Numerical simulation of the propagation of stress disturbance in shock-loaded granular media using the discrete element method

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Summary. By using the discrete element method, we perform numerical simulations of the propagation of stress disturbance in granular media composed of spherical particles placed in a vertical channel with a rectangular cross section. The results show that (1) the shock loads subjected to the uppermost granular particles are transferred through certain paths randomly distributed in the granular medium, (2) the loads are not evenly distributed on the bottom wall as postulated in earlier works, (3) the locally averaged, internal force per unit area introduced in the present work is helpful to understand macroscopic load transfer processes inside shock-loaded granular columns, and (4) the maximum stress is not observed on the bottom wall, but in the upper or middle of the granular column.

1 Introduction

Granular materials can be defined as any materials composed of many individual, macroscopic solid particles, irrespective of particle size [1]. They are ubiquitous in nature and of great importance in applications as diverse as pharmaceutical, food and semiconductor industries [2]. Due to dissipative interactions between grains, granular matter exhibits unique and complex behaviors, which are much different from those of the other familiar forms of matter — solids, liquids or gases [3].

One of the most prominent properties of granular media might be quick dissipation of kinetic energy. In fact, there are many practical applications of granular media as absorbents of mechanical perturbations such as vibration, sound and shock wave. Thus granular materials are expected to be used for protection of people and plants against shock and blast waves. Mainly from this standpoint, the subject of shock wave interaction with granular media has attracted increasing interest in the shock wave research community. Britan and Levy [4] have reviewed recent investigations into this subject in detail. Although earlier efforts such as reviewed in their paper have revealed many important phenomena inside shock-loaded granular media, the dynamics of contact force transfer is still poorly understood.

In the present study, numerical simulations of the propagation of stress disturbance in shock-loaded granular media composed of spherical particles are performed by using the discrete element method (DEM) [5, 6]. The main purpose of the present work is to examine how stress disturbance propagates through the granular media placed in a vertical channel container after shock loading, which is extremely difficult to understand from earlier shock tube experiments (e.g., [7]).

2 Numerical simulation

2.1 Discrete element method (DEM)

DEM is the time-dependent numerical solution of Newton's equation of motion for all particles of which the granular material consists. In the present simulations, the granular media are modeled as assemblies of rigid spherical particles. The equations of translational and rotational motion of the i-th particle are given by

$$m_i \frac{d^2 \boldsymbol{u}_i}{dt^2} = \sum_{j=1}^{N} \boldsymbol{F}_{ij} + m_i \boldsymbol{g}, \qquad I_i \frac{d^2 \boldsymbol{\theta}_i}{dt^2} = \sum_{j=1}^{N} \boldsymbol{M}_{ij},$$

respectively. Here t is the time, m_i is the mass, I_i is the moment of inertia, $\boldsymbol{u}_i$ is the displacement, $\boldsymbol{\theta}_i$ is the angular displacement, $\boldsymbol{F}_{ij}$ and $\boldsymbol{M}_{ij}$ are the force and moment, respectively, exerted on the i-th particle by the j-th particle, N is the total number of particles which contact with the i-th particle, and $\boldsymbol{g}$ is the gravitational acceleration. These equations are numerically integrated based on Cundall's algorithm [5].

2.2 Contact force model

In the present simulations, the normal and tangential contact forces between two particles are calculated using the Voigt model with a no-tension joint [6]. For the tangential force model, a friction slider of Coulomb-type is inserted between the particles to take account of the slip at the contact point. The spring constant in the normal direction is obtained from Hertz's solution of the contact problem [8]. The viscosity coefficient in the normal direction was determined so as to reproduce the results of free fall experiments. On the other hand, the spring constant in the tangential direction is determined using Mindlin's theory [9, 10] and the viscosity coefficient in the tangential direction is evaluated with the critical dumping value.

2.3 Granular media model and simulation parameters

The granular media are modeled as assemblies of 1.5 mm-diameter spherical particles made of a polyacetal resin, which are placed in a vertical channel container with a rectangular cross section ($w \times w$) and a height (d_{L}) such as shown in Fig. 1. The side walls and the bottom wall of the container are made of a polyacetal resin and brass, respectively. The dimensions of the containers are summarized in Table 1. In order to examine the effects of the particle arrangement on the load transfer process, we conducted simulations both for body-centered cubic (BCC) and random assemblies.

The shock loading is achieved by applying a downward step force equally on the uppermost particles in the present simulations. The magnitude of the applied force is estimated from the gas pressure behind a shock wave normally reflected from a flat wall. It is assumed that the Mach number of the incident shock wave is 1.33 and the initial pressure is 0.1 MPa, which correspond to those in shock tube experiments [7].

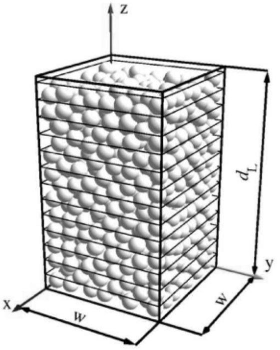


Fig. 1. Granular media model

Table 1. Dimensions of the granular media models

	d_L (mm)	w (mm)
Case 1	20	12
Case 2	20	24
Case 3	60	12

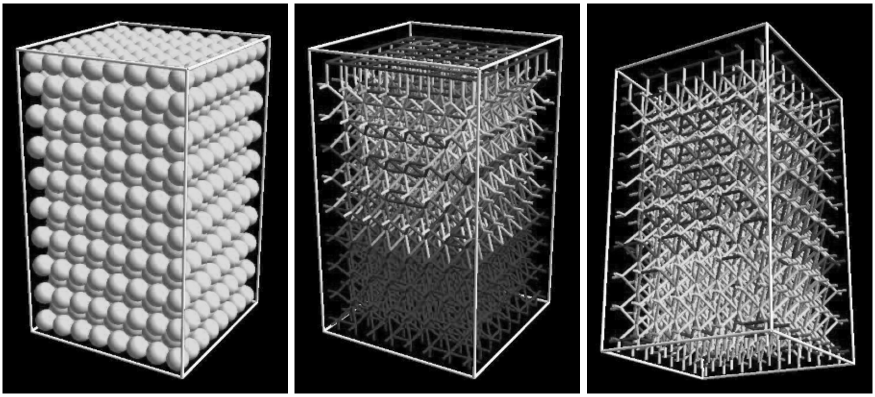
3 Results and discussion

3.1 Contact force distributions

Representative results for BCC and randomly arranged assemblies (both Case 1) are shown in Fig. 2. In this figure, the normal contact force distributions within these granular media 60 and 120 μs after the shock loading are visualized along with the initial arrangements of spherical particles. The pipe segments drawn between the centers of two particles which contact each other designate the load transfer paths and their color indicates the magnitude of the normal contact force.

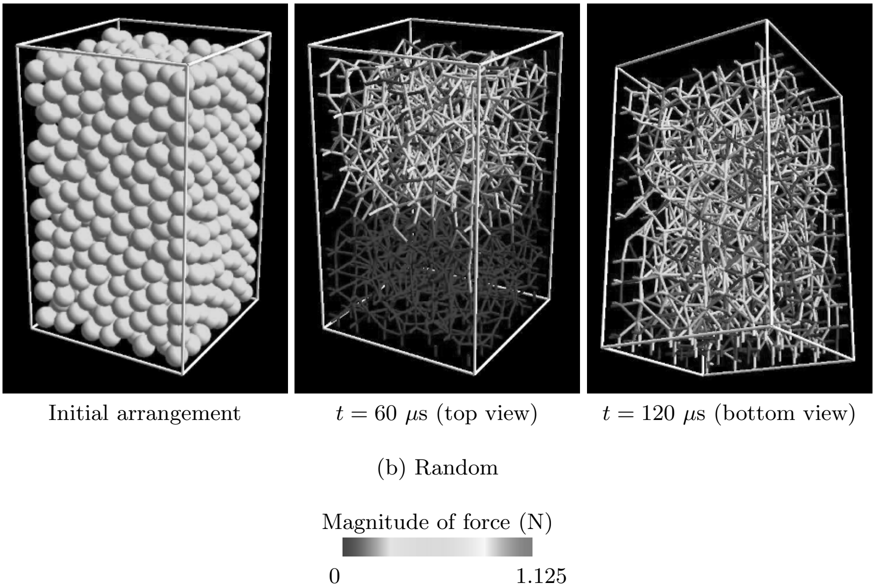
For the BCC case, the loads are regularly transferred from the top to the bottom as expected. The front of the load transferred region is clearly visible in the snapshot at $t = 60$ μs. When the load front reaches the bottom wall at $t = 120$ μs, the normal contact force on the bottom wall peaks. It should be noted here that the loads are not evenly distributed on the bottom wall as postulated in shock tube experiments [4,7]; the contact forces near the corners of the container are smaller than those in the center area. This fact clearly suggests the great importance of the dimension and the shape of the container to the load transfer processes.

For the randomly arranged case, the situation is more complicated; the shock loads subjected to the uppermost particles are transferred through certain paths randomly distributed in the granular medium called stress chains [11]. Although the load transfer front is barely visible in the snapshot at $t = 60$ μs, the wave-like load transfer process cannot be observed from the snapshot at $t = 120$ μs any longer. We should mention here that several, highly concentrated load paths (designated by the red pipes in the figure) can be seen in the upper region of the granular column. As pointed out in [11], increased contact force along these paths may lead to localized fracture and subsequent friction of



Initial arrangement $t = 60$ μs (top view) $t = 120$ μs (bottom view)

(a) BCC



Initial arrangement $t = 60$ μs (top view) $t = 120$ μs (bottom view)

(b) Random

Fig. 2. Arrangement of spherical particles and normal contact force distributions in shock-loaded granular media (Case 1). Here t is the time after the shock impact

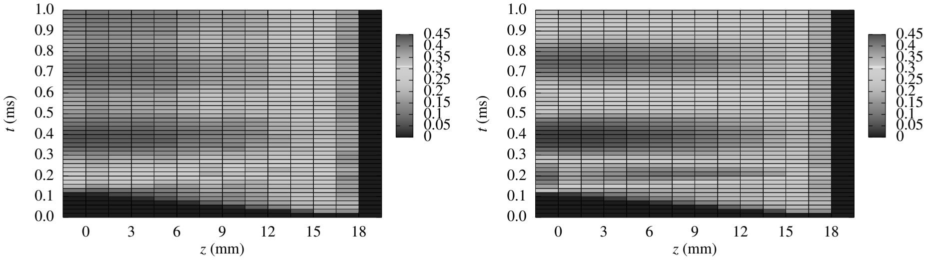
newly formed surfaces, which is a possible source of hot spot within reactive granular materials.

3.2 Propagation of the stress disturbance

In order to better understand the load transfer processes in shock-loaded granular media, we calculate the internal normal force per unit area that balances to the external load applied to the particles inside a sliced region (1.5 mm height) of granular columns, such as illustrated in Fig. 1 (referred to hereafter as "stress"). Figure 3 shows temporal evolutions of the stress distributions inside granular media with different dimensions (Cases 1 to 3). Note that the value depicted in the negative region in the z-axis coordinate represents the stress on the bottom wall.

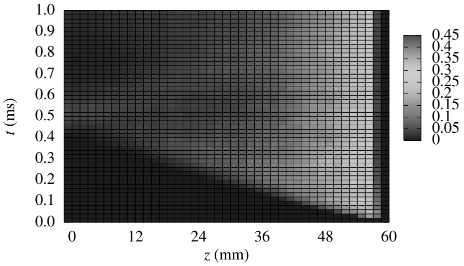
We can clearly observe wave-like propagation of stress disturbance in granular media from these figures. For relatively shallow granular media (Cases 1 and 2), the loads are transferred to the bottom wall and the oscillations of the stress are observed anywhere in the granular column. For deeper granular media case (Case 3), on the other hand, the load cannot be transferred to the bottom any longer, but rather dumped in the upper region. These results qualitatively agree with those from stress measurements conducted in shock tube facilities [4, 7].

Thus, the proposed stress (locally averaged, internal force per unit area) is found to be helpful to understand macroscopic load transfer processes inside shock-loaded granular media. It is worth pointing out that the maximum stress is not observed on or near the bottom wall even for relatively shallow media cases, but in the upper or middle of the granular column (see Figs. 3 (a) and (b)). It seems to be extremely difficult to explain why this happens by continuum mechanical approaches.



(a) Case 1

(b) Case 2



(c) Case 3

Fig. 3. Temporal evolutions of stress distribution (unit: MPa)

4 Conclusion

Numerical simulations of the propagation of stress disturbance in granular media were performed by using the discrete element method to shed light on complex behavior of shock-loaded granular media. The granular media were modeled as assemblies of spherical particles, which were placed in a vertical channel container with a rectangular cross section. The main conclusions derived from the present work are summarized as follows:

1. The shock loads subjected to the uppermost granular particles are transferred through certain paths randomly distributed in the granular medium (stress chains).
2. The loads are not evenly distributed on the bottom wall as postulated in shock tube experiments conducted previously.
3. The locally averaged, internal force per unit area introduced in the present work is helpful to understand macroscopic load transfer processes inside shock-loaded granular columns.
4. The maximum stress was not observed on the bottom wall even for relatively shallow media cases, but in the upper or middle of the granular column.

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