A calculator for shock wave reflection phenomenon

M. Sun

Center for Interdisciplinary Research, Tohoku University, Sendai 980-8578, Japan *

Summary. We develop a numerical tool, as simple as a calculator, by integrating all numerical related issues, such as grid generation, boundary and initial conditions setup as a whole, for simulating shock wave phenomenon. Unlike available numerical codes that generally require the knowledge of mathematics to run them, the present calculator is developed for the phenomenon and set physical quantities as input parameters. As a result, for obtaining a numerical result of shock reflection, one can run it by only specifying shock Mach number, the wedge angle and the ratio of specific heats. The software is available from website *http* : *www.flowwatch.net**dl**swr.html*. The methodology is applicable for the simulation of other basic shock phenomenon as well.

1 Introduction

Numerical techniques for the simulation of shock wave phenomenon have grown matured especially in the last two decades, as a key research field of computational fluid dynamics (CFD). Numerous commercial software and non-commercial algorithms have been developed in laboratories over the world. The use of these software requires the knowledge of mathematical modeling of physical flows, such as boundary and initial conditions. In practice, in order to get a numerical result, one has to spend days or weeks to master them, so that the CFD is sometimes forbidden for these who do not want to spend time on numerics although they want to get numerical results for reference.

This work is devoted to constructing a *virtual* shock tube that is based on physical models, such as wedges, cones, similar to those used in a real lab. We try to integrate all numerical related issues, such as grid generation, the setup of boundary and initial conditions as a whole, for a given model, and hide these numerical issues from end users. Consequently, one can obtain a numerical result by only specifying phenomenon-related physical parameters, as one performed an experiment shot.

The methodology is applicable to broad classes of shock wave phenomena. In this paper, only a two-dimensional wedge and an axisymmetric cone is considered to demonstrate the methodology. The shock interaction with them is called shock wave reflection. Shock wave reflection, a basic phenomenon in shock dynamics, can be fully determined by three parameters as

$$f = f(M_s, \theta_w, \gamma), \tag{1}$$

where M_s is the Mach number of the incident shock wave, and θ_w is the angle of the wedge for a 2-D reflection or the semi-apex angle of a cone for an axisymmetric reflection.

* Also Interdisciplinary Shock Wave Laboratory, Transdisciplinary Fluid Integration Research Center, Institute of Fluid Science, Tohoku University, Sendai 980-8577, Japan

Relation (1) is valid for the inviscid and the ideal gases with the constant adiabatic index of γ. A sketch of the phenomenon is shown in Fig. 1.

The phenomenon of shock wave reflections has been investigated for over one century, and a recent state-of-the-knowledge review was given by Ben-Dor (2006), and more details have been studied and reported by those therein referred to. The phenomenon has been documented according to observed reflection patterns, and the well-known analytical models are based on the invicid assumption. Therefore, these reflection pattern can be well predicted by the Euler equations. In fact, the Euler equations provide the set of solutions of the phenomenon more accurate and more general than any invicid analytical model. Nowadays, it is fairly easy to obtain a numerical solution of the equations by CFD.

In this paper, we attempt to construct a virtual lab that can visualize and display the reflection pattern in a computer, taking full advantage of the advanced solution techniques developed in CFD for compressible flows. The software is made so simple that anyone can easily get a quick image of reflection pattern. Once the three parameters are specified, the corresponding numerical image will be shown by one click. It is as simple as a "calculator "that frees people from evaluating basic algebraic operations. It is hoped that the calculator for shock reflection can free people from studying CFD and running shock tubes for the phenomenon. The calculator and a few numerical examples are introduced in section 2.

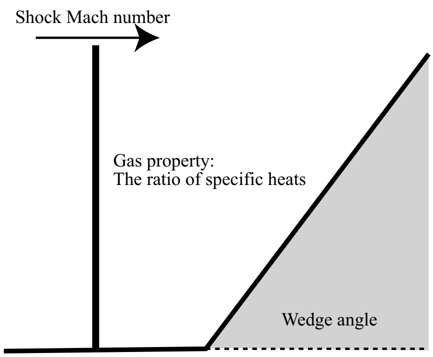

Fig. 1. Three parameters in shock wave reflection over a 2-D wedge or an axisymmetric cone

2 Overview of the calculator

The calculator has been designed and built in the way that can be easily applied to other shock wave phenomena in addition to shock wave reflections. It consists of two main modules, Problem Module and CFD Module, and a graphical user interface (GUI), as depicted in Fig. 2.

2.1 Problem Module

A Problem Module selects and optimizes the computational model for a given flow phenomenon according to the input parameters. Physical parameters must be defined for a

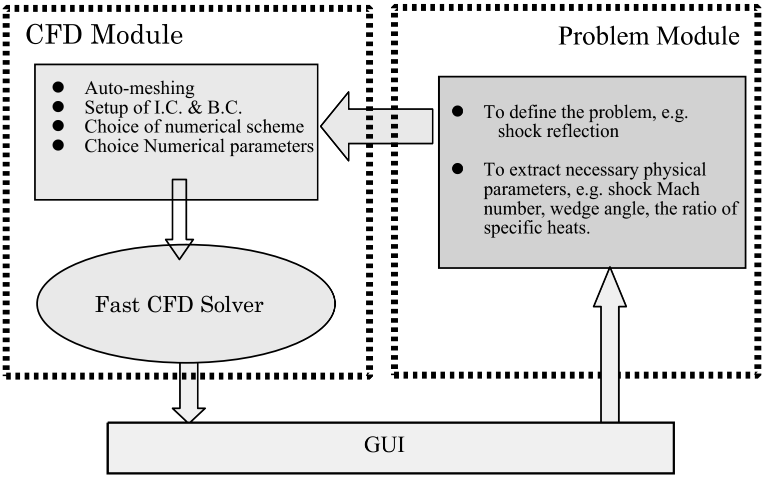

Fig. 2. Schematic of the calculator for shock wave dynamics

user to input. For the sake of users, the parameters should be simple, and the number of them should march with to the mathematical model to be solved. Three parameters are required for shock wave reflections over a wedge or a cone, shock Mach number (M_s), wedge angle (θ_w) and the ratio of specific heats (γ), when solving compressible Euler equations for the ideal gases, as written in Figure 2. The algebraic range of the parameters are set as follows: $M_s \in [1.001, 10]$, $\theta \in [0°, 90°]$, and $\gamma \in [1.001, 2.]$. Since the assumption of the ideal gas is not accurate for high temperature, the upper limit of shock Mach number is set to be 10.

Problem Module further defines what required for initializing the CFD Module, such as setting the computation domain, boundary and initial conditions, the CFL number for the numerical scheme, etc. These are unusually done by a CFD person to start a computation using any software or algorithm.

Problem Module can contain numerous flow cases that represent typical flow phenomena, such as flows over a circular cylinder. For each case, one needs to select the minimum flow parameters in order to complete the theoretical model that approximates the flow phenomenon, and defines other numerical issues that are required by the CFD module for obtaining a result efficiently, accurately and robustly. Notice that an end-user of the present calculator does not need to specify these issues, which requires a thorough understanding of CFD.

2.2 CFD Module

Once the parameters for a flow phenomenon are specified in Problem Module, CFD module takes them as input parameters and show interactive results automatically through the GUI. It requires the kernel algorithm of CFD module to be able to realize all functions that a good commercial software can do, such as grid generation, flow simulation, and plotting results. Among many other technical issues in CFD module, the robustness of a flow solver is the most challenging in the realization of the calculator working for any input parameters.

The CFD solver used for simulation shock wave reflection is based on an in-house code, VAS2D. It was developed based on solution-adaptive unstructured quadrilateral grids for easy vector and parallel processing on supercomputers, while maintaining its high efficiency on personal computers [2]. VAS2D can simulate unsteady/steady, inviscid/viscid, frozen/equilibrium compressible flows for arbitrary geometries with two spatial coordinates (2-D planar and axisymmetric). The conservation laws are discretized by the finite volume method. The second-order spatial accuracy is achieved by a MUSCL-type linear reconstruction, and the MINMOD limiter is used for suppressing possible oscillations around large gradients. A few Riemann solvers has been included in the VAS2D, such as the exact Riemann solver, HLL and HLLC approximate Riemann solvers [4]. In the calculator, the AUFS solver [3] is used by default. The two-step Runge-Kutta scheme was used for achieving second order temporal accuracy.

A grid is automatically generated for any given wedge angle. Totally 19 grids were generated using a meshing tool for every 5 degrees from 0° to 90°, and stored as a grid database. For other angles between, the grid is obtained by modifying and smoothing the grid having the closest wedge angle from the grid database. This strategy is extremely robust, and it can create a grid for any wedge angles, although it can be replaced by a meshing tool that however may possibly crash for some unknown wedge angles.

In fact, constructing the good Problem and CFD modules is much tougher than making CFD software as those available in the market. In the case of commercial codes, one needs to setup parameters, and to optimize the parameters as well for obtaining a good result. In the present calculator, all these are conducted automatically on the background. Extensive experience on CFD and a thorough understanding of fluid dynamics are essential to realizing the automatic optimization.

2.3 Graphical User Interface

The graphical interface of the calculator is, as shown in Fig. 3, designed for parameter input and displaying results. An end-user just needs to specify three parameters, shock Mach number, the angle and the ratio of specific heats on the left panel of the main window interface as shown in Fig. 3a, and the numerical result will be shown on the right as shown in Fig. 3b.

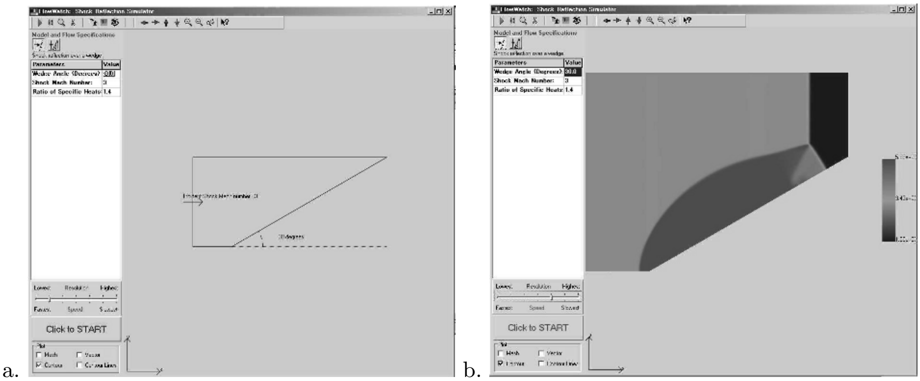

Fig. 3. Graphical user interface of the calculator: a) three parameters are defined on the left panel; b) the dynamic numerical result will be shown on the right window.

The intermediate numerical results during computation and the final result can be depicted. One can stop, restart or give pause to the process of computation by clicking the corresponding icon above. The displayed image is created by an efficient post-processing package that provides a few commonly-used functions, including plotting color/line contours, velocity vectors, drawing numerical mesh, or any combination of them. An example of solution-adaptive grid together with density contours is shown in Fig. 4 for the shock wave of $M_s = 3$ reflecting over a 30° wedge. An image can be moved or zoomed in or out, and one can also zoom in the interested portion of the image by mouse. The image shown on the right window of the calculator can be copied, and pasted directly in Microsoft Word, Adobe Photoshop, and others.

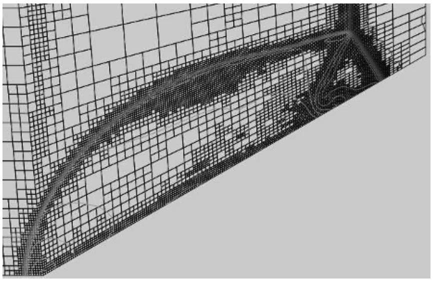

Fig. 4. Density contours with correspoding 4-level adaptive grids

The calculator provides a control for the end-user to select the resolution needed. It appears as a horizontal scroll bar on the left panel. In the CFD Module, it actually reflects the level of refinement. If the scroll bar is moved to the leftmost end, the computation will use the initial grid without any grid refinement. If it is set to the rightmost end, a 6-level refinement will be used. It is set to 1-level refinement by default.

A high resolution result requires more computer time to complete, so that one has to make a trade-off between resolution and speed. Fig. 5 shows a typical result. The computation is conducted on a desktop computer with Window XP operating system and Intel Xeon(R) 1.6GHz CPU. A high resolution result can resolve the wave pattern in more details, while requires more computer time. Notice that it takes about 4 times as much computer time for the increase of one level refinement. For a uniformly structured grid it takes 8 times instead, so that the calculator is about one order faster than the algorithms using uniform grid to achieve the high-resolution result.

3 Concluding remarks

A calculator for simulating shock wave reflections has been proposed. All numerics-related issues have been carefully considered inside. Anyone interested in shock wave phenomena can use it without learning CFD, for example, pure experimenters, engineering students, and so on. The software is available from our website $http:\backslash\backslash www.flowwatch.net$. The methodology followed is applicable to other flow phenomenon. Toward the buildup of a virtual shock wave laboratory, a software package including more models that represents other shock wave phenomenon is under development, and will be distributed through the website.

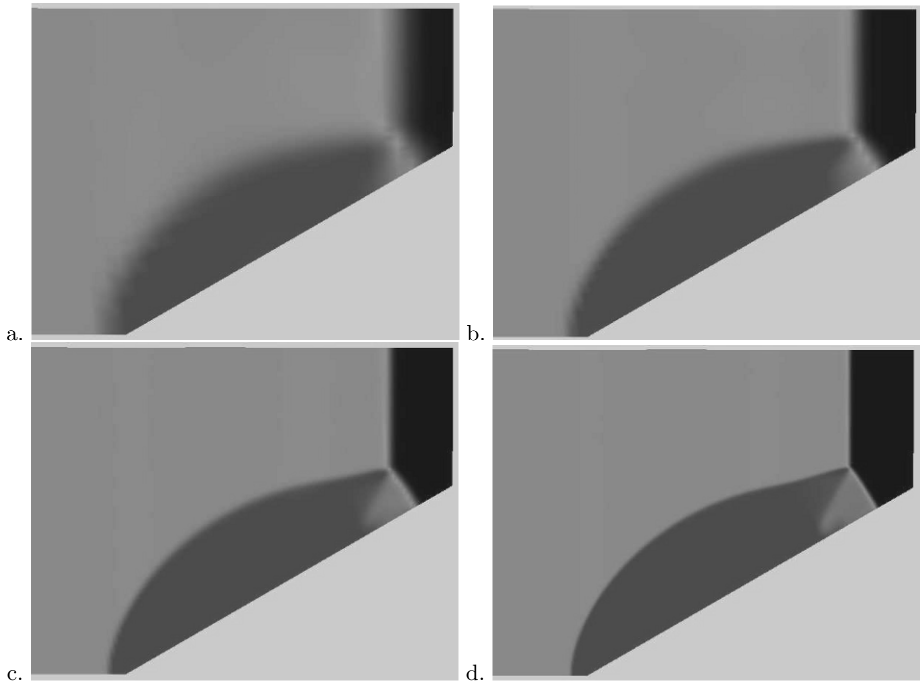

Fig. 5. Results for a shock wave of $M_s = 3$ over a wedge of $\theta_w = 30^\circ$, resolution versus speed: a) 1-level refinement, desktop time required for the computation $T \approx 1s$; b) 2-level refinement, $T \approx 4s$; c) 3-level refinement, $T \approx 16s$; d) 4-level refinement, $T \approx 65s$.

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