Two dimensional structure and onset Mach number of condensation induced shock wave in condensing nozzle flows

M. Yu[1], **M.L. Wang**[1], **B. Huang**[1], **H. Xu**[1], **Y.J. Zhu**[1], **X. Luo**[2], **J.M. Yang**[1]

[1] *Department of Modern Mechanics, University of Science and Technology of China,230027 Hefei, P.R. China*
[2] *Shock Wave Laboratory, RWTH Aachen University, 52056 Aachen, Germany*

Summary. The two dimensional structure of condensation induced shock wave in condensing nozzle flows is investigated numerically and experimentally. By means of the dimensional analysis method, two similarity laws of the onset Mach number and structure of condensation shock are obtained for a same parameter, which is the compromise of condensation strength and the energy consumed before condensation. The similarity law of onset Mach number agrees well with the experimental results from other authors.

1 Introduction

Phase transitions of vapor-liquid are often observed in daily life, such as the formation of clouds in the atmosphere and the dew on the leaves. Since the cooling process is often very slow, condensation occurs in equilibrium in these phenomena where significant number of foreign agents (e.g. ions, aerosols, particles) act as condensation nuclei. These phenomena are called heterogeneous condensation. When the characteristic timescale of the cooling is small compared to the timescale of the agglomeration of vapor molecules at foreign agents, the metastable supersaturated state is achieved, and then the condensation nuclei form in the vapor phase itself. This is called homogeneous nucleation. A fast expansion of vapor would cause a fast drop of temperature and a spontaneous condensation, of which the development to liquid phase is dominated by homogeneous nucleation.

Vapor-liquid transitions are also important in many fields of science and technology, such as transonic and supersonic flows, turbo-machinery and propulsion engines. Much work has been done in these fields [1–4].

Since the condensation induced weak shock in the nozzle flow was firstly visualized by Prandtl [5], the studies of non-equilibrium condensation in transonic/supersonic nozzle have been widely carried out by both experiments and numerical simulations [6, 7].The states of nozzle flow are easy to measure; the effects of condensation are easily derived from one-dimensional frictionless gas dynamic models [8]. These models were applied in the dependence of flow states on relative humidity in the supply (Φ_0) and agree well in most experiments. But difficulties arise when the stagnation temperature is varied, and the structure of condensation induced shock wave in the nozzle with high wall curvature hasn't been well studied.

The motivation of present work is to study the structure of condensation shock and the combined dependence of onset Mach number on T_0 and Φ_0 in two-dimensional nozzle with high wall curvature.

2 Experimental setup

We study the condensation of moist air in two-dimensional nozzle both experimentally and numerically. The schematic of experimental setup was shown as Fig. 1, in which the half height of nozzle throat, $y^* = 5.22$ mm; the radius of the wall, $R^* = 24.9$ mm and the width of the nozzle is 13.6 mm. The position and structure of condensation shock are obtained by schlieren method.

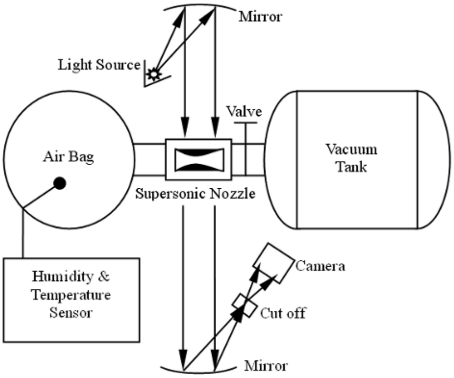


Fig. 1. Schematic of experimental setup

3 Numerical method

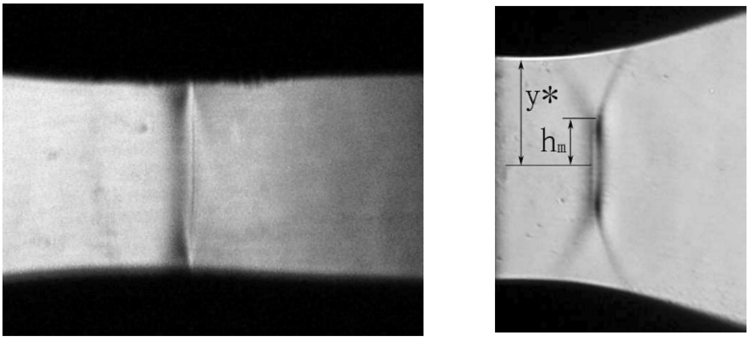
In our numerical study we applied the numerical method of ASCE2D [3]. The governing equation for the complete system can be considered as the combination of the moment equations of the size distribution with the Euler equations for time-dependent two-dimensional flow. This method adopts a condensation model consisting of the reduced *Internally Consistent Classical Nucleation Theory* nucleation model and the explicit droplet growth formulation by Gyarmathy [9]. The temperature difference between the droplets and the surrounding gas has been taken into account by applying a wet-bulb approximation. For a detailed description about the condensation model and the numerical method, the reader is referred to [3].

4 Results and analyses

4.1 Different structures of condensation shock

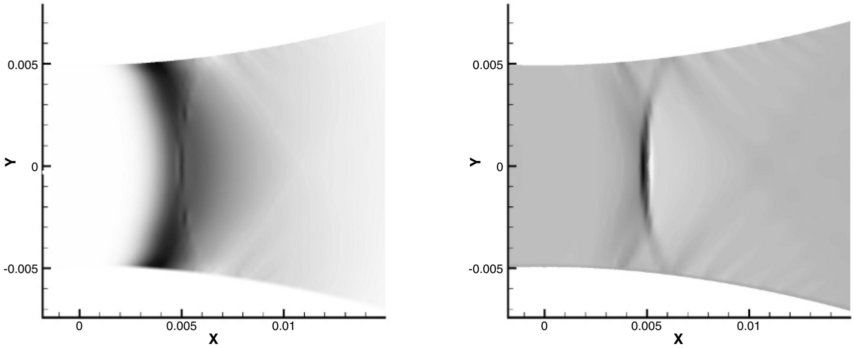
As shown in Fig. 2, the condensation shock trends to be normal shock in slender nozzles, while double λ-shaped shock connected with a Mach stem in the middle appear in the nozzles with high wall curvature. This kind of double-λ structure can be characterized by dimensionless length of Mach stem h_m^*(half Mach stem length h_m normalized by the half nozzle throat height y^*).

The complex structure of condensation shock is the result of non-uniformed expansion of moist air. As shown in Fig. 3(a), the flow near the wall expands faster than that near



(a) Condensation induced normal shock **(b)** Condensation shock with complex structure

Fig. 2. Different structures of condensation induced shock



(a) X-gradient of condensate mass fraction **(b)** Digital schlieren

Fig. 3. Numerical simulation of condensation shock with complex structure

the axis and consequently drops faster in temperature. As a result, the condensation happens earlier and more dramatically near the wall, which produces an oblique shock wave (or compressive wave) from each side of the walls. The two oblique shocks meet together near the axis to form a Mach stem. This Mach stem compress the flow and slow down the condensation near the axis. So the non-uniformity of expansion is aggravated. In comparison, if the relative saturation rate is higher, condensation may also induce normal shock in the nozzles with high wall curvature. In this case, the condensation trends to be more uniformed in the flow field.

4.2 Dependence of onset Mach number and dimensionless length of Mach stem on stagnation states

We take the position of Mach stem as where condensation shock onset. Then, the onset Mach number can be calculated based on the one dimensional theory of compressible flows. Results and some simple relations can be found in Table 1 and Fig. 4. It is found

Table 1. Experiment results of Condensation onset Mach number M_c and dimensionless length of Mach stem h_m^*

Φ_0 (%)	69.0	73.5	75.0	77.0	82.0	85.0	92.0	91.0	80.0	80.0	79.5	67.0
T_0 (^{o}C)	18.2	18.3	18.1	18.1	18.6	18.2	18.6	21.4	21.7	25.2	29.1	26.9
M_c	1.39	1.37	1.33	1.31	1.26	1.28	1.23	1.22	1.33	1.26	1.23	1.43
h_m^*	0.095	0.094	0.193	0.225	0.509	0.480	0.538	0.726	0.324	0.517	0.739	0.098

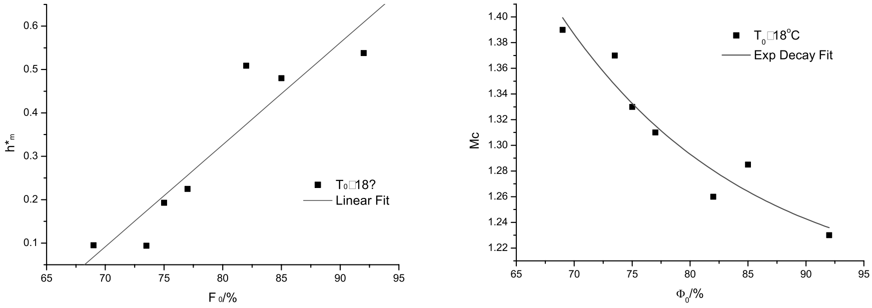


Fig. 4. Simple relations between h_m^*, M_c and Φ_0 at almost constant T_0.

that when Φ_0 goes up, the condensation shock moves upstream and the Mach stem lengthens due to stronger condensation. When T_0 goes up, the Mach stem lengthens at almost the same position. This is a compromise of stronger condensation (pushing the shock forward) and higher temperature (delaying the condensation).

4.3 Dimensional analysis

Firstly, let's analyze the dimensionless length of Mach stem h_m^*. The parameters that may influent h_m^* are stagnation pressure p_0, stagnation temperature T_0, stagnation density ρ_0, relative humidity in the supply $\Phi_0 = p_{v0}/p_{s0}$, where p_{v0} is the vapor partial pressure in the supply, $p_{s0} = p_{s0}(T_0)$ is the saturation pressure for a plane boundary; Geometric parameters: $l = (y^* R^*)^{1/2}$, where y^* is the half height of throat and R^* is the radius of throat wall; Heat capacity C_p and ratio of heat capacity γ; latent heat released by unit mass of vapor L. Besides, the relaxation time τ (duration of condensation) and ΔT_c (the difference between T_0 and condensation temperature) are also very important. Then,

$$h_m^* = F_1(p_0, T_0, \rho_0, p_{v0}, \Phi_0, \Delta T_c, l, y^*, C_p, L, \tau, \gamma) \tag{1}$$

Taking T_0, l and p_0 as basic dimensions, we get:

$$h_m^* = F_2\left(\frac{p_{v0}}{p_0}, \Phi_0, \frac{L}{C_p T_0}, \frac{\Delta T_c}{T_0}, \frac{y^*}{l}, \frac{\tau a^*}{l}, \gamma\right), \tag{2}$$

Where p_{v0}/p_0 is the ratio of vapor in the supply; $L/(C_p T_0)$ is the ratio of latent heat and the heat capacity of unit carrier gas; $y^*/l = (y^*/R^*)^{1/2}$ is the effect of nozzle wall curvature; $a^* = a^*(T_0)$ is the local sound speed of nozzle throat, l/a^* is the characteristic time of flow near throat,so $\tau a^*/l$ is the ratio of relaxation time and characteristic time.

Since the p_0 is constant, ρ_0 only depends on T_0. C_p, γ and L can also be taken as constant for the ratio of vapor in air is actually very little. Usually 10^{-5} s $< t < 10^{-4}$ s[11] , here we assume $\tau = 5 \cdot 10^{-5}$.

Based on 1-D theory, if T_0 is around 300 K, the temperature near throat will drop to about 240 K and pressure drop to about 0.52 atm. Even if we count in the effect of condensation shock, water should still be in solid phase. Wegener[[1]] had proved that the product of condensation in supersonic nozzle is solid. The critical temperature of the vapor-solid transition is about 273 K, so we take $\Delta T_c = T_0 - 273$.

At last, only y^*, l, T_0 and Φ_0 are left to be control variables. If we define $q = [L/(C_p T_0)] * [p_{v0}/p0] * [\rho_v/\rho_{v0}]$ as the ratio of latent heat and the heat capacity of unit mass unit time at the throat, then we get:

$$h_m^* = F_3\left(\frac{L}{C_p T_0} * \frac{p_{v0}}{p_0} * \frac{\rho_v}{\rho_{v0}}, \frac{\Delta T_c}{T_0}, \frac{\tau a^*}{l}, \frac{y^*}{l}\right) = F_4\left(\frac{q T_0}{\Delta T_c}, \frac{\tau a^*}{l}, \frac{y^*}{l}\right) \tag{3}$$

By linear fit, we can get the similarity law:

$$h_m^* = A\left(\frac{y^*}{l}\right)^{\alpha 1}\left(\frac{\tau a^*}{l}\right)^{\alpha 2}\frac{q T_0}{\Delta T_c} - B \tag{4}$$

Where $A = 3.87, B = 1.02, \alpha 1 = 0.53, \alpha 2 = 0.10$. If we let $X = \left(\frac{y^*}{l}\right)^{\beta 1}\left(\frac{\tau a^*}{l}\right)^{\beta 2}\frac{q T_0}{\Delta T_c}$, which can be view as the compromise of condensation strength and the energy consumed before condensation, the results of table 1 can be redisplayed as Fig. 5 (Left part).

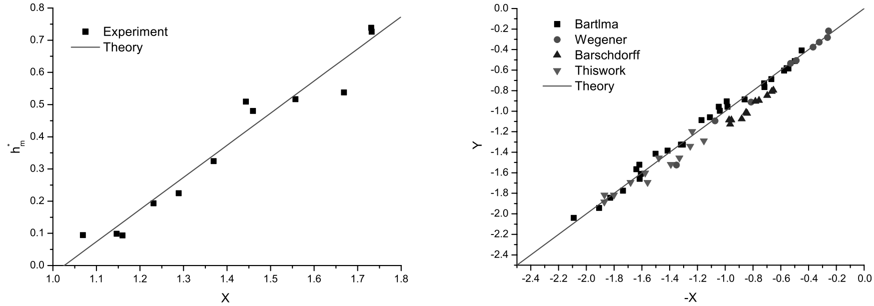


Fig. 5. h_m^* dependence of the compound parameter X (Left) and relation between compound parameter X and Y (Right)

Using the same method, we can also get the similarity law of M_c, which is proved to be a good approximation to the experimental results for several different nozzles, as shown in Fig. 5 (Right part).

$$Y = \ln\left((M_c - C_2)/\left(C_1\left(\frac{y^*}{l}\right)^{\beta 1}\left(\frac{\tau a^*}{l}\right)^{\beta 2}\right)\right), \tag{5}$$

where $C_1 = 0.22, C_2 = 1.00, \beta 1 = -0.51, \beta 2 = 0.67$. C_2 can be viewed as the onset Mach number when the mixture in supply is extremely easy to condense.

5 Conclusions

The two dimensional structure of condensation induced stable shock wave in condensing nozzle flows is investigated both experimentally and numerically. Based on schlieren method, the different structures of condensation shock are characterized and found to be the results of non-uniformity expansion of the flow and the interaction of Mach stem with flow.

With the dimensional analysis method, we found two similarity laws of the onset Mach number and structure of condensation shock which are both depend on the same compound parameter X : the compromise of condensation strength and the energy consumed before condensation. The similarity law of onset Mach number agrees well with the experimental results from other authors. But there is no much work on the 2-D structure of condensation shock for comparison.

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