α_1, β_1, β_2 - root characteristics of multiply possible theoretical solutions of steady Mach reflections in perfect diatomic gases

J.-J. Liu[1] **and T.-I. Tseng**[2]

[1] *Department of Engineering Science, National Cheng Kung University, Tainan, Taiwan ROC*
[2] *National Center for High-Performance Computing, Hsinchu, Taiwan ROC*

Summary. The existence of the jump in β_1=β_2 double roots in the three-shock theoretical solutions of steady Mach reflections on the (θ_1, M_0) plane is determined for the first time, θ_1 is flow deflection behind the incident shock, M_0 is the incident shock Mach number. This jump behavior of β_1=β_2 double roots explains the occurrence of newly found α_1=β_2 double roots of multiple theoretical solutions of steady MR in perfect diatomic and triatomic gases. The critical condition determining the location of M_0 of this jump, for gas specific heats ratio of 1.4, is obtained to occur at α_1= β_1= β_2 triple roots where M_0=3.41485. There are two findings closely associated with this jump behavior of β_1=β_2 double roots: 1, the α_1 solutions cease to become physically meaningful for $M_0 > 3.41485$; 2, α_1=β_2 double roots do not exist for $M_0 < 3.41485$.

1 Introduction

This work is an extension of Liu [1-3] and Liu et al. [4] reporting on multiplicity regimes of three-shock theoretical solutions of steady Mach reflections (MR) in perfect diatomic and triatomic gases. The existence and basic characteristics of newly found α_1= β_2 double roots of three-shock solutions of steady MR are verified and analyzed in perfect diatomic and triatomic gases [3.4]. As noted earlier [1], the problem of three-shock confluences of MR phenomena in steady flow is important both theoretically and practically. Academically, the problem of weak pseudo-steady MR has attracted attentions of researchers in areas of physics, mathematics, and engineering for more than half of century. A change of coordinates from laboratory to self-similarly propagating triple-point makes pseudo-steady MR equivalent to steady MR in the vicinity of the triple-point. Henderson [5] showed that the equation of motion of a confluence of perfect-gas three-shock waves (i.e. steady MR) could be reduced to a single polynomial equation of tenth degree with the pressure ratio across the Mach stem as the variable. Henderson [5] then gave the maps of the multiplicity of theoretical three-shock solutions on the (θ_1, M_0) plane for $\gamma = 1.1$, 9/7, 7/5 and 5/3 where m = 0, 1, 2 for the former two gases and m = 0, 1, 2, 3 for the latter two (m, the number of physically significant roots). Liu [1,2], on the other hand, pointed out that there exist regimes of m = 3 for $\gamma = 1.1$ and 9/7, and there are erratum in Henderson's multiplicity map for $\gamma = 7/5$. The reason for these mistakes are incorrectness in computed β_1=β_2 double-root lines. Henderson [5] reported that the β_1=β_2 double-root line and triple-root (II) curve coincide over the entire M_0 range considered for γ=1.1, and they merge for $M_0 > 3$ for $\gamma = 7/5$. Liu [1,2], however, showed that these two lines coincide only at M_0 = 2.41 and 3.117 for γ = 9/7 and 7/5, respectively. We show here that these two lines affect the map of the multiplicity of three-shock theoretical solutions of steady MR significantly. Needless to say, the newly found α_1=β_2 double roots not only change the β_1=β_2 double-root line, but dramatically alter the multiplicity map of steady MR. Liu [3] remarked that the occurrence of α_1=β_2 roots implies the existence of a possible discontinuity behavior in the β_1=β_2 double-root line. The aim of this work is the determination of the location of the jump in β_1=β_2 double-root line of theoretical solutions of steady MR on the (θ_1, M_0) plane. Another motivation of this work stems from the above-mentioned link between theoretical solutions of a steady MR, where exact solutions are available, and those of a pseudo-steady MR. Henderson's 1987 work [6] on pseudo-steady MR, which was based on his 1964 work [5] on steady MR, is the most frequently referred theoretical analysis for describing the problem of the von Neumann paradox of weak pseudo-steady MR. However, are these two Henderson's works correct?

2 Analysis

The tenth degree polynomial equation of perfect-gas three-shock confluences given by Henderson [5] is of the form: $R_iT_i = 0$ (i=1,2,..9), where R_i and T_i are algebraic polynomials of degree 6. The tenth degree polynomial equation obtained by Liu [1] is of the form $\sum_{n=0}^{10} C_n x^n = 0$, where C_n *'s* are too lengthy to be given here. For example, the numbers of different terms in coefficients C_3 and C_4 are 748 and 778, respectively. This tenth degree polynomial equation is used for calculating all possible three-shock theoretical solutions for a given condition of a steady MR. An intermediate form of this equation is given in [1].

According to Henderson [5], a physically significant solution of a steady MR requires that the pressure behind the Mach stem be larger than that behind the incident shock. There exist double incident Mach line degeneracies, D_1, double reflected Mach line degeneracies, D_2, a physically possible α_1 solution, a physically impossible α_2 solution. This gives a total of six roots with two pairs of complex roots left. He used symbols β_1, β_2 to represent a second and third appearances of physically possible solutions when one of the two pairs becomes real. In the following, we explain nomenclature used in obtaining multiple three-shock solutions by applying it to a steady MR of $M_0 = 4.015$, $P_1 = 6.215$, $\gamma = 1.4$. Figure 1(a) shows (p - θ) shock polar diagrams illustrating multiply possible three-shock solutions for this steady MR, where m = 2. There is one regular reflection solution RR (weak), one backward-facing reflected shock solution β_1, one forward-facing reflected shock solution β_2. The α_1 root is not physically realizable in this case. The physical plane corresponding to β_1 is shown in Fig. 1(b). The wave configuration is drawn according to calculated results of the β_1 solution, and it agree reasonably well with that of the actual experiment of this case reported in JFM (2002, 459). Note the vast differences among β_1, β_2 and α_1 solutions, and not all intersection solutions lead to physically meaningful results. Liu et al. [4] reported the link between the occurrence of α_1=β_2 roots and a possible jump behavior in β_1=β_2 root-line. More specifically, possible jump behaviors of β_1=β_2 root-line were found to be located in narrow ranges near M_0=3.5 and 2.3 for $\gamma = 7/5$ and $9/7$, respectively. Searching for the exact location of the jump of the β_1=β_2 double-root line on the (θ_1, M_0) plane are carried out by systematically examining multiply possible solutions at different M_0 *'s* by varying θ_1 from upper forbidden sonic conditions to their minimum incident Mach angle conditions. Three series of sequential solutions of M_0 = 3.4, 3.41484 and 3.45 are reported.

3 Locating the jump of β_1=β_2 double roots line by examining theoretical solutions of steady MR at different M_0's

Earlier works of one of the authors [1-4] have explained mathematical and physical meanings for the occurrences of possible theoretical α_1, β_1, β_2, D_2, D_1 and α_2 solutions of perfect-gas steady MR. Owning to the usefulness of (p - θ) shock polar representation of theoretical MR solutions, the results illustrating in Figs. 2-4 are self-explanatory. In particular, solutions of the upper sonic forbidden condition are marked as M_1=1, D_2=α_1, marked as Triple-root (I), D_2=β_1 as Triple-root (I) or (II), D_2=β_2 as Triple-root (II). Solutions of forward- and backward-strong/weak separating, forward- and backward-sonic, forward/backward separating, von Neumann and limiting incident shock Mach angle conditions, β_1=β_2, α_1=β_2 are correspondingly marked. Therefore, only brief descriptions regarding main features of multiply possible solutions are provided for M_0 = 3.4 and 3.45. Detailed discussions on behaviors of α_1, β_1 and β_2 solutions are given for M_0 = 3.41485, where the jump in β_1=β_2 is located. Figures 2 and 4 give systematic shock polar solutions for M_0 = 3.4 and 3.45, respectively. There are similarities and differences in these two series theoretical solutions. Similarities are θ_1 locations of the solutions of triple-root (I) and (II), forward/backward separating, forward- and backward- sonic and von Neumann conditions. However, significant differences are θ_1 locations of β_1=β_2 double roots. Particularly, α_1=β_2 double roots, which occur twice in M_0 = 3.45, do not exist in M_0 = 3.4. The reason for this apparently large difference in θ_1 location in such a narrow variation of M_0 and the occurrence of α_1=β_2 may be understood when one compares calculated solutions of M_0 = 3.4 and 3.5 with those of M_0 = 3.41485. The critical condition for occurring the jump in β_1=β_2 double roots is now found to be the α_1=β_1=β_2 triple-root condition at M_0=3.41485, θ_1=36.4344, shown in Fig. 3(b). One observes that the α_1 solutions cease to become physically realistic, for they are unable to move above D_2 for M_0 > 3.41485. After the occurrence of α_1=β_1=β_2, α_1 and β_2 disappear immediately, and the β_1 moves towards the forward-facing, then backward-facing branches of the reflected shock polar. From the incident polar viewpoint, the β_1 moves from the weak to the strong branches. As for the various specific MR solutions, they are marked in Fig. 3. Most interesting cases are α_1=β_2 roots whose first appearance is the merge of the α_1 and β_2 roots and their second appearance is the separation of these two roots, when M_0 > 3.41485. Finally, this obtained jump behavior of β_1=β_2 double-root line, occurring at M_0=3.41485, 29.8438° ≤ θ_1 ≤ 36.4344°, is added to the map of the multiplicity of theoretical three-shock solutions of steady MR of perfect diatomic gases on the (θ_1, M_0) plane. This is shown in Fig. 5(a). Important locations delineating different regimes of multiplicities of steady MR are marked as points a to k. Their definitions are given accordingly there. Locally enlarged view near the β_1=β_2 jump of 5(a) illustrating various separating or limiting properties of steady MR solution curves are shown in Fig. 5(b).

4 Conclusions

The existence of the jump in β_1=β_2 double roots in the three-shock theoretical solutions of steady Mach reflections on the (θ_1, M_0) plane is determined for the first time, θ_1 is flow reflection behind the incident shock, M_0 is the incident shock Mach number. This jump behavior of β_1=β_2 double roots explains the occurrence of newly found α_1=β_2 double roots of multiple three-shock theoretical solutions of steady MR in perfect diatomic and triatomic gases. The critical condition determining the location of M_0 of this jump, for gas specific heats ratio of 1.4, is obtained to occur at $\alpha_1 = \beta_1 = \beta_2$ triple roots where M_0=3.41485. There are two findings closely associated with this jump behavior of β_1=β_2 double rots: 1, the α_1 solutions cease to become physically meaningful for $M_0 > 3.41485$; 2, α_1=β_2 double roots do not exist for $M_0 < 3.41485$.

References

1. Liu J. J. : A map of multiplicity of perfect-gas three-shock theoretical solutions of steady Mach reflection in diatomic gases, The 5th Int'l Workshop on Shock/Vortex Interactions. Kaohsiung, Taiwan, 120-127, 2003
2. Liu J. J. : Multiply possible three-shock theoretical solution of steady Mach reflections in triatomic perfect-gases, The 5th Int'l Workshop on Shock/Vortex Interactions. Kaohsiung, Taiwan, 105-111, 2003
3. Liu J. J. : A new kind of double roots of three-shock theoretical solutions of steady Mach reflections in perfect triatomic gases, The 28th National Conference on Theoretical and Applied Mechanics, 659-666, 2004
4. Liu J. J., Lin C. C., Chou C. A. : The existence of α_1=β_2 double roots in the three-shock theoretical solutions of steady Mach reflections in perfect diatomic gases, AASRC/CCAS Joint Conference, 2005
5. Henderson L. F. : On the confluence of three shock waves in a perfect gas, Aeronautical Quarterly, vol. 15, 181-197, 1964
6. Henderson L. F. : Regions and boundaries for diffracting shock wave systems, ZAMM, vol. 67, 2, 1987

.

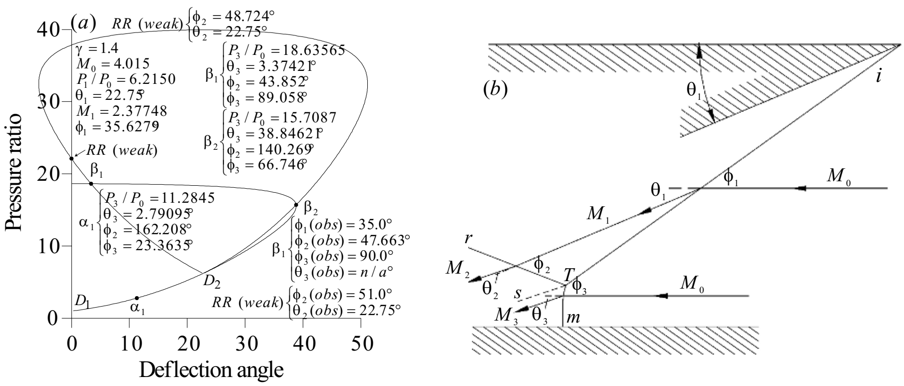

Fig. 1. Steady MR shock polar diagram (a) showing multiple solutions and the physical plane of the β_1 solution for M_0 =4.015, θ_1=22.75°, γ=1.4, where i, incident shock; r, reflected shock; m, Mach stem; s, slipstream. (0), (1), (2), (3), respectively, are flow regions as defined, ϕ_n, wave angles, θ_n, deflection angles, T, triple-point, χ, triple-point angle, M_s, incident shock Mach number, θ_w, wedge angle, M_n, flow Mach number.

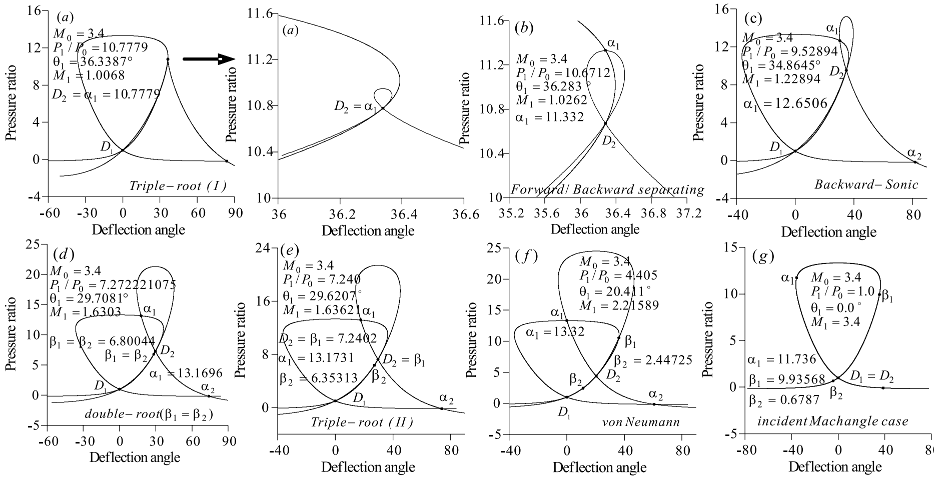

Fig. 2. Sequential (p-θ) shock polar solutions of steady MR of $M_0 = 3.4$

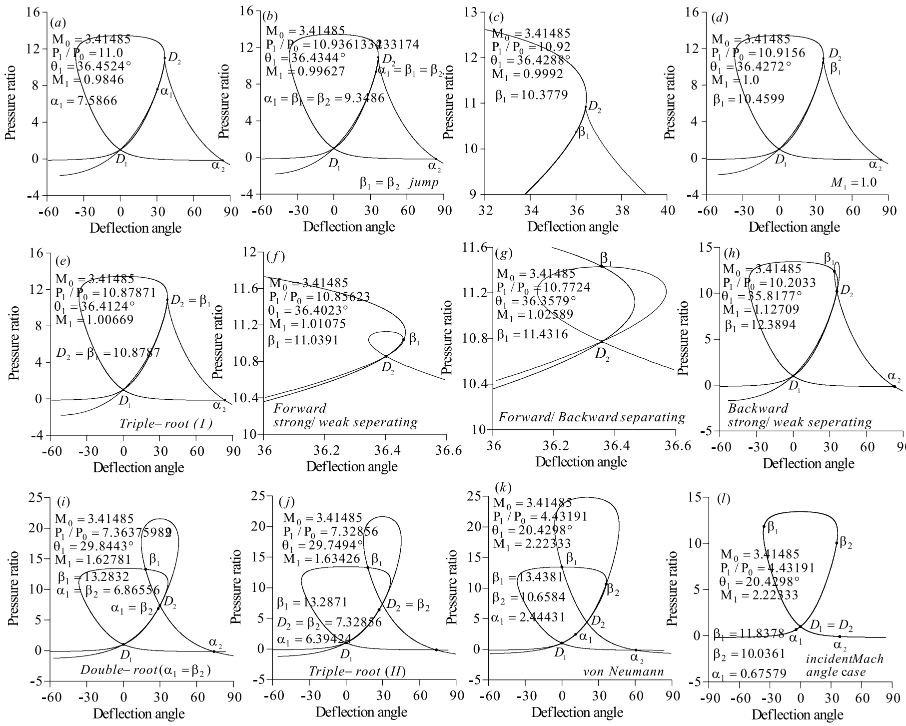

Fig. 3. Sequential (p-θ) shock polar solutions of steady MR of $M_0 = 3.41485$

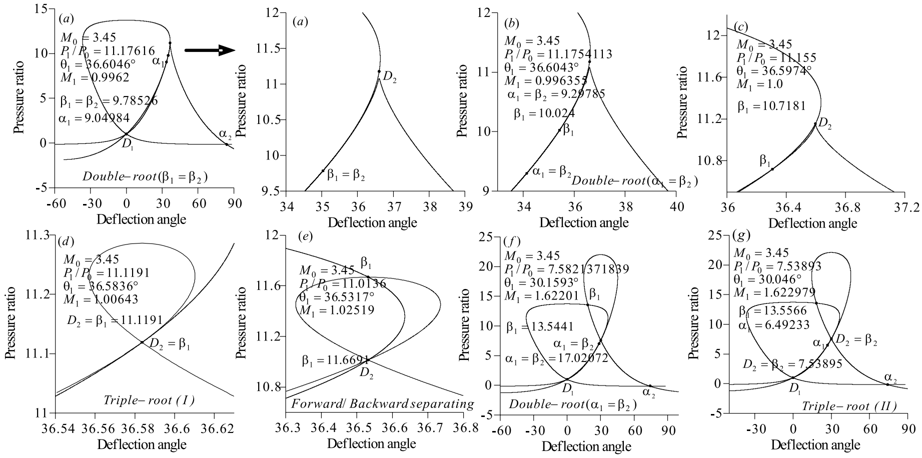

Fig. 4. Sequential (p-θ) shock polar solutions of steady MR of M_0 = 3.45

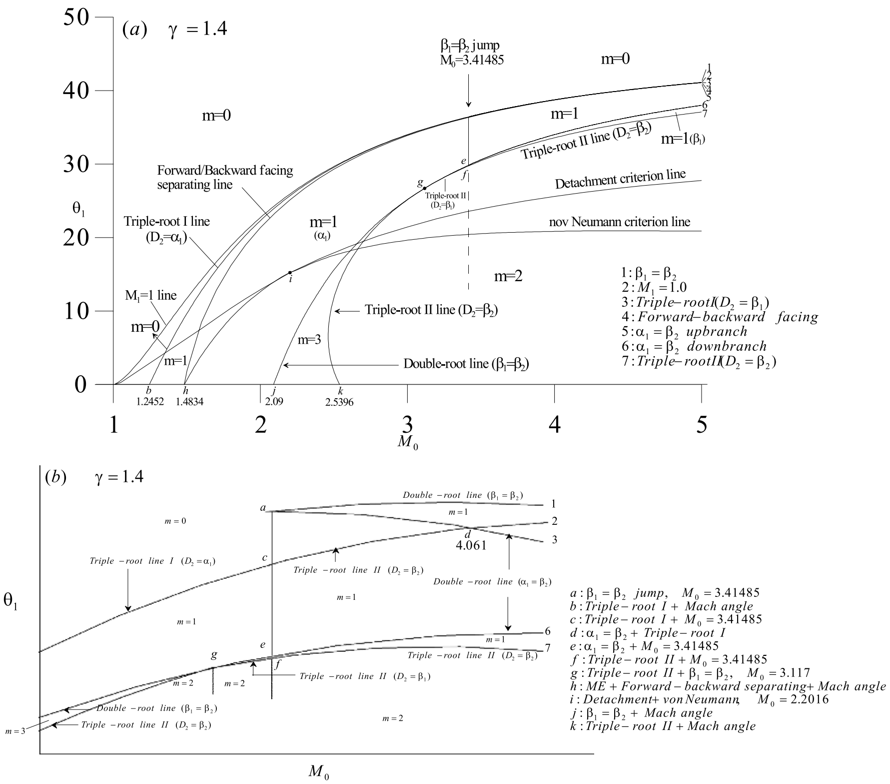

Fig. 5. The map of multiplicity of theoretical three-shock solutions of steady MR in perfect diatomic gases on the (θ_1, M_0) plane. (b) Locally enlarged view (not in proportional to actual curves) near the β_1=β_2 jump showing solution curves of β_1= β_2 jump and various separating and limiting conditions of different resimes of (a).