step sub-goals are generated, and in the final step the first sub-goal that can be attained by an available operator is executed. In the Tower-of-Hanoi problem the large disk must be put onto peg C – a sub-goal is generated. To do this, it is first necessary to remove the smaller disk, that is a sub-sub-goal (moving the smaller disk from peg A to peg B) must be completed, and so on. The problem with MEA is that the number of sub-goals can become quite large which limits its usefulness if one considers the narrow capacity limitations of human working memory.

As already mentioned before, PST is hard to apply to insight problem solving because one core assumption is that problem solving proceeds in a stepwise fashion. Another problem is the implicit assumption that problem solvers generate a representation of the full problem space. Looking at human problem solving, it seems necessary to distinguish between a "subjective" problem space and an "objective" problem space. The latter unfolds all possible states in well-defined problems (like in the Tower of Hanoi). It can only be defined if all operators are known and all states can be computed. The subjective problem space of an individual problem solver can be inadequate, e.g., the wrong elements of a problem are considered. Furthermore, problem solvers may apply the wrong heuristics. Cognitive accounts of insight problem solving have explored both possibilities and they have suggested corresponding additions and modifications to PST to account for insight and restructuring. In the following, we will discuss these different accounts.

6 Heuristics and Insight

Kaplan and Simon (1990) assume that insight problems are extremely difficult, because initially they are "over-represented". In other words, they assume that many irrelevant or even misleading features and properties are incorporated into the initial problem representation whereas crucial problem aspects are omitted. In the latter case problem solvers need to change their representation of the problem space (Kaplan and Simon, 1990, p. 377):

> "Within a given problem space, the trick lies in searching for the right operator to apply next. But if no operators seem to yield progress, one must search for a new problem space to explore."

Kaplan and Simon suggest that looking for a new problem representation is a conscious process. When the problem is very difficult solvers may generate a number of different problem representations. In this case, successful problem solvers apply heuristics that enable them to detect which aspects remain invariant across different problem representations (Kaplan and Simon, 1990, p. 404):

> " . . . noticing invariants is a widely applicable rule of thumb for searching in ill-defined domains, [but] there can be no guarantee that those noticed will be the critical ones for the particular problem. Nevertheless, the constraints offered by the notice-invariant heuristic are a vast improvement over blind trial and error search."

They investigated these assumptions in a study of the mutilated checkerboard problem (Wickelgren, 1974, Figure 3). The task consists of an 8×8 checkerboard with two diagonally opposite corners removed. The task is to find out whether it is possible to cover the remaining 62 squares with 31 dominos, or to prove that this is impossible. A domino can cover two fields horizontally, or vertically, but not diagonally (Fig. 3). The solution is that it is impossible to cover the mutilated checkerboard with 31 dominos, because the removed corners have the same parity (same color). However, a domino can only cover two adjacent squares (black and white), and adjacent squares always have different colors.

This problem is extremely hard, even for very smart students. Only few of them were able to solve it and some of them took several days. In order to demonstrate that it is crucial to represent the parity of the two removed squares, Kaplan and Simon introduced solution hints. They found, for example, that a bread and butter version was easier to solve (Fig. 3) because this made it easier to detect that the removed fields were of the same parity ("bread").

A further theoretical approach that also emphasizes the important role of heuristics for insight problem solving is based on criteria for satisfactory progress (MacGregor *et al.*, 2001; Ormerod *et al.*, 2002; Chronicle *et al.*, 2004). The criterion for satisfactory progress theory (CSPT) postulates that successful problem solving requires two basic principles: First, problem solvers seek to maximize the consequences of each move such that the move results in a state that is as close as possible to the desired goal. This is basically the hill-climbing heuristic. Second, problem solvers constantly monitor their progress and only select moves that meet a criterion of progress – when a selected move fails to meet the criterion there is an impulse to seek alternative solutions (cf. Ormerod *et al.*, 2002, p. 792).

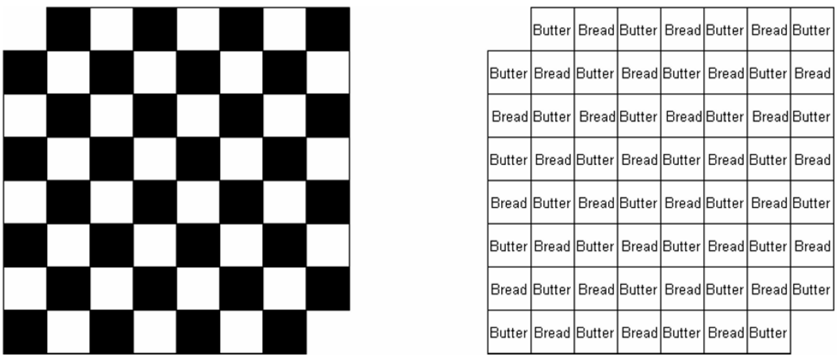

Fig. 3. The mutilated checkerboard (Wickelgreen, 1974). On the left the original problem, on the right the bread-and-butter version.

According to Ormerod and colleagues, maximization and progress monitoring together may trigger insight because they prompt the discovery and retention of so-called promising states that meet the progress-monitoring criterion. In their model previously unexpected solution paths may arise when problem solvers backtrack and start with a new "promising state". The probability of meeting an impasse (getting stuck in a solution path that does not work) also depends on a person's look-ahead. It is defined as the number of potential moves a person can consider. Of course, this varies across different individuals. People with a high look-ahead will realize more quickly that alternative ways of looking at the problem are needed because they will more quickly run out of moves that meet the progress-monitoring criterion and therefore start more quickly to look for alternative moves.

The experiments by MacGregor and colleagues on the nine-dot problem (Scheerer, 1963) illustrate the core assumptions of CSPT. The task is to connect the nine dots, arranged in a 3×3 matrix, with four straight lines without lifting the pen. The solution requires drawing lines beyond the boundary of the matrix (Fig. 4a). For a long time this was believed to be the main source of difficulty in this problem. However, MacGregor and colleagues think that the high difficulty is mainly due to the fact that problem solvers apply inappropriate heuristics.

According to their account people use the following two criteria to solve the problem: First, the maximization criterion is to connect as many dots as possible with each line. Second, the progress-monitoring criterion is to determine the ratio between the remaining strokes and dots after each move. For example, after connecting three dots with the first stroke, there are still three strokes left to connect the remaining six dots. Evaluating these criteria, a person with an exceptionally high look-ahead may immediately recognize that the problem is unsolvable within the boundaries of the 3×3 dot matrix. The reason is that no configuration of three strokes will satisfy the criteria if one stays within the matrix. This, in turn, may then trigger the search for a promising state such as the non-dot point extension outside the matrix that is required to solve the problem.

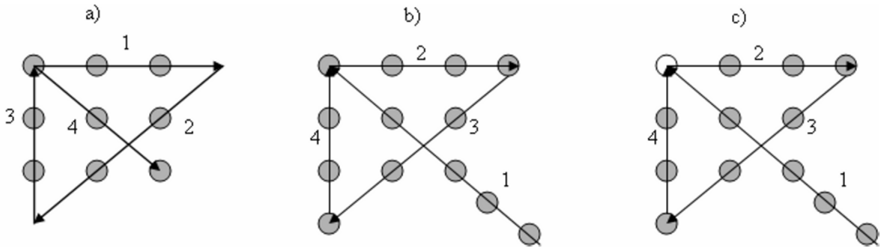

Fig. 4. a) The 9-dot problem and its solution; numbers indicate the sequence of moves. b) The 13-dot variant and its solution. c) The 12-dot variant and its solution.

To test their assumptions empirically, MacGregor and colleagues used several variants of the nine-dot problem. Using the 13-dot problem (Fig. 4b) they demonstrated that problem solvers tended to select such moves that connect as many dots as possible (verifying the maximization criterion). Introducing the 12-dot problem (Fig. 4c) they had no problem to use non-dot extensions, if they satisfied the progress-monitoring criterion. In further studies, MacGregor and colleagues successfully applied the assumptions of maximization, progress-monitoring, and look-ahead to the eight-coin problem (MacGregor *et al.*, 2001; Ormerod *et al.*, 2002), and to another set of coin problems (Chronicle *et al.*, 2004).

The two accounts discussed in this section emphasize the importance of heuristics for problem solving. Both views can be considered as direct extensions of the classical PST. Insight requires "nothing special" except particular strategies. Thus, heuristics are believed to be the driving force behind problem solving. If the appropriate heuristics are known and applied, then insight problems do not differ from other problems (Chronicle *et al.*, 2004, p. 26):

> "Our view of the processes of solution discovery in insight problem solving indicates links between insight and conventional problem solving, suggesting that accounting for insight lies within the scope of unitary cognitive architectures ... and adaptive control of thought ... ."

7 Representational Change and Insight

Another account that builds on PST, but postulates that there are "special" processes of restructuring, is Ohlsson's (1992) representational change theory (RCT) of insight problem solving. In this theory insight problems are considered to be special because they tend to trick the problem solvers into representing the problem in a way that does not allow them to solve it. In many cases the faulty initial representation needs to undergo a fundamental change before the problem can be solved.

Why do people generate inadequate problem representations? The reason is that the initial encoding of the problem depends on the problem solver's prior knowledge. During encoding problem solvers try to apply seemingly appropriate knowledge to the problem, and this affects how the problem elements are grouped perceptually and conceptually. Prior knowledge also dictates which problem elements are selected and which elements are ignored, and constrains what is considered as a solution for the problem. As a consequence, the initial problem representation can be misleading in many ways. It may not contain the crucial problem elements, the elements may be grouped in the wrong manner, or the goal of problem solving may be too narrow in order to allow a solution.

If problem solvers have the wrong problem representation they will sooner or later hit an impasse, a state of mind where problem-solving behavior ceases and where they do not know what to do next. A representational change becomes necessary before new solution paths can be generated. Depending on what is wrong with the initial problem representation, different perceptual and memory processes can lead to a representational change. It is assumed that these processes operate outside of consciousness (Schooler *et al.*, 1993) and thus create the impression of a sudden insight once the representation has changed and new solution paths become available.

The assumptions of RCT have been tested using simple matchstick arithmetic tasks (Knoblich *et al.*, 1999; Knoblich *et al.*, 2001; Knoblich *et al.*, 2005; Knoblich and Öllinger, 2006; Öllinger *et al.*, 2006; Öllinger *et al.*, 2008). Matchstick arithmetic tasks present the problem solver with an equation consisting of numerals and the arithmetic operators $+$, $-$, and $=$. The task is to move a single matchstick in order to generate a true (correct) expression.

Problem type		Initial state	Chunk description	Solution	Problem difficulty
formal	Base	$X = Y - Z$	loose chunks	$X = Y - Z$ (true)	+
example		**VIII=IV+VI**	**VIII=IV+VI**	**VIII=IV+IV**	
formal	Chunk dec.	$X = Y - Z$	tight chunks	$X = Y - Z$ (true)	++
example		**VI=VI+V**	**VI=VI+V**	**XI=VI+V**	

Table 1. Problem types of the matchstick arithmetic task that require a change of the goal representation: chunk decomposition.

Two characteristics of these problems are worth noting. The first is that single matchsticks are perceptually grouped to form meaningful chunks. For example, in the problem displayed in the first row of Tab. 1 the two slanted matchsticks are automatically grouped to form the letter **V**. Similarly, $+$, $=$, and so on are immediately recognized as “meaningful chunks”. There are two types of chunks. *Loose chunks* are meaningful entities that can be easily decomposed, for instance **VI** into **V** and **I**. The single Roman numeral **I** can be easily moved within the equation because it is meaningful in itself. By contrast, a *tight chunk* consists of constituents that have no meaning by themselves. For instance, decomposing **X** results in two slanted sticks that have no meaning within the context of the task.

Applying RCT one can predict that problem solvers will initially treat tight chunks as being non-decomposable. That is, the single units forming a tight chunk will initially not be represented separately. For some problems this is an inadequate problem representation because they require transforming the Roman numeral **V** into the Roman numeral **X**. This can be achieved by moving one match, but only if the tight chunk **X** is decomposed into its constituents **** and **/**. We call the process that triggers this change *chunk decomposition*.

A second characteristic of the problems worth noting is that they are likely to activate the problem solver's knowledge of simple arithmetic. This knowledge will lead to an initial goal representation that constrains the space of possible solutions that problem solvers will potentially consider. The goal representation determines which operations can be applied to the encoded problem elements and guides the problem solving process. Imposing such constraints is an automatic and unconscious process that often helps to reduce the problem space. However, these constraints can also narrow the problem space to an extent that a solution becomes impossible.

Problem type		Initial state	Constraint relaxation	Solution	Problem difficulty
formal	Base	$X = Y - Z$	—	$X = Y - Z$ (true)	+
example		**VIII=IV+VI**	—	**VIII=IV+IV**	
formal	Operator	$X = Y - Z$	$X\mathrm{Op}_1 Y\mathrm{Op}_2 Z$	$X\mathrm{Op}_1 Y\mathrm{Op}_2 Z$	++
example		**IX=VI-III**	**IX=VI-III**	**IX-VI=III**	
formal	Tautology	$X = Y - Z$	$X\mathrm{Op}_1 Y\mathrm{Op}_2 Z$ and $\mathrm{Op}_1 = \mathrm{Op}_2$	$X\mathrm{Op}_1 X\mathrm{Op}_1 X$	+++
example		**VI=VI+VI**	**VI=VI+VI**	**VI=VI=VI**	

Table 2. Problem types of the matchstick arithmetic task that require a change of the goal representation: constraint relaxation.

Consider the following example. Prior knowledge of simple arithmetic suggests that values change and operators do not change (see Tab. 2). Therefore, it is likely that problem solvers form an initial goal representation that represents values as variable and operators as constant (Var1 = Var2 + Var3). For some problems this representation is inappropriate to obtain a successful solution. Ohlsson (1992) postulated that a second unconscious restructuring process relaxes the self-imposed constraints on the goal of problem solving when problem solvers hit an impasse. This process is called *constraint relaxation*. It generates a more flexible goal representation. For instance, representing arithmetic operators as variable activates moves that manipulate the operators. The tautology type in Tab. 2 illustrates that certain tasks require the relaxation of two or more constraints. In addition to conceiving the operator as variable, a second constraint – equations consist of two different kinds of operators – has to be overcome. Only in this case one can conceive of solutions where both operators are equal signs.

Knoblich *et al.* (1999) provided empirical evidence supporting the assumptions outlined above. In several experiments they asked participants to solve different types of matchstick arithmetic tasks that differed in their need to decompose tight chunks and to relax initial constraints on the goal. The results show that chunk decomposition and constraint relaxation are two independent processes that can lead to representational change. In particular, problems requiring the decomposition of tight chunks are much more difficult than problems that only require the decomposition of loose chunks. Moreover, the problem difficulty increases dramatically with the number of self-imposed constraints that must be relaxed to solve a particular problem. The tautology type is more difficult than the operator type, and the operator type is more difficult than the base type (see Tab. 2; Knoblich *et al.*, 1999; Knoblich *et al.*, 2001; Öllinger *et al.*, 2006; Öllinger *et al.*, 2008).

Recently, Öllinger and colleagues (2008) raised the question whether set effects (Luchins, 1942) can be interpreted as resulting from representational change. Their basic idea was that the repeated solution of similar problems induces a gradual representational change which continuously narrows the space of solutions considered. Imagine you are asked to solve a variety of matchstick problems which, conforming to your prior knowledge of arithmetic, can be solved by manipulating values. Will this increase the difficulty of problems that require constraint relaxation (manipulating operators) even more? According to Luchins this should be the case because the same solution procedure was used repeatedly when solving problems that require manipulating values.

Öllinger and colleagues found that this is not the case. Repeatedly solving value problems does not increase the difficulty of problems that require constraint relaxation. The reason is that they afford an initial goal representation consistent with one's prior knowledge of arithmetic that is already dominant and cannot be narrowed further. However, if participants were asked to solve a number of problems that require manipulating operators, the solution of a subsequent problem that requires manipulating values was strongly impaired. In this case, the repeated manipulation of operators biased the goal representations towards representing the operators as variable and the values as constant. As a consequence it became more difficult to solve problems that require a manipulation of values. This demonstrates that a new insight, repeatedly applied, can overwrite existing prior knowledge.

Another interesting finding is that having one insight is likely to reduce the likelihood of having another insight. If people solved problems that repeatedly required the decomposition of tight chunks (**X** → **/** → **V**) the solution of problems that required producing a tautology became almost impossible: Chunk decomposition (one insight) reduced the likelihood of constraint relaxation (another insight). This provides further evidence that chunk decomposition and constraint relaxation are two different processes. Chunk decomposition pertains to perceptual aspects of the problem element whereas constraint relaxation pertains to the solutions a problem solver can conceive of conceptually. Problem solvers found it difficult to switch from being flexible with regard to the percept to being flexible with regard to conceptualizing the problem in a new way.

Another promising approach to address insight empirically is recording people's eye movements while they solve problems (Knoblich *et al.*, 2005). Eye movements provide a more fine-grained behavioral measure than solution times or solution rates. Therefore they allow one to test more specific predictions that could not be tested with performance measures. For instance, Knoblich *et al.* (2001) used this technique to test predictions derived from RCT. Participants attempted to solve matchstick arithmetic tasks (base, operator, tautology, and chunk decomposition) that were more or less likely to require a representational change.

Three predictions were tested: First, during an impasse, the problem solving behavior should cease to some extent. Therefore problem solvers should more often stare at a problem without testing particular solution ideas. The results confirmed this prediction. Mean fixation duration increased for problems that required a representational change. This provides evidence for the assumption that people do encounter impasses during insight problem solving.

The second prediction was that the initial goal representation should be biased towards the values, and therefore values should initially receive more attention (eye movements) than operators. Indeed, participants spent much more time looking at the values than looking at the operators during the initial stages of problem solving.

The third prediction pertained to differences between successful and unsuccessful problem solvers. Successful solvers of insight problems should gradually spend more time looking at the crucial problem elements than unsuccessful problem solvers. It was found that in later stages of problem solving successful problem solvers gazed longer on the operators and the critical tight chunk.

These results support the concepts of impasse and representational change. The problem representation determines which parts of a task problem solvers attend to. Successful problem solvers differ from unsuccessful problem solvers in their ability to shift their attention to previously neglected parts of the problem. Knoblich *et al.* (2001) showed that the shift of attention likely results from a preceding change in the problem representation.

Grant and Spivey (2003) addressed the complementary question whether an externally triggered shift in attention to a crucial problem element could affect the solver's problem representation. They performed two experiments in which they asked participants to solve Duncker's tumor problem (Duncker, 1945): "Given a human being with an inoperable stomach tumor, and lasers which destroy organic tissue at sufficient intensity, how can one cure the person with these lasers and, at the same time, avoid harming the healthy tissue that surrounds the tumor ?" The solution is to use two lasers radiating at the tumor from different angles, so that their beams meet at the location of the tumor. The addition of the intensities of the beams provides the necessary energy to destroy the tumor, while the reduced intensity of the single lasers leaves the surrounding tissue unharmed.

Grant and Spivey provided a simple schematic drawing the problem solvers were looking at while attempting to solve the problem. The tumor was simply depicted as a small solid oval, with a circumscribing oval representing the skin. In a first experiment they found that successful problem solvers looked significantly longer at the skin than unsuccessful problem solvers who looked longer at the tumor. In the second experiment, they tested whether drawing a problem solver's attention to the skin would increase the solution rates. They introduced three conditions. In the first condition the skin pulsated slightly, in the second condition the tumor pulsated slightly, and in the control condition they presented a static display. They idea was that participants' attention was attracted by the pulsating portion of the display. In line with their hypotheses they found that solution rates were significantly increased in the pulsating skin condition compared to the other conditions. This result is quite astonishing because this simple manipulation was much more effective in increasing the solution rates than a variety of explicit verbal hints that had been tried in previous research.

Although RCT can quite well explain why people encounter impasses and which processes can help to resolve these impasses, it is less successful in explaining what happens before and after an impasse. Furthermore there is growing evidence that insight problems often have multiple sources of difficulty that need to be disentangled, some related to heuristics, some related to representational change. This has led to attempts to compare and integrate the two types of accounts which we will discuss in the next section.

8 Heuristics, Representational Change, and Insight

Jones (2003) contrasted the predictions of CSPT and RCT (see also Knoblich *et al.*, 2005). He asked participants to solve problems taken from the car park game while he tracked solvers' eye movements. In this game, one needs to maneuvre a taxi car out of a car park with other cars blocking the exit way. In particular, one needs to figure out how to clear the exit way so that the taxi can leave the car park. In some problems, the taxi itself needs to be moved back and forth before an exit way has been created. Jones (2003) suggested that these problems require insight because problem solvers impose the constraint that the taxi can only be moved after an exit way has been created. Accordingly, in line with the assumptions of RCT, he expected for such problems that problem solvers encounter impasses.

Furthermore, he tested the assumption of CSPT by determining problem solvers look-ahead value. He expected that problem solvers having a greater look-ahead value (see above) should be more successful in solving insight problems, because they should encounter impasses earlier than problem solvers with a smaller look-ahead value (cf. Sec. 6). A higher look-ahead value should be a predictor for a successful solution.

From the eye movements it was determined whether problem solvers encountered impasses before they carried out the crucial taxi move, and how many moves they could plan ahead. Jones found that all participants who successfully solved the problem encountered one or more impasses before they moved the taxi for the first time, and that participants with a greater look-ahead completed the problem significantly faster and needed significantly fewer moves than participants with a smaller look-ahead value.

These results led Jones (2003) to propose that insight problem solving can be best understood if one integrates CSPT and RCT rather than treating them as competing explanations (Jones, 2003, p. 1026):

> "The dynamical constraint theory essentially covers insight up to the point at which insight is sought. [. . .] The representational change theory on the

> other hand covers how insight will be achieved, and, therefore, the point at which insight is sought is the beginning point of the theory."

Öllinger *et al.* (2006) further investigated the interplay between heuristics and representational change. They modified the matchstick arithmetic and added to each problem type (see Table 2) an additional value move. Thus the new tasks required two moves for a successful solution. Moreover, they created two sets of problems. The first set consisted of problems that required an additional value transformation reducing the distance to the goal. The second set consisted of problems that required an additional value transformation that initially increased the distance to the goal. Distance was defined as the numerical difference between the left-hand side and right-hand side of the equation. For instance, the equation **VI** = **IV** + **VI** has a distance of four. Solving the problem requires two steps, first to apply a value move that changes the **IV** into a **VI**, thus increasing the distance to six. The solution is a tautological structure **VI** = **VI** = **VI** with distance zero.

This task modification allowed Öllinger and colleagues to test assumptions of CSPT and RCT. According to CSPT, the distance measure is nothing else than a maximization criterion. Reducing the distance makes the right and left side of the equation more similar (hill climbing). The criterion for progress is based on assessing, after each move, whether there is an available consecutive move that can equalize the left and right side of the equation. CSPT predicts that tasks requiring two moves that reduce the distance between the left and the right side of the equation should be easier than tasks requiring a move that increases the distance. RCT predicts that the problem difficulty is driven by the degree of representational change required. That is, problems requiring a value move plus a move that produces a tautological structure should be significantly more difficult than problems requiring a combination of two value moves.

Öllinger *et al.* (2006) found that the problem difficulty varied according to whether or not a representational change was required. Problem difficulty was not influenced by the kind of value move. It did not matter whether the moves increased or reduced the distance. Thus, it seems that problem solvers did not apply the maximization criterion of CSPT. In addition, the outcomes indicated that two-move problems were much more difficult than one-move problems (Knoblich *et al.*, 1999; Knoblich *et al.*, 2001; Öllinger *et al.*, 2006; Öllinger *et al.*, 2008). This shows that the larger problem space in two-move matchstick arithmetic tasks was an additional source of problem difficulty. Öllinger and colleagues suggest that the main source of difficulty in insight problems is the necessity to change the problem representation. Heuristics sometimes help to reduce large problem spaces and may therefore reduce the time a problem solver spends exploring fruitless solution paths.

A recent study of the nine-dot problem (Fig. 4a) by Kershaw and Ohlsson (2004) further underlines that the number of sources of difficulty a problem poses must not be underestimated. The classical explanation for the high problem difficulty of the nine-dot problem is that problem solvers initially do not consider moves that go beyond the virtual square formed by the dots (Ohlsson, 1992; Scheerer, 1963). Accordingly, the insight needed for the solution is to realize that the lines can be extended to non-dotted locations outside the virtual square – that is, a representational change (draw beyond the barriers) can solve the problem. However, Weisberg and Alba (1981) showed that people did not benefit from hints that told them to relax this constraint. As described above, MacGregor *et al.* (2001) claimed that the main source of difficulty is the application of the appropriate heuristics.

Kershaw and Ohlsson compiled the contradictory evidence on the nine-dot problem and concluded that it entails four sources of difficulty. First, in line with the classical account an essential amount of the problem difficulty is the necessity of drawing beyond the virtual boundaries. The second source of difficulty is the shape of the solution. The configuration of lines required is quite extraordinary and therefore both hard to find and hard to apply. The third source of difficulty is the size of the solution space: The four consecutive moves create a large problem space and moving beyond the virtual boundaries considerably increases this problem space. Finally, using variants of the standard nine-dot problem, they found that it is difficult to change direction at locations that do not contain dots. They concluded that insight problems often have a number of sources of problem difficulty that require the contribution of different cognitive processes. This indicates the necessity to construct problems for insight research that permit systematic variations of particular sources of difficulty.

In the next section we will give a short review of current findings on neural correlates of insight problem solving. Although this research has been caried out for less than a decade, it has already provided some interesting results that can help to improve functional explanations of insight.

9 Neural Correlates of Insight Problem Solving

Luo and Nicki (2003) performed the first functional magnetic resonance imaging (fMRI) study on insight in order to determine whether particular brain regions are activated in insight problem solving. They asked participants to solve Japanese riddles that either require a reinterpretation of the concepts involved or not. A typical riddle is like this: "What is the thing that can move heavy logs but cannot move a small nail?" The answer is a river. In a first step a number of participants were shown the solution to the riddles and asked whether the solution was surprising. Those riddles that had a surprising solution were used in the "insight" condition and those that were not surprising were used in the "non-insight" condition.

The fMRI technique allows one to infer from magnetic activations in a particular brain site how much oxygen the blood stream currently transports in this region. It is postulated that large oxygen consumption is an indicator for involvement of a particular brain area in a particular task (hemodynamic response). By contrasting the blood flow between insight and non-insight riddles they found that the right hippocampal system was more strongly activated when solutions of insightful riddles were presented to the problem solvers.

The hippocampus is suspected to be the gateway to the long-term memory system. That is, this structure might be responsible for encoding and addressing new information and conveying it, after a delay, to long-term memory (McClelland *et al.*, 1995). Luo and Nicki provided three possible roles the hippocampal system might play. First, the stronger activation for insightful solutions could be due to the formation of novel associations among already existing concepts. Second, the hippocampus might be involved in breaking unwarranted mental fixation. Third, because the hippocampus plays an important role in spatial-orientation tasks it is conceivable that the stronger activation in the insight condition reflects the formation of a new reference frame.

The common denominator of all three possibilities is the involvement of the hippocampus in building or permitting "something new" which would imply an important role of the hippocampal system in representational change. A further study emphasizing the importance of the hippocampal system in insight was conducted by Wagner *et al.* (2004). Participants were presented with digit strings and were asked to apply two rules to these sequences. However, all sequences could be solved according to a third, hidden rule that greatly reduced the difficulty of the task.

Wagner and colleagues investigated whether the likelihood that participants discovered the hidden rule increases after sleep. They pointed out that such strategy changes are very similar to insight insofar as they happen very suddenly and without any recognizable effort on the part of the person. After a long training phase participants in one condition slept for 8 hours, the other participants stayed awake and waited for 8 hours to continue with the task. There were further conditions controlling for the effects of fatigue. Surprisingly, it was found that the group that had slept detected the hidden rule much more often than people who had not slept for the same time. Wagner and colleagues explained the finding as a consequence of consolidation and restructuring new memory representation during nocturnal sleep. Converging with Niki and Luo's (2003) results, the hippocampus was suspected as the crucial region where restructuring takes place.

A further fMRI study (Jung-Beeman *et al.*, 2004) revealed that the hippocampus may not be the only region involved in restructuring. In the experiments by Jung-Beeman and colleagues participants solved a number of remote association tasks. Three words are presented and the task is to find a target word that in combination with the given words results in a meaningful new word or phrase (e.g., given the words pine, crab, and sauce, the target word is apple). After finding the solution they indicated whether or not it was accompanied by an aha-experience. The responses were classified into insight and non-insight solutions and the hemodynamic activations of both conditions were contrasted. The results showed that insightful solutions were accompanied by activation in the right anterior superior temporal gyrus relative to non-insight problems. Jung-Beeman and colleagues argued that this brain region is putatively responsible for linking mental concepts in a novel way and may foster representational change.

There is also evidence from EEG studies that insight problem solving acquires other neural resources than the solution of conventional or non-insight problems. EEGs are recordings of cortical electrical activity. Although their spatial resolution is fairly poor, the temporal resolution of EEGs is very high. A study of event-related potentials (ERP; the averaged EEG signal triggered by a particular event), conducted by Lavric *et al.* (2000), compared the activation patterns between tasks requiring either analytic reasoning (the Wason selection task) or creative problem solving (Duncker's candle problem, see above). Furthermore, participants were asked to count simultaneously auditory stimuli – the events that triggered the onset of the ERP signal.

Lavric and colleagues predicted that counting would disturb analytic reasoning, because it recruits the same brain sites, but not creative problem solving. This was what they found. The main result was that two factors in the P300 component could be extracted (one located frontally, the other left-lateralized) that differed between analytic and creative problem solving. Moreover, the P300 was located more frontally during analytic problem solving compared to creative problem solving (see also Lavric *et al.*, 1998; Mai *et al.*, 2004). This suggests that insight involves non-analytic modes of thinking.

Jung-Beeman *et al.* (2004) reported a further EEG study where they found that a sudden burst of high-frequency (gamma-band) neural activity precedes insight solutions by about 300 ms and could therefore be a neural marker of the subjective aha-experience. Mai *et al.* (2004) investigated Chinese riddles that either had an expected solution ("no-aha" condition) or an unexpected solution ("aha" condition). They found that between 250 and 500 msec after the onset of the answer "aha" solutions elicited a more negative ERP signal than "no-aha" solutions. The difference wave was located over the central electrode site (Cz) with a peak latency of N380. They speculated that the anterior cingulate cortex may generate this component and that the N380 may reflect conflict detection in "aha" answers that require overcoming constraints imposed by prior knowledge.

Finally, there is neuropsychological evidence that yet another area is involved in insight problem solving. Reverberi *et al.* (2005) investigated the impact of brain lesions on the solution of matchstick arithmetic tasks. Patients with a frontal lesion and healthy controls solved different types of matchstick arithmetic problems (cf. Tab. 2). Surprisingly, patients with a lesion in the dorsolateral prefrontal cortex (DLPFC) turned out to be more successful than healthy controls in solving the difficult insight problems that require to produce a tautology. Reverberi and colleagues argued that the DLPFC might be the site that constrains the space of possible solutions a problem solver considers. They suggest that a lesion in this area reduces top-down control and therefore increases the likelihood that prior knowledge overly constrains the goal of problem solving (however, at the cost of successful analytic thinking). Therefore, it is conceivable that DLPFC is the brain area where a goal representation is formed that integrates elements of a problem situation and prior knowledge.

10 Conclusions

Cognitive psychologists and cognitive neuroscientists build on early research initiated by Gestalt psychology to unravel the mystery of how solutions to difficult problem sometimes appear out of the blue. They work under the assumption that important scientific discoveries are not qualitatively different from groundbreaking insights that have allowed great scientists like Pauli or Einstein to fundamentally change our understanding of natural laws.

Current research suggests that insight problem solving can be described as a process that passes through three phases, the phase preceding an impasse, the impasse phase and its resolution, and the phase after an impasse (Ohlsson, 1992; Knoblich *et al.*, 1999; Öllinger *et al.*, 2006). During the initial phase a problem representation is established. This representation is perceptually and conceptually constrained by the problem solvers' prior knowledge and their experiences. Problem solvers use heuristics to effectively search for a solution in the space of possible solutions defined by the initial representation. At some point, problem solvers fail to find new solution paths. This happens earlier for problem solvers who have a large look-ahead (MacGregor *et al.*, 2001; Jones, 2003).

Then problem solvers get stuck in an impasse, doing nothing (Knoblich *et al.*, 2001) or trying the same unsuccessful solution paths over and over again (Knoblich *et al.*, 1999). During inactive phases, the activation of the initial problem representation gradually drops (Öllinger *et al.*, 2008) and unconscious perceptual and memory processes start to affect different aspects of the problem representation. Chunk decomposition can lead to a regrouping of perceptual elements and constraint relaxation leads to a more flexible goal representation. It is likely that there are other processes that can also affect the problem representation.

Once the problem representation has been altered new solution paths become available and stepwise problem solving is resumed. Heuristics play an important role before and after an impasse but it is not clear to which extent they can actually trigger representational change. There is an ongoing debate on this issue, and further research will tell whether people can develop strategies to change problem representations.

Research on insight problem solving in cognitive neuroscience has just begun, and it is already clear that no single area is responsible for representational change. One neural mechanism that could be important for representational change is memory consolidation in the hippocampus. Such a

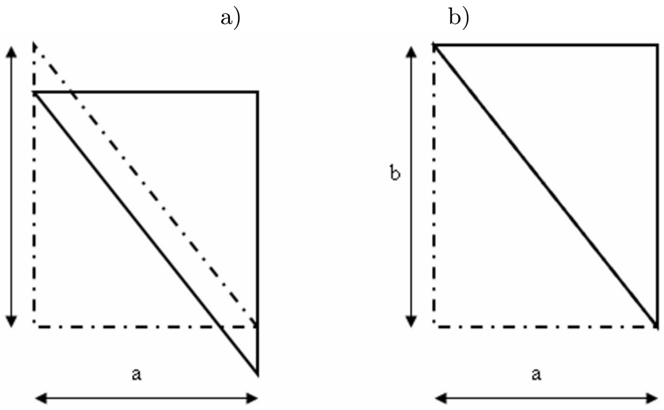

Fig. 5. Solution of Wertheimer's square-parallelogramm problem discussed in Sec. 3: a) restructuring, b) solution.

consolidation could result in conceptual change or in the detection of previously unnoticed regularities. Inferior temporal cortex may also contribute to conceptual change but it is far from clear why the hippocampus is activated in some tasks, why the inferior temporal cortex is activated in others, and whether the results can be generalized at all. Obviously, much more research is needed.

In addition, frontal brain areas seem to be crucially involved in insight problem solving. The anterior cingulate cortex is involved in the detection of conflict and might be involved in making problem solvers realize that they have encountered an impasse. In particular, the repeated failure with an inappropriate representation may be detected and evaluated by the anterior cingulate cortex. Dorsolateral prefrontal cortex seems to be involved in defining the constraints with respect to the goal of problem solving. One of the many issues that will be crucial to address in future research is how different brain areas cooperate in creating new ideas in the problem solver's mind.

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