Pressure waves interference under supersonic flow in flat channel with relief walls

M.-C. Kwon, V.V. Semenov, and V.A. Volkov

Moscow state Aviation Institute, Dept. of Rocket engines, Volokolomskoe Shosse 4, 125493 Moscow, Russia

Summary. In a number of cases, surfaces of channel walls are distorted in the process of operation so that periodically reiterative dimples and convexities are formed. Thus, additional wave drag appears when such relief structures are flowed by the supersonic gas stream. There are theoretical data on relief surfaces wave drag only for some simplest forms of relief, but there are practically no experimental data. This work suggests the method which allows obtaining within the limits of linear approximation the exact formula for wave drag of channel walls with the arbitrary plate in the first and subsequent interference zones. By the example of sinusoidal relief it is demonstrated that pressure waves interference can lead to both increasing and decreasing of wave drag.

1 Formulation of the problem

Consider flat stationary supersonic flows of ideal gas in the channel between walls with flat relieves are represented schematically on Fig. 1.

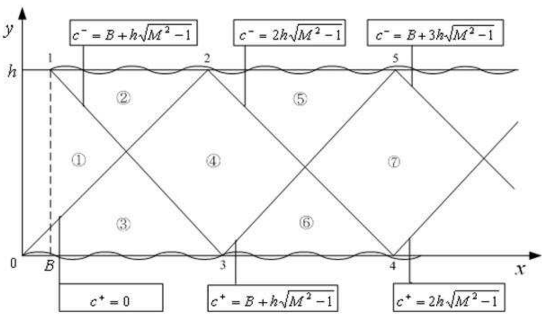

Fig. 1. Schema of relief channel and interference zones

Let functions $\xi_0(x)$ and $\xi_h(x)$ to describe relieves in bottom and upside wall.

$$y_{wo} = \xi_x(0),\ x \in [0, L_0]; \quad y_{wh} = \xi_h(x),\ x \in [B, B + L_h]. \tag{1}$$

It is obvious that functions $\xi_0(x)$ and $\xi_h(x)$ should be continuous. As it is shown in work [1], generally their derivatives of function $\xi_0^{'}(x)$, $\xi_h^{'}(x)$, $\xi_0^{''}(x)$, $\xi_h^{''}(x)$ are possible to suppose sectionally continuous.

In addition assume that functions $\xi_0(x)$, $\xi_h(x)$, $\xi_0^{'}(x)$, $\xi_h^{'}(x)$ in the range of definition satisfies the restrictions:

$$\xi_0(x) \ll L_0,\ \xi_h(x) \ll L_h,\ \ |\xi_0'(x)| \ll 1,\ \ |\xi_h'(x)| \ll 1. \tag{2}$$

Usually suppose that dimensionless pressure disturbance, density and speed are too small:

$$\frac{\delta p}{\kappa p} = \Psi \ll 1; \quad \frac{\delta \rho}{\rho} = E \ll 1; \quad \frac{\delta U}{U} = \Phi \ll 1; \quad \frac{\delta V}{V} = \Theta \ll 1. \tag{3}$$

If the restrictions (2), (3) are executed, the boundary conditions on walls of the channel can be set in linearized form:

$$\frac{\delta V}{U}|_{y=0} = \Theta|_{y=0} = \xi_0'(x), \quad \frac{\delta V}{U}|_{y=h} = \Theta|_{y=h} = \xi_h'(x). \tag{4}$$

Thus local and total wave drag coefficients on walls, it is possible to express through the value of function Ψ:

$$c_{px_0} = \frac{2}{M^2}\Psi_{wo}(x)\xi_0'(x), \quad c_{px_h} = -\frac{2}{M^2}\Psi_{wh}(x)\xi_h'(x). \tag{5}$$

$$c_{pL_0} = \frac{2}{M^2 L_0}\int_0^{L_0} \frac{2}{M^2}\Psi_{wo}(x)\xi_0'(x)dx,\ c_{pL_h} = -\frac{2}{M^2 L_h}\int_B^{B+L_h} \frac{2}{M^2}\Psi_{wh}(x)\xi_h'(x)dx. \tag{6}$$

where M is Mach number of uniform flow.

For the decision of assigned task, it is enough to solve the two equations of two unknown functions Θ, Ψ. If conditions (2), (3) are executed, this system can be written down according to [5]:

$$(M^2-1)\frac{\partial \Psi}{\partial x} + M^2\frac{\partial \Theta}{\partial y}, \quad M^2\frac{\partial \Theta}{\partial x} + \frac{\partial \Psi}{\partial y}. \tag{7}$$

These equations were derived by D.E. Blohinzeva [4] . However, direct application methods in the works [1-3] for decision of the boundary problems (4), (7) appear impossible, so it is difficult to generalize because of intricateness of calculations. Therefore in the given work instead of the D.E. Blohinzeva's method [4], the initial boundary problems (4), (7) are solved by means of the method characteristics [5].

General solutions of linearized system

In case of $M > 1$, we can define the reduced transverse coordinate $\tilde{y}$ and the reduced function of pressure $\tilde{\Psi}$, such that:

$$\tilde{y} = y\sqrt{M^2-1}, \quad \tilde{\Psi} = \Psi\sqrt{M^2-1}/M^2. \tag{8}$$

Substituting (8) into (7), we have:

$$\frac{\partial \tilde{\Psi}}{\partial x} + \frac{\partial \Theta}{\partial \tilde{y}} = 0, \quad \frac{\partial \Theta}{\partial x} + \frac{\partial \tilde{\Psi}}{\partial \tilde{y}} = 0. \tag{9}$$

The general solutions of system (9) agree to [5], such that:

$$\Theta = I^+(c^+) + I^-(c^-), \quad \tilde{\Psi} = I^+(c^+) - I^-(c^-), \quad c^{\pm} = x \mp \tilde{y}. \tag{10}$$

Functions $I^+(c^+)$ and $I^-(c^-)$ are the Riemann intervals, and it's lines agree to $c^{\pm} = const$, where $c^{\pm}$ are characteristics. When we solve the boundary problems, concrete view of functions $I^+(c^+)$ and $I^-(c^-)$ are defined by the method of characteristics [5] on the given boundary conditions (4).

Linearized models of discontinuous characteristics According to definition of shock as gas dynamics parameters, discontinuous characteristics for any function $f(c^+, c^-)$ can be written as:

$$[f]_{c^+=a} = \lim_{c^+\to a+0} f(c^+, c^-) - \lim_{c^+\to a-0} f(c^+, c^-)\ ,$$
$$[f]_{c^-=b} = \lim_{c^-\to b+0} f(c^+, c^-) - \lim_{C^-\to b-0} f(c^+, c^-). \qquad (11)$$

Substituting the general solution (10) into definitions (11), we have:

$$[I^-]_{c^+=a} = 0 \Rightarrow [\Theta]_{c^+=a} = [I^+]_{c^+=a},\quad [\tilde{\Psi}]_{c^+=a} = [I^+]_{c^+=a}\ ,$$
$$[I^+]_{c^-=b} = 0 \Rightarrow [\Theta]_{c^-=b} = [I^-]_{c^-=b},\quad [\tilde{\Psi}]_{c^-=b} = -[I^-]_{c^-=b}. \qquad (12)$$

From (12) only one of invariants has break in discontinuous characteristic, namely the sign coincides with a sign on this characteristic.

2 Wave drag and interference in channel

Wave drag in channel walls depends not only on their relief, but also on initial disturbance entering into the channel. In our study, it is supposed that the interference of waves is caused only by relief wall and that initial disturbance into the channel is equal to zero. This assumption makes the boundary problems (4), (9) explicitly same as [1]. When slope angles of leading edges in both upside and bottom wall are distinct from zero, also as well as in break point, oblique shock wave or expansion wave is appeared. It is modeled by "discontinuous" characteristics [1] within the limits of linear approximation. Therefore it can be supposed that disturbance from the bottom wall is extended from its leading edge to downward stream along the characteristic $c^+ = 0$, and from upside wall- along the characteristic $c^- = B + \tilde{h}$. All flow areas are designated on Fig. 1.

According to the characteristics method and boundary problems, we can obtain pressure distributions in each wall of channel:

$$\tilde{\Psi}_{w0} = \xi_0'(x),\quad 0 < x < B + \tilde{h}\ ,$$
$$\tilde{\Psi}_{w0} = \xi_0'(x) - 2\xi_h'(x - \tilde{h}),\quad B + \tilde{h} < x < 2\tilde{h}. \qquad (13)$$

$$\tilde{\Psi}_{wh} = -\xi_h'(x),\quad B < x < \tilde{h}\ ,$$
$$\tilde{\Psi}_{wh} = -\xi_h'(x) + 2\xi_0'(x - \tilde{h}),\quad \tilde{h} < x < B + 2\tilde{h}. \qquad (14)$$

Substituting (13), (14) into (6), we can receive the coefficients of wave drag on walls:

$$c_{pL_0} = \frac{2}{L_0\sqrt{M^2-1}} \left\{ \int_0^{B+\tilde{h}} [\xi_0'(x)]^2 dx + \int_{B+\tilde{h}}^{L_0} \xi_0'(x)[\xi_0'(x) - 2\xi_h'(x-\tilde{h})]dx \right\},$$
$$\tilde{h} \le L_0 \le 2\tilde{h}\ ,$$
$$c_{pL_h} = \frac{2}{L_h\sqrt{M^2-1}} \left\{ \int_B^{\tilde{h}} [\xi_h'(x)]^2 dx + \int_{\tilde{h}}^{B+L_h} \xi_h'(x)[\xi_h'(x) - 2\xi_0'(x-\tilde{h})]dx \right\},$$
$$\tilde{h} \le L_h \le 2\tilde{h}\ . \qquad (15)$$

In each of formulas (15), the first term- it is the contribution of region where disturbance does not come from other wall, and the second - the contribution of the first interference zone on the given wall.

Interference of wave drags in channel As an example, we consider the particular case that leading edges are not displaced relatively to each walls and both lengths and relieve of them are identical.

$$B = 0, \quad L_0 = L_h = L, \quad \xi_0(x) = \xi_h(x) = \xi(x) \;\Rightarrow\; \xi_0^{'}(x) = \xi_h^{'}(x) = \xi^{'}(x). \tag{16}$$

Then we can obtain the coefficient of total wave drag of channel:

$$c_{pL_\Sigma} = \frac{4}{L_0\sqrt{M^2-1}} \left\{ \int_0^{\tilde{h}} [\xi^{'}(x)]^2 dx + \int_{\tilde{h}}^{L} \xi^{'}(x)[\xi^{'}(x) - 2\xi^{'}(x-\tilde{h})]dx \right\}, \; \tilde{h} \le L \le 2\tilde{h}. \tag{17}$$

Consider the particular case of sine wave relieves:

$$\xi(x) = A\cos(2\pi/\lambda) \;\Rightarrow\; \xi^{'}(x) = -2\pi A/\lambda \sin(2\pi/\lambda). \tag{18}$$

And define two dimensionless parameters and the total wave drag of channel in case of periodic relieves depends on these parameters:

$$\Lambda = \frac{L}{h\sqrt{M^2-1}} = \frac{L}{\tilde{h}}, \quad K = \frac{L}{\lambda}. \tag{19}$$

ParameterΛ is the interference number, K is the wave number. It fully characterizes modes of interference. Actually, when $\Lambda < 1$, any disturbances are generated by one wall can not reach to the other walls so that interference zones on both walls are absent. In this case the total coefficient of wave drag of the channel can be written:

$$\Lambda \le 1 \;\Rightarrow\; c_{pL_\Sigma} = \frac{16\pi^2}{L\sqrt{M^2-1}} \int_0^L [\xi^{'}(x)]^2 dx. \tag{20}$$

Substituting profile (18) into (20), we have:

$$\Lambda \le 1, \; c_{pL_\Sigma} = \frac{16\pi^2(\frac{A}{\lambda})^2}{L\sqrt{M^2-1}} \int_0^L \sin^2(2\pi\frac{x}{\lambda})^2 dx = \frac{8\pi^2}{\sqrt{M^2-1}}(\frac{A}{\lambda})^2[1 - \frac{\sin(4\pi K)}{4\pi K}]. \tag{21}$$

When $1 < \Lambda \le 2$, each region of walls has the first interference zone so the characteristics are appeared in these regions. Substituting profile (18) into (17), and using replacement of integration variable, we have:

$$c_{pL_\Sigma} = \frac{16\pi^2(\frac{A}{\lambda})^2}{L\sqrt{M^2-1}}\{\int_0^1 \sin^2(2\pi K\eta)d\eta - 2\int_{1/\lambda}^1 \sin(2\pi K\eta)\sin[2\pi K(\eta - \frac{1}{\lambda})]d\eta\}. \tag{22}$$

Both integrals in curly bracket in (22) are expressed through elementary functions. This final result can be presented as:

$$1 < \Lambda \le 2, \quad c_{pL_\Sigma} = \frac{16\pi^2}{M^2-1}\left(\frac{A}{\lambda}\right)^2 N(K,\Lambda),$$

$$N(K,\Lambda) = \frac{1}{2}\left(1 - \frac{sin(4\pi K)}{4\pi K}\right) - \left(1 - \frac{1}{\Lambda}\right)\cos(2\pi\frac{K}{\Lambda}) + \frac{\cos(2\pi K)}{2\pi K}\sin\left[2\pi K\left(1 - \frac{1}{\Lambda}\right)\right]. \tag{23}$$

Function $N(K, \Lambda)$ is enough complex. Flow satisfies the value $\Lambda = 2$ when the characteristics beginning on leading edges, after reflection precisely reach on edge assignment of the same wall. In this case the first interference zone is closed, consequently resonance effects take place. In fig. [2], [3], [4], we reduce some graphics illustrating behavior of function $N(K, \Lambda)$ in regions $1 < \Lambda \leq 2$.

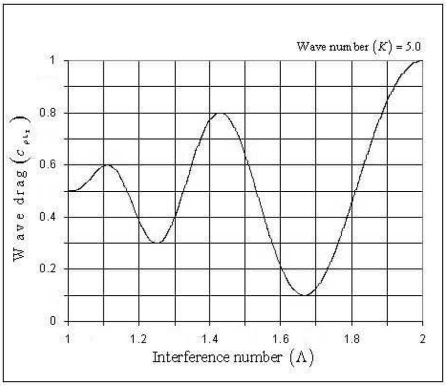

Fig. 2. Value of wave drag at odd wave number($K = 5.0$) has maximum value that is equal to unit. It is exceeding twice as much as total wave drag value in same two plates. (at $\Lambda = 2.0$)

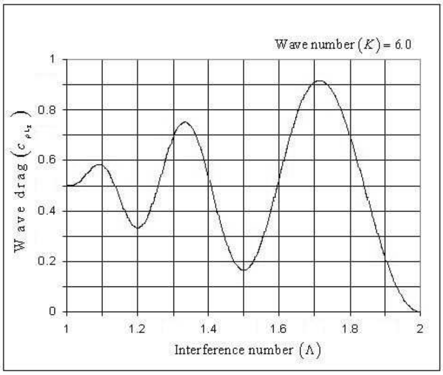

Fig. 3. Value of wave drag at even wave number($K = 6.0$) has minimum value that is equal to zero. (at $\Lambda = 2.0$)

In Fig. [4], it is shown that the minimum value of wave drag is equal to zero at all even values of wave number, when interference number is $\Lambda = 2$. Also we can see that the overall maximum value of wave drags is approximately equal to 1.06, it is realized at $K \approx 0.8$. Consequently, the local maximum value of wave drags little bit exceeding unit and monotonously tends it with growth K, when wave number is odd natural numbers. As we said, in the plane of parameters Λ and K, there are set of numerable points $K_n \approx n$

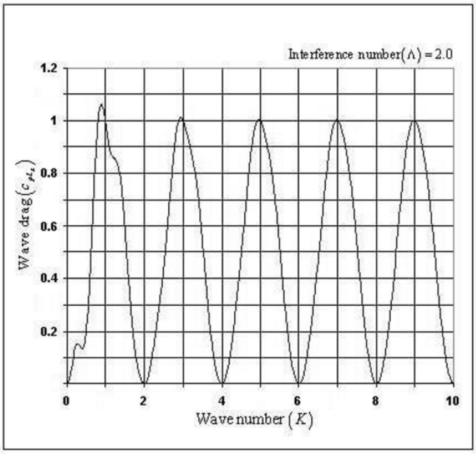

Fig. 4. Interference number (Λ) = 2.0

at $\Lambda = 2$, and resonant effects are shown as that wave drags have the local maximum and minimum values.

3 Conclusion

Flow regimes with interference number are formulated in the range $\Lambda \leq 2$. It is possible to calculate analytically or numerically the factors of wave drag in rectilinear flat channels with any relief walls, which have finite number of break points. In our study, theories are based on linearized equations of gas dynamics. Its predictive ability now is not doubted, because it was successfully applied to the decision of many problems of applied aerodynamics [3], [4], [5]. It is necessary to proof the results of his study; not for confirmation of the theory, and for definition of the boundary area which was considered in this work.

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