Dynamics of unsteady shock wave motion

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Summary. An experimental study of a normal shock wave subject to unsteady periodic forcing in a parallel walled duct has been conducted. Measurements of the pressure rise across the shock wave have been taken and the dynamics of unsteady shock wave motion have been analysed from high speed schlieren images. The velocity of shock wave motion is shown to be related to the instantaneous pressure ratio across a shock wave. The relative importance of geometry and pressure perturbation frequency on dynamic shock wave behaviour is considered and the concept of a critical frequency is proposed, which relates the two. In the absence of flow separation the effects of viscosity on the dynamics of unsteady shock wave motion appear to be small. Further work establishing non-dimensional relations is proposed to improve the general applicability of the findings of this study.

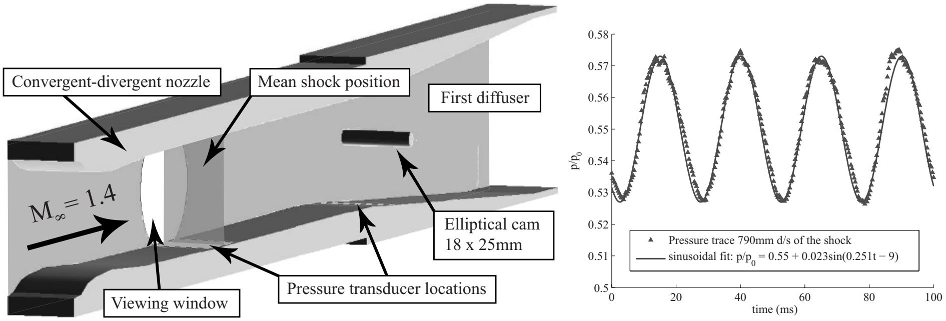
1 Introduction

Shock wave / boundary layer interactions (SBLIs) occur in many high speed aerodynamic applications and often have detrimental effects on performance. In many instances, interactions have been observed to exhibit significant unsteady behaviour. Due to the large changes in local flow properties that occur across shock waves, the presence of unsteady effects can lead to large and undesirable local fluctuations in properties such as pressure and the rate of heat transfer. Current understanding of the mechanisms that govern unsteady SBLIs has not yet reached the level where unsteady effects can be reliably predicted. For this reason, modern design methods use rules of thumb and large safety margins to avoid designing applications where unsteady SBLIs are likely to be present. This cautious approach of avoiding, rather than designing for, SBLI unsteadiness is a limiting factor in the design of modern high speed aerodynamic applications.

It is widely accepted that many examples of unsteady shock wave behaviour, such as buffeting of transonic aerofoils [1] and buzz or engine unstart in supersonic engine intakes, are caused by periodic pressure perturbations downstream of the shock wave. On a transonic aerofoil, these may come from the aerofoil's wake while, in an aero-engine, they might originate from the face of the compressor. Despite the serious safety and performance implications of phenomena such as transonic buffeting and engine unstart, no reliable techniques for predicting their characteristics exist. In a previous study [2], it was found that unsteady SBLIs are influenced by a combination of viscous and inviscid factors but their relative effect on the oscillation amplitude was not studied. At very high frequencies, the amplitude of shock wave motion is known to be very small and of little concern. With decreasing frequency however, the amplitude of unsteady shock wave motion has been shown to increase [3], though the reasons for this are not well understood. It is the aim of the present study to address this particular issue.

2 Experimental Methods

Experiments have been performed in the blowdown-type supersonic wind tunnels of the University of Cambridge. The tunnels have a rectangular working section with a constant cross section 114 mm wide and 178 mm high. For the present study, an elliptical cam was mounted at the beginning of the first diffuser, 790 mm downstream of the mean shock position, as shown in fig. 1(a). During tunnel runs, the tunnel stagnation pressure was held constant and the cam was rotated at frequencies between 8 and 46 Hz to produce a periodic variation in tunnel back pressure at a frequency double that of cam rotation. Fig. 1(b) shows measurements of static pressure beneath the rotating cam at a cam rotational frequency of 20 Hz. It can be seen that the pressure varies almost sinusoidally, and this was observed to be the case at all frequencies tested. This fluctuating back pressure caused the position of the tunnel's normal recovery shock wave to oscillate about its mean position. The tunnel operating parameters were chosen so that the shock wave was located at the centre of the working section window under steady flow conditions.



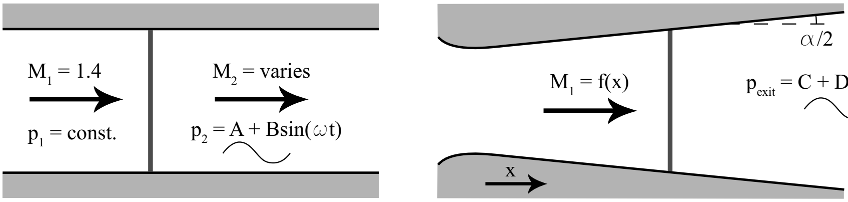
a) Wind tunnel working section b) Tunnel back pressure

Fig. 1. Experimental arrangement

Wall static pressure measurements were obtained using eight Druck PDCR-200 series pressure transducers located directly beneath 0.5 mm diameter holes in the areas of the tunnel floor highlighted in fig. 1(a). High speed schlieren images were obtained using a Photron FASTCAM-ultima APX high speed camera at shutter speeds of 1/6000 s and a resolution of 512x512 pixels.

Analytical and computational study

In addition to experiments, a simple analytical and computational study of a normal shock wave of strength $M_\infty = 1.4$ in a duct subject to downstream pressure variations has been conducted. Parallel and diverging duct geometries were investigated, as shown in fig. 2. In all cases, the magnitude of downstream pressure perturbations have been scaled to match those measured experimentally.



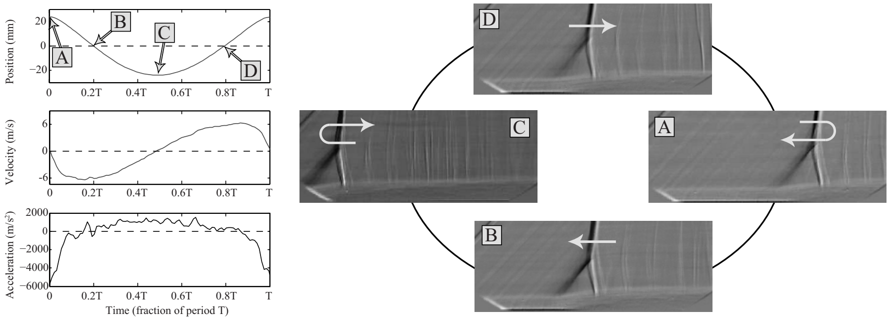
(a) Parallel walls

(b) Diverging walls

Fig. 2. Duct geometries used for the analytical and computational study

3 Results

When subjected to sinusoidal variations of downstream pressure, the normal shock wave undergoes consistent and repeatable periodic motion. Fig. 3(a) shows plots of position, velocity and acceleration for a complete cycle of periodic shock wave motion at a shock wave oscillation frequency of 40 Hz. Fig. 3(b) shows a selection of schlieren images that correspond to the points in the cycle marked A, B, C and D. All images measure approximately 110 mm by 40 mm and the flow is from left to right. The arrows on the images indicate the direction of shock wave motion.



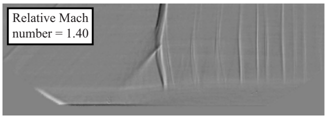
(a) Dynamics

(b) Schlieren images

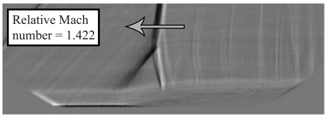
Fig. 3. Results for a shock wave oscillation frequency of 40 Hz

The skewed velocity variation is caused by bunching and spreading of compression and expansion waves, respectively, as they travel upstream from the cam. This is most clearly seen by the spike in the acceleration plot at position A, which is due to bunching of compression waves leading to a very rapid rise in pressure behind the shock wave which encourages upstream motion. Results at all frequencies are broadly similar to those presented in fig. 3. The main difference is that amplitudes decrease and accelerations increase with increasing frequency. Peak velocities at all frequencies are around 7 m/s in both the upstream and downstream directions, which is of the order of ±2% of the freestream velocity, $U_\infty = 410$ m/s. This corresponds to a change in the relative Mach number of the shock wave of the order of ±2%.

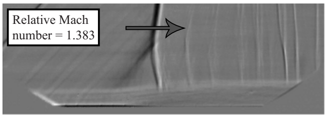
Fig. 4 shows the pressure rise at two points of the unsteady SBLI cycle when the shock wave is in same place but traveling in different directions (points B and D in fig. 3) compared to the pressure rise through a steady SBLI at $M_\infty = 1.4$. Also marked on the graph are two predicted downstream pressures. These were calculated based on the expected pressure rise across the interaction for the instantaneous relative Mach numbers shown on the images, scaled to the same constant upstream pressure as the steady interaction.



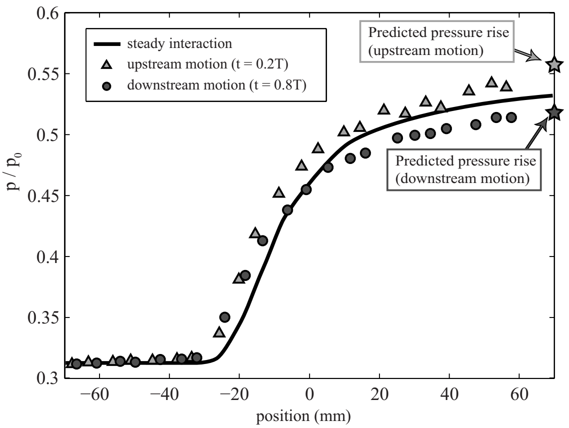
(a) Steady shock wave



(b) Upstream shock wave motion (t = 0.2T)



(c) Downstream shock wave motion (t = 0.8T)



(d) Pressure rise across the interaction

Fig. 4. Comparison of pressure rise during upstream and downstream shock wave motion for a shock wave oscillation frequency of 70 Hz

The predicted pressure rises closely match the experimental measurements in fig. 4 and this suggests that the pressure jump across the unsteady SBLI depends only on the flow Mach number relative to the shock wave. This implies that the velocity of shock wave motion can be determined analytically for a given (varying) pressure ratio. Furthermore, given the pressure variation driving shock wave motion, the shock wave trajectory can easily be calculated by integrating the predicted shock wave velocities, thus yielding the amplitude of shock wave motion. Fig. 5 shows the results of such a calculation for various frequencies of pressure fluctuation, together with experimentally measured amplitudes.

The agreement between prediction and experiment is very good. This clearly suggests that unsteady shock wave motion is simply the mechanism by which a shock wave changes its strength to satisfy an imposed varying pressure ratio. For a given pressure fluctuation, shock wave velocities should therefore be independent of frequency and this is indeed the case, as previously observed. Experimentally measured amplitudes are in general around 20% below the analytical prediction, which suggests that other, most likely viscous, factors are also of some importance.

Fig. 5 predicts that oscillation amplitudes become infinitely large for frequencies tending to zero. While this is correct for a truly inviscid parallel duct, it clearly cannot be

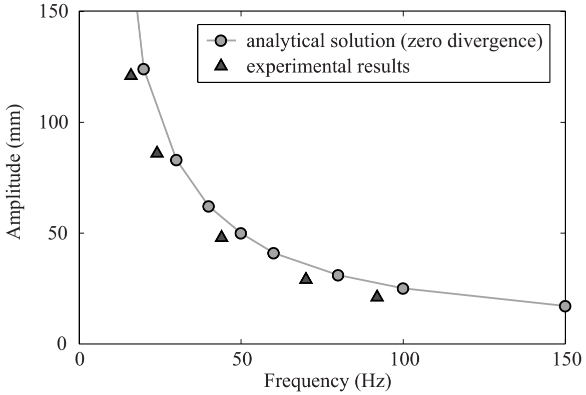


Fig. 5. Comparison of analytical and experimental results

the case in a diverging duct. The effect of divergence has been studied computationally using a simple 1D Euler scheme. The results for a number of different duct divergence angles are presented in fig. 6.

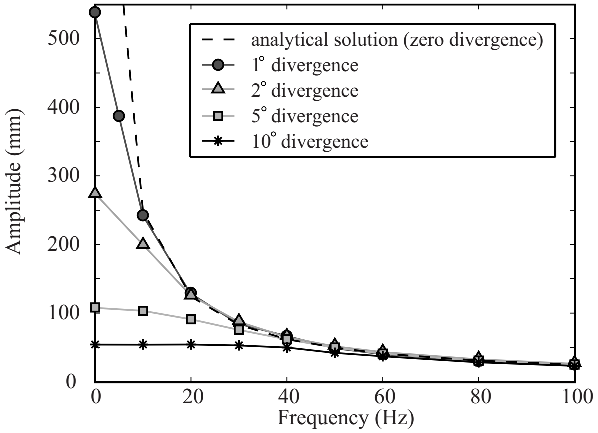


Fig. 6. Effect of divergence

The results show that at low frequency, the amplitude of shock wave motion tends toward limit amplitudes. These correspond to the difference in the steady shock wave positions at the extremes of the duct exit pressures. These effectively set the 'steady state' upstream and downstream positions of the shock wave and hence the 'zero-frequency amplitude'. At higher frequencies, the amplitude of shock wave oscillation is largely independent of divergence angle and very closely matches the analytical prediction for parallel ducts. Fig. 7 shows the general shock wave behaviour.

For a given geometry and imposed downstream pressure perturbation, a critical frequency exists such that: At frequencies below f_{crit}, the amplitude of shock wave motion is primarily determined by the divergence of the duct and is independent of frequency, while

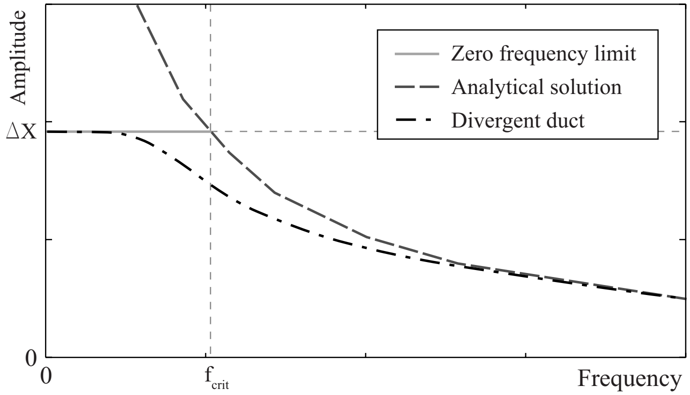


Fig. 7. Amplitude - frequency relation in a diverging duct

at frequencies above f_{crit}, the amplitude becomes almost independent of duct divergence and is only a function of frequency. The implications of this are potentially significant, and the ability to predict the critical frequency could be of great use to designers.

4 Conclusion

Experiments studying the dynamics of unsteady shock wave motion have been performed. The relationship between the amplitude and frequency of unsteady shock wave motion has been measured and a model for the situation of a normal shock wave in a diverging duct has been outlined. A critical frequency is proposed that relates the relative importance of geometry and the frequency of pressure perturbation to dynamic shock wave behaviour. Simple inviscid analytical and computational schemes have been observed to capture the overall physics of the situation, suggesting the effects of viscosity are small in the current setup. The findings of this work have implications for the unsteady performance of applications such as transonic diffusers, mixed compression engine inlets and transonic aerofoils. Further work is needed to establish non-dimensional relations and allow a more fundamental analysis of the problem and greater applicability to real-world situations.

References

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