A Rough Set Approach to Multiple Criteria ABC Analysis

Ye Chen[1], Kevin W. Li[2], Jason Levy[3], Keith W. Hipel[1], and D. Marc Kilgour[4]

[1] Department of Systems Design Engineering, University of Waterloo, Waterloo, Ontario, N2L 3G1, Canada
{y3chen,kwhipel}@uwaterloo.ca
[2] Odette School of Business, University of Windsor, Windsor, Ontario, N9B 3P4, Canada
kwli@uwindsor.ca
[3] Huxley College of the Environment, Western Washington University, Washington, 98225, US
Jason.Levy@wwu.edu
[4] Department of Mathematics, Wilfrid Laurier University, Waterloo, Ontario, N2L 3C5, Canada
mkilgour@wlu.ca

Abstract. A dominance-based rough set approach (DRSA) to multiple criteria ABC analysis (MCABC) is designed and compared to other approaches using a practical case study. ABC analysis is a well-known inventory planning and control approach, which classifies inventory items, or stock-keeping units (SKUs), based solely on their annual dollar usage. Recently, it has been suggested that MCABC can provide more managerial flexibility by considering additional criteria such as lead time and criticality. This paper proposes an MCABC method that employs DRSA to generate linguistic rules to represent a decision maker's preferences based on the classification of a test data set. These linguistic rules are then applied to classify other SKUs. A case study is used to compare the DRSA with other MCABC approaches to demonstrate the applicability of the proposed method.

Keywords: Inventory management, ABC analysis, multiple criteria decision analysis, rough set theory, dominance-based rough set approach.

1 Introduction

In response to demand for mass customization, firms often increase inventories of components, work-in-progress, and spare parts [30]. The different items in an inventory system, referred to as stock-keeping units (SKUs), typically number in the thousands. Corner convenience stores, for instance, may have several thousand SKUs. In such a large inventory system, specific control schemes for individual SKUs are simply not practical, as they would leave no resources for other management activities [5]. Instead, a general practice in industry is to

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aggregate SKUs into several groups and apply control policies that are uniform across each group [1].

One commonly used approach to classifying SKUs is ABC analysis. In the traditional ABC analysis, SKUs are ranked in descending order of annual dollar usage, the product of unit price and annual demand. The top few SKUs, with the highest annual dollar usage, are placed in group A, which will receive the most management attention; the SKUs with least annual dollar usage are placed in group C and will receive the least management attention; and the remaining SKUs are placed in group B. Figure 1 captures the essence of this rule.

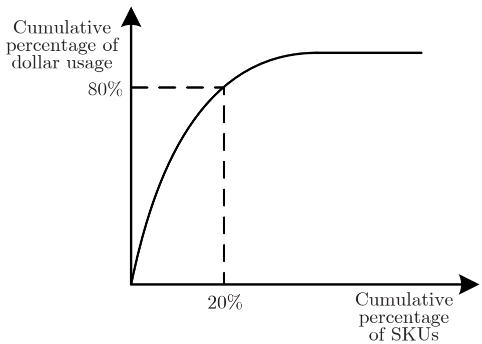


Fig. 1. Example of Dollar Usage Distribution Curve [2]

Traditional ABC analysis can be viewed as an implementation of Pareto's famous observation about the uneven distribution of national wealth [19]: the majority of national wealth is controlled by a few, and the majority of the population controls only a small portion of the wealth. Applications similar to ABC analysis are found in many managerial areas [32]; for instance, in marketing it is often observed that the majority of sales come from a few important customers, while a significant proportion of total sales is due to a large number of very small customers.

Classical ABC analysis has been criticized because of the amount of attention that management pays to an SKU depends on a single criterion, the annual dollar usage of the SKU at the time of classification [9]. However, other attributes of an SKU sometimes play a significant role in prioritization. For instance, suppose that two SKUs are virtually identical except that one is easy to replace while the other is unique and has only one specific supplier. Understandably the SKU with higher substitutability should receive less management attention. Other criteria that could be accounted for include obsolescence, repairability, criticality, and lead time [7], [8].

To carry out multiple criteria classification of SKUs, a variety of approaches has been proposed. One of the first attempts was the Flores and Whybark's bi-criteria matrix method [7]. This approach begins by selecting another critical criterion, in addition to dollar usage, depending on the nature of the industry. Some examples are obsolescence, lead time, substitutability, reparability, criticality and commonality [7]. Next, the model requires that SKUs be divided into three levels of importance, A, B, and C, for each of the two criteria, respectively. The model then reclassifies SKUs into three categories, AA, BB, and CC, representing the three new groups, according to some rules jointly determined by the new criterion and the dollar usage. The structure of the model can be conveniently represented as a joint criteria matrix as shown in Figure 2, adapted from [7]. A general guideline as indicated by the arrows is to regroup AB and BA as AA, AC and CA as BB, and BC and CB as CC.

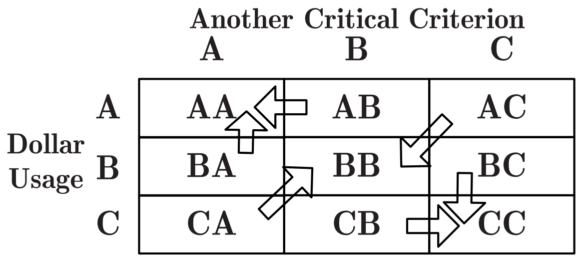


Fig. 2. The Joint Matrix for Two Criteria

Other approaches include the analytic hierarchical process (AHP) [9,21], genetic algorithm [15] and artificial neural networks [20]. Recently, based upon the same case study as described in [9], Ramanathan [27], Ng [17], Zhou and Fan [33], and Chen et al. [2,4] proposed various new approaches to MCABC. For example, Chen et al. [4] proposed a multiple criteria ABC analysis (MCABC) method that employs DRSA to generate linguistic rules for representing a decision maker's preferences based on the classification of a test data set.

In this paper, we refine the previous work in [4] and provide a comprehensive analysis procedure to demonstrate how DRSA can be applied to MCABC. Our results are then compared with other approaches using a practical case study. More specifically, we show how DRSA, a recent advance in rough set theory [11], can be applied to extract information about a decision maker's preferences from the classification of test data and then generate a set of decision rules to classify other SKUs. In addition, the compatibility of DRSA to generate decision rules with other methods are tested and the comparison of classification ability is explored.

The reminder of the paper is organized as follows. Section 2 provides some background pertaining to multiple criteria decision analysis, while Section 3 describes the DRSA in the context of MCABC. An illustrative example is furnished in Section 4, followed by some concluding remarks in Section 5.

2 Multiple Criteria Decision Analysis

Multiple criteria decision analysis (MCDA) is a set of techniques to assist a single decision maker (DM) to *choose*, *rank*, or *sort* a finite set of alternatives according to two or more criteria [28]. The first step of MCDA is to establish a basic structure of the decision problem: define the objectives, arrange them into criteria, identify all possible alternatives, and measure the consequences of each alternative on each criterion. A consequence is a direct measurement of the success of an alternative against a criterion (e.g. cost in dollars). Note that a consequence is usually a physical measurement or estimate; it should not include preferential information.

Figure 3, adapted from [3], shows the basic structure of an MCDA problem. In this figure, $\mathbf{N} = \{N^1, N^2, \cdots, N^i, \cdots, N^n\}$ is a set of alternatives, and $\mathbf{Q} = \{1, 2, \cdots, j, \cdots, q\}$ is a set of criteria. The consequence of alternative N^i over criterion j is denoted $c_j(N^i)$, which can be shortened to c_j^i when there is no possibility of confusion. Note that there are $n > 1$ alternatives and $q > 1$ criteria.

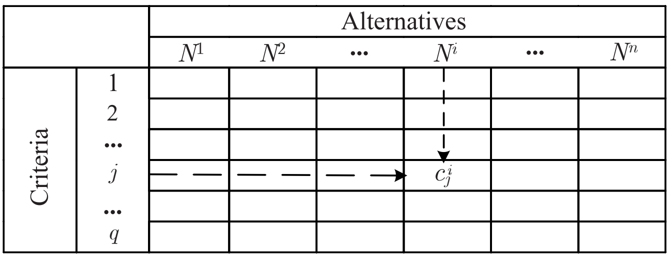


Fig. 3. The Structure of MCDA

Several approaches are available for a DM to structure a decision problem as per Figure 3. Roy [28] suggested that MCDA can be organized into three **problématiques**, or fundamental problems, as follows:

- α, **Choice problématique**. Choose the best alternative from **N**.
- β, **Sorting problématique**. Sort the alternatives of **N** into predefined, relatively homogeneous groups, arranged in preference order.
- γ, **Ranking problématique**. Rank the alternatives of **N** from best to worst.

MCABC is a special kind of sorting problématique: the alternatives are SKUs, and they are to be arranged into three groups, **A**, **B** or **C**. The preference order $\mathbf{A} \succ \mathbf{B} \succ \mathbf{C}$ signifies that an SKU in **A** is to receive more management attention than an SKU in **B**, for instance. It is understood that SKUs in the same group are to receive equal management attention; in this sense, they are indifferent.

The DM's preferences are crucial to the solution of any MCDA problem; moreover, different ways of expressing them may lead to different results. Pareto-Superiority [19] may be used to identify some inferior alternatives, but almost always a more elaborate preference construction is needed to carry out any of the problématiques. Generally speaking, there exist two kinds of preference expressions: *v*alues, which are preferences on consequences, and *w*eights, which are preferences on criteria.

After the structure of an MCDA problem is determined and the DM's preferences are acquired, a model must be constructed to aggregate preferences, thereby permiting the chosen problématique to be investigated. Some methods, such as multiattribute utility theory (MAUT) [18], are direct models in which explicit numerical functions are constructed to evaluate alternatives; others, including Outranking methods [28] and AHP [29], employ pair-wise comparison procedures rather than explicit functions to conduct the evaluation; still others, such as rough set theory [11], tackle the MCDA problem implicitly using linguistic rules.

3 A Rough Set Approach to MCABC

Pawlak [22,23] introduced Rough Sets as a tool to describe dependencies among attributes and to evaluate the significance of individual attributes. Because of its ability to handle the inherent uncertainty or vagueness of data, rough set theory complements probability theory, evidence theory, fuzzy set theory, and other approaches. Recent advances in rough set theory have made it a powerful tool for data mining, pattern recognition, and information representation. For example, Pawlak and Skowron [24] provided a comprehensive literature review of rough set theory including different research directions and various applications. Some theoretical extensions of rough set theory are proposed in [25], and the hybrid of rough set theory and Boolean reasoning with different applications are discussed in [26].

An important principle of rough sets is that all relevant information about alternatives, which may include both condition and decision attributes, can be expressed in a data set [22]. Condition attributes refer to the characteristics of the alternatives; for instance, condition attributes describing a firm can include size, financial characteristics (profitability, solvency, liquidity ratios), market position, and so on. Decision attributes define a partition of the alternatives into groups reflecting the condition attributes in some way. In terms of MCDA, condition and decision attributes are regarded as criteria and decision choices, respectively.

3.1 A Dominance-Based Rough Set Theory for MCABC

As pointed out in [11,14], the original rough set approach cannot efficiently extract knowledge from the analysis of a case set. In MCDA problems, preferences over groups and indiscernibility or similarity must be replaced by the *dominance* relation [14] (also see [10,12] for a detailed discussion of the relationship between the classical rough set approach and DRSA).

To apply rough set theory to MCABC, we treat SKUs as alternatives and relevant data about SKUs as criteria (conditions). We select a non-empty case set $\mathbf{T} \subseteq \mathbf{N}$ and ask the DM to decide how to partition the case set into three non-overlapping classes, $\mathbf{A}'$, $\mathbf{B}'$ and $\mathbf{C}'$, with a preference order $\mathbf{A}' \succ \mathbf{B}' \succ \mathbf{C}'$. (Typically, $\mathbf{T}$ is much smaller than $\mathbf{N}$. For convenience, we assume that $\mathbf{T} = \{N^1, \ldots, N^m\}$.) Then we use rough set theory to extract a set of linguistic rules, $\mathbf{R}$, that capture preferential information in the case set classification, and apply $\mathbf{R}$ to all elements of $\mathbf{N}$ to extend $\mathbf{A}'$ to $\mathbf{A}$, $\mathbf{B}'$ to $\mathbf{B}$, and $\mathbf{C}'$ to $\mathbf{C}$. Thus, $\mathbf{N}$ is

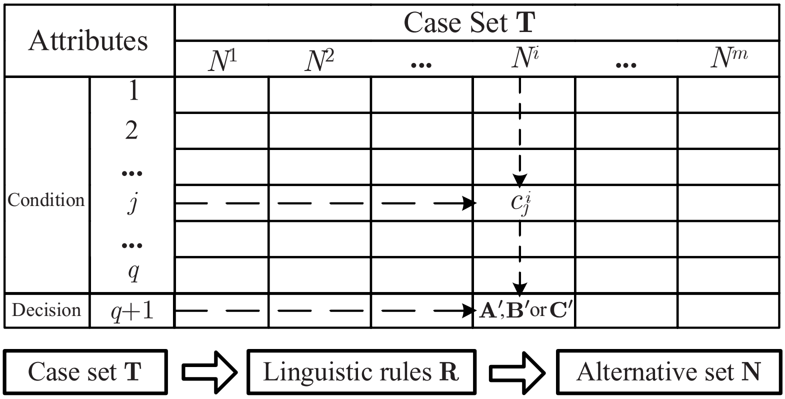


Fig. 4. The Structure of the Case Set

sorted into three classes, **A**, **B**, and **C**, with a preference order $\mathbf{A} \succ \mathbf{B} \succ \mathbf{C}$. This classification procedure is illustrated in Figure 4.

Let S_j be a binary preference relation with respect to criterion $j \in \mathbf{Q}$, such that $N^i S_j N^l$ means that "N^i is at least as good as N^l with respect to criterion j", where $N^i, N^l \in \mathbf{T}$ are alternatives. We assume that S_j is a complete preorder, i.e. a strongly complete and transitive binary relation, and that $\mathbf{S} = (S_1, S_2, \ldots, S_q)$ is a comprehensive preference relation on **N**, i.e. $N^i \mathbf{S} N^l$ means $N^i S_j N^l$ for every criterion $j \in Q$, for $N^i, N^l \in \mathbf{N}$.

The *upward union* and *downward union* [11,14] with respect to the classes in the test set is defined next. Upward unions are denoted by subscript "$\geq$", and downward unions by subscript "$\leq$".

- $\mathbf{C}'_{\geq} = \mathbf{C}' \cup \mathbf{B}' \cup \mathbf{A}'$; $\mathbf{C}'_{\leq} = \mathbf{C}'$.
- $\mathbf{B}'_{\geq} = \mathbf{B}' \cup \mathbf{A}'$; $\mathbf{B}'_{\leq} = \mathbf{C}' \cup \mathbf{B}'$.
- $\mathbf{A}'_{\geq} = \mathbf{A}'$; $\mathbf{A}'_{\leq} = \mathbf{C}' \cup \mathbf{B}' \cup \mathbf{A}'$.

For example, $\mathbf{C}'_{\geq}$ consists of those test items that at least belong to group $\mathbf{C}'$, and $\mathbf{C}'_{\leq}$ those test items that at most belong to group $\mathbf{C}'$.

N^i dominates N^l with respect to criterion set $\mathbf{P} \subseteq \mathbf{Q}$ and is written as $N^i D_{\mathbf{P}} N^l$, iff $N^i S_j N^l$ for all $j \in \mathbf{P}$. Relative to N^i, the **P**-dominating set is defined by

$$D^{+}_{\mathbf{P}}(N^i) = \{N^l \in \mathbf{T} : N^l D_{\mathbf{P}} N^i\},$$

and the **P**-dominated set by

$$D^{-}_{\mathbf{P}}(N^i) = \{N^l \in \mathbf{T} : N^i D_{\mathbf{P}} N^l\}.$$

With respect to $\mathbf{P} \subseteq \mathbf{Q}$, we say that N^i belongs to $\mathbf{G}'_{\geq}$ unambiguously, where $\mathbf{G}' = \mathbf{A}'$, $\mathbf{B}'$ or $\mathbf{C}'$, iff $N^i \in \mathbf{G}'_{\geq}$ and, for any $N^l \in D^{+}_{\mathbf{P}}(N^i)$, $N^l \in \mathbf{G}'_{\geq}$. More generally, the **P**-lower approximation to $\mathbf{G}'_{\geq}$ is

$$\underline{\mathbf{P}}(\mathbf{G}'_{\geq}) = \{N^i \in \mathbf{T} : D^+_{\mathbf{P}}(N^l) \subseteq \mathbf{G}'_{\geq}\},$$

and the **P**-upper approximation to $\mathbf{G}'_{\geq}$ is

$$\overline{\mathbf{P}}(\mathbf{G}'_{\geq}) = \bigcup_{A^l \in \mathbf{G}'_{\geq}} D^+_{\mathbf{P}}(N^l).$$

Similarly, the **P**-lower approximation to $\mathbf{G}'_{\leq}$ is

$$\underline{\mathbf{P}}(\mathbf{G}'_{\leq}) = \{N^l \in \mathbf{N}' : D^-_{\mathbf{P}}(N^l) \subseteq \mathbf{G}'_{\leq}\},$$

and the **P**-upper approximation to $\mathbf{G}'_{\leq}$ is

$$\overline{\mathbf{P}}(\mathbf{G}'_{\leq}) = \bigcup_{N^l \in \mathbf{G}'_{\leq}} D^-_{\mathbf{P}}(N^l).$$

The **P**-boundaries (**P**-doubtful regions) of $\mathbf{G}'_{\leq}$ and $\mathbf{G}'_{\geq}$ are

$$BN_{\mathbf{P}}(\mathbf{G}'_{\leq}) = \overline{\mathbf{P}}(\mathbf{G}'_{\leq}) - \underline{\mathbf{P}}(\mathbf{G}'_{\leq}),$$

$$BN_{\mathbf{P}}(\mathbf{G}'_{\geq}) = \overline{\mathbf{P}}(\mathbf{G}'_{\geq}) - \underline{\mathbf{P}}(\mathbf{G}'_{\geq}).$$

The quality of the sorting of the case set $\mathbf{T}$ with respect to $\mathbf{P} \subseteq \mathbf{Q}$ is

$$\gamma_{\mathbf{P}}(\mathbf{G}') = \frac{\left|\mathbf{N} - \left\{\left(\bigcup_{\mathbf{I}'=\mathbf{A}',\mathbf{B}',\mathbf{C}'} BN_{\mathbf{P}}(\mathbf{I}'_{\leq})\right) \cup \left(\bigcup_{\mathbf{I}'=\mathbf{A}',\mathbf{B}',\mathbf{C}'} BN_{\mathbf{P}}(\mathbf{I}'_{\geq})\right)\right\}\right|}{m},$$

where m is the size (cardinality) of the case set $\mathbf{T}$. Thus, $\gamma_{\mathbf{P}}(\mathbf{G}')$ represents the proportion of alternatives in the case set $\mathbf{T}$ that are accurately sorted using only the criteria in $\mathbf{P}$.

Each minimal subset $\mathbf{P} \subseteq \mathbf{Q}$ such that $\gamma_{\mathbf{P}}(\mathbf{T}) = \gamma_{\mathbf{Q}}(\mathbf{T})$ is called a *reduct* of $\mathbf{Q}$. A case set $\mathbf{T}$ can have more than one reduct; the intersection of all reducts is called the *core* [11,14].

3.2 Decision Rules for MCABC

The approximations obtained through dominance can be used to construct decision rules capturing preference information contained in the classification of a case set [11]. Assume that all criteria are benefit criteria, i.e. that $c_j(N^i) \geq c_j(N^l)$ implies $N^i S_j N^l$ for all $j \in \mathbf{Q}$ and $N^i, N^l \in \mathbf{N}$. Then three types of decision rules can be generated from a non-empty set of criteria $\mathbf{P} \subseteq \mathbf{Q}$ and are used to sort $\mathbf{N}$ into $\mathbf{G}$ and $\mathbf{H}$, respectively, where $\mathbf{G} \neq \mathbf{H}$ and $\mathbf{G}, \mathbf{H} \in \{\mathbf{A}, \mathbf{B}, \mathbf{C}\}$, as required.

- $\mathbf{R}_{\geq}$ decision rules, which have the syntax

  $$\text{If } c_j(N^i) \geq r_j \text{ for all } j \in \mathbf{P}, \text{ then } N^i \in \mathbf{G}_{\geq},$$

  where, for each $j \in \mathbf{P}$, $r_j \in \mathbb{R}$ is a consequence threshold for criterion j. Rules of this form are supported only by alternatives from the **P**-lower approximations of class $\mathbf{G}'_{\geq}$.
- $\mathbf{R}_{\leq}$ decision rules, which have the syntax

$$\text{If } c_j(N^i) \le r_j \text{ for all } j \in \mathbf{P}, \text{ then } N^i \in \mathbf{G}_{\le},$$

where, for each $j \in \mathbf{P}$, $r_j \in \mathbb{R}$ is a consequence threshold for criterion j. Rules of this form are supported only by alternatives from the **P**-lower approximations of class $\mathbf{G}'_{\le}$.

- $\mathbf{R}_{\ge\le}$ decision rules, which have the syntax

$$\text{If } c_j(N^i) \ge r_j \text{ for all } j \in \mathbf{O} \text{ and } c_j(N^i) \le r_j \text{ for all } j \in \mathbf{P} - \mathbf{O},$$

$$\text{then } N^i \in \mathbf{G} \cup \mathbf{H},$$

where $\mathbf{O} \subseteq \mathbf{P}$ such that both $\mathbf{O}$ and $\mathbf{P} - \mathbf{O}$ are non-empty, and $r_j \in \mathbb{R}$ is a consequence threshold for criterion j for each $j \in \mathbf{P}$. Rules of this form are supported only by alternatives from the **P**-boundaries of the unions of the classes $\mathbf{G}'_{\ge}$ and $\mathbf{H}'_{\le}$.

A set of decision rules is *complete* if, when it is applied to alternatives in the case set **T**, all of them can be reclassified to one or more groups and there is no alternative for which rules cannot be applied for classification. Furthermore, alternatives are consistent when they are classified to the original groups; alternatives are inconsistent when they are assigned to a different group or more than one group. A set of decision rules is *minimal* if it is complete and non-redundant, i.e. exclusion of any rule makes the set incomplete [11]. Fortunately, software is available (see below) that produces sets of minimal decision rules.

4 Application

4.1 Background

We now employ a case study on a hospital inventory system, based on data in [9], to demonstrate the proposed procedure. In the reference, 47 disposable SKUs used in a respiratory therapy unit are classified using AHP-based method [29] for MCABC analysis. Table 1 lists data on the SKUs, referred to as S1 through S47. Four criteria as listed in Column 2-5 of Table 1 are considered to be relevant to the MCABC analysis: (1) average unit cost ($), ranging from $5.12 to $210.00; (2) annual dollar usage ($), ranging from $25.38 to $5840.64; (3) criticality, described by numerical values (1, for high or very critical, 0.5, for moderate or important, and 0.01, for low or non-critical); (4) lead time (weeks), the normal time to receive replenishment after an order is placed, ranging from 1 to 7 weeks. The last column of Table 1 shows the AHP-based classification results.

As indicated earlier, in addition to the initial work of Flores et al. [9], several MCABC methods, including Chen et al. [2,4], Ramanathan [27], Ng [17], and Zhou and Fan [33], have been proposed and used the same data set as listed above for demonstration purposes. Here, two types of comparisons of DRSA with other approaches are conducted to show the applicability of DRSA in MCABC:

- *Decision rule generation comparison*: The approaches by Flores et al.[9], Ramanathan [27], Ng [17], and Zhou and Fan [33] rely on direct sorting in which explicit numerical functions are constructed and employed to classify SKUs. To carry out the comparison, DRSA is used to generate decision rules based on the data set including the condition attributes as shown in Table 1 and the decision attributes calculated by other methods. Then, employing the generated decision rules, the reclassification of the training set is done to examine the compatibility of DRSA with other approaches. To a certain extent, this comparison employs a practical example to validate the conclusions drawn by Slowinski et al. [31] and Greco et al. [13] that DRSA is the most general MCDA methodology and other MCDA approaches can be represented in terms of decision rules.
- *Comparison of classification results*: Next, the classification results obtained by using the AHP-based approach in [9] are adopted as benchmark data and are compared with those generated by the herein proposed DRSA approach. To apply the DRSA procedure, a training set, consisting of three **A** items, five **B** items, and seven **C** items, is randomly selected from the 47 SKUs. The training set is then fed into the DRSA-based MCABC procedure to generate decision rules for classifying all 47 SKUs in the inventory system. The sampling is conducted 20 times and the final classification results are then reconciled with those in [9] to examine how well our proposed approach can extract the inherent knowledge imbedded in the training set.

4.2 Decision Rule Generation Comparison

Firstly, based on the classification results obtained by an AHP-based approach [9] as shown in the last column of Table 1, DRSA is utilized to generate decision rules to reflect the DM's subjective judgement, and these rules are employed to reclassify the data set to verify the compatibility of these two approaches. Then, a summary of similar comparisons is provided with other MCABC models. The purpose of these comparisons is to examine whether the reclassification can reproduce the results obtained with other MCABC approaches, thereby confirming the claim in [31] and [13] that DRSA is the most general MCDA methodology and other MCDA approaches can be represented in terms of decision rules. Note that, in general, the case (training, test) set information can be provided by the expert directly and, hence, the generated rules should express the knowledge of the expert used to give his/her classification information.

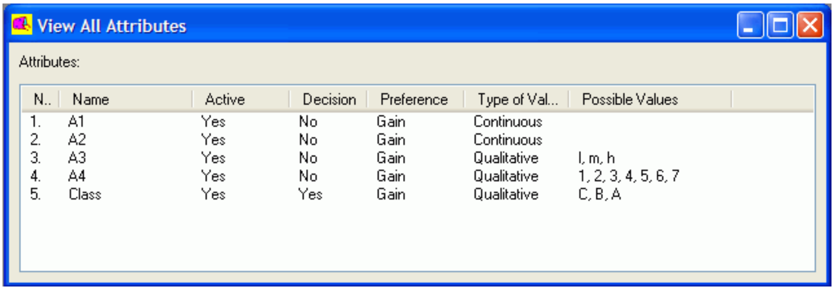
Analysis Procedures. The software 4eMka2 [16] is employed to conduct the calculations and the analysis procedures are given as follows:

(1) Criteria specification

The detailed criteria specification using 4eMka2 is shown in Figure 5. All of the criteria are interpreted to be benefit criteria: for example, lead time is a gain criterion since the greater the lead time, the higher the level of management attention required. Hence, their preferences are all set as "Gain" as shown in the fifth column of the figure. Note that A1, A2, A3 and A4 represent the criteria of average unit cost, annual dollar usage, criticality and lead time, respectively.

Table 1. Listing of SKUs with multiple criteria, adapted from [9]

SKUs	Criteria				
	Average unit cost ($)	Annual dollar usage ($)	Critical factor	Lead time (week)	Group
S1	49.92	5840.64	1	2	**A**
S2	210.00	5670.00	1	5	**A**
S3	23.76	5037.12	1	4	**A**
S4	27.73	4769.56	0.01	1	**C**
S5	57.98	3478.80	0.5	3	**B**
S6	31.24	2936.67	0.5	3	**C**
S7	28.20	2820.00	0.5	3	**C**
S8	55.00	2640.00	0.01	4	**C**
S9	73.44	2423.52	1	6	**A**
S10	160.50	2407.50	0.5	4	**B**
S11	5.12	1075.20	1	2	**B**
S12	20.87	1043.50	0.5	5	**B**
S13	86.50	1038.00	1	7	**A**
S14	110.40	883.20	0.5	5	**B**
S15	71.20	854.40	1	3	**A**
S16	45.00	810.00	0.5	3	**C**
S17	14.66	703.68	0.5	4	**B**
S18	49.50	594.00	0.5	6	**A**
S19	47.50	570.00	0.5	5	**B**
S20	58.45	467.60	0.5	4	**B**
S21	24.40	463.60	1	4	**A**
S22	65.00	455.00	0.5	4	**B**
S23	86.50	432.50	1	4	**A**
S24	33.20	398.40	1	3	**A**
S25	37.05	370.50	0.01	1	**C**
S26	33.84	338.40	0.01	3	**C**
S27	84.03	336.12	0.01	1	**C**
S28	78.40	313.60	0.01	6	**C**
S29	134.34	268.68	0.01	7	**B**
S30	56.00	224.00	0.01	1	**C**
S31	72.00	216.00	0.5	5	**B**
S32	53.02	212.08	1	2	**B**
S33	49.48	197.92	0.01	5	**C**
S34	7.07	190.89	0.01	7	**C**
S35	60.60	181.80	0.01	3	**C**
S36	40.82	163.28	1	3	**B**
S37	30.00	150.00	0.01	5	**C**
S38	67.40	134.80	0.5	3	**C**
S39	59.60	119.20	0.01	5	**C**
S40	51.68	103.36	0.01	6	**C**
S41	19.80	79.20	0.01	2	**C**
S42	37.70	75.40	0.01	2	**C**
S43	29.89	59.78	0.01	5	**C**
S44	48.30	48.30	0.01	3	**C**
S45	34.40	34.40	0.01	7	**B**
S46	28.80	28.80	0.01	3	**C**
S47	8.46	25.38	0.01	5	**C**


View All Attributes

Attributes:

N..	Name	Active	Decision	Preference	Type of Val...	Possible Values
1.	A1	Yes	No	Gain	Continuous	
2.	A2	Yes	No	Gain	Continuous	
3.	A3	Yes	No	Gain	Qualitative	l, m, h
4.	A4	Yes	No	Gain	Qualitative	1, 2, 3, 4, 5, 6, 7
5.	Class	Yes	Yes	Gain	Qualitative	C, B, A

Fig. 5. The Criterion Settings

Average unit cost and annual dollar usage are identified as continuous criteria shown in the sixth column of Figure 5, while criticality and lead time are identified as discrete criteria along with all possible values shown in the last column of the figure. Note that criticality in Flores et al. [9] is represented using numerical values, 1, 0.5 and 0.01, for high critical, moderate and low critical, respectively. Considering the ordinal nature of this criterion, in the DRSA, the linguistic expressions, h, m and l are used instead of 1, 0.5 and 0.01. The same setting is applied to other comparisons. The last row of the figure is the decision attribute, class, which indicates three sorting groups, A, B, and C, for MCABC.

(2) Input data

All data in Table 1 are input into the software for training as shown in Figure 6.

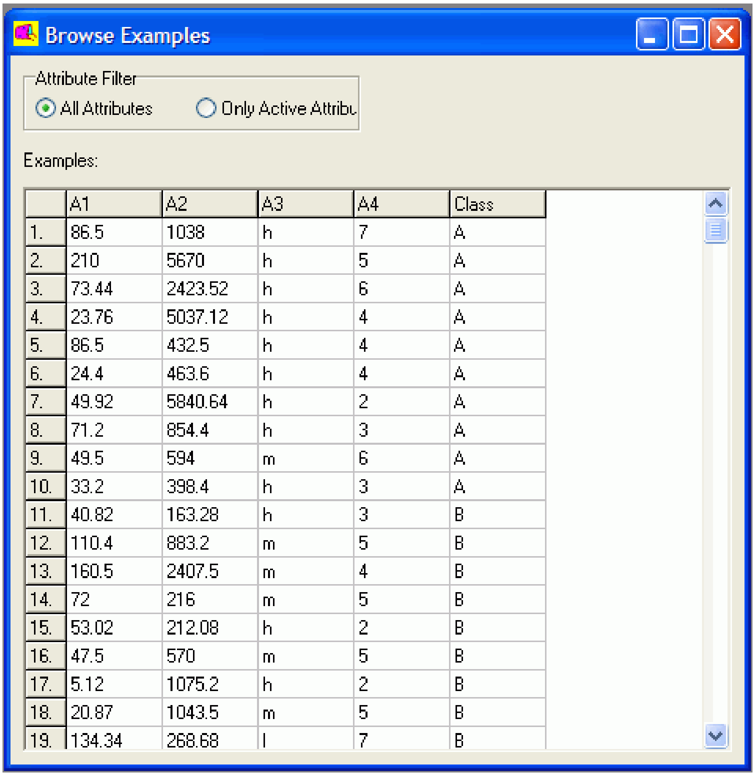
(3) Calculation of unions

All upward unions, downward unions, and boundaries for each class, $\mathbf{A}'$, $\mathbf{B}'$, and $\mathbf{C}'$, are calculated by the software and shown in Figure 7. There are no cases in each group boundary, indicating that the case set has been classified consistently.

(4) Rule generation

As shown in Figure 8, 17 rules are generated based on the algorithm, DomLEM as described in [12], to construct a minimal cover. These rules can be regarded as experts' knowledge in linguistic expressions generated by rough set theory and may help a DM to identify and explain his or her preferences using natural languages. The DM can check and update them as necessary and then apply them to classify any remaining SKUs. For convenience, these 17 rules are reproduced below:

- Rule 1. (A1 $\leq$ 7.07) & (A2 $\leq$ 197.92) $\Rightarrow$ (Class at most **C**);
- Rule 2. (A2 $\leq$ 150) & (A4 $\leq$ 6) $\Rightarrow$ (Class at most **C**);
- Rule 3. (A3$\leq$1) & (A4$\leq$ 6) $\Rightarrow$ (Class at most **C**);
- Rule 4. (A1$\leq$ 31.24) & (A3 $\leq m$) & (A4$\leq$3) $\Rightarrow$ (Class at most **C**);
- Rule 5. (A2$\leq$ 2936.670000) & (A3 $\leq m$) & (A4 $\leq$ 3) $\Rightarrow$ (Class at most **C**);
- Rule 4. (A2$\leq$ 2936.670000) & (A3 $\leq m$) & (A4 $\leq$ 3) $\Rightarrow$ (Class at most **C**);
- Rule 5. (A1$\leq$ 45) & (A2 $\leq$ 810) & (A3 $\leq m$) & (A4 $\leq$ 3) $\Rightarrow$ (Class at most **C**);
- Rule 6. (A2 $\leq$ 370.5) $\Rightarrow$ (Class at most **B**);



	A1	A2	A3	A4	Class
1.	86.5	1038	h	7	A
2.	210	5670	h	5	A
3.	73.44	2423.52	h	6	A
4.	23.76	5037.12	h	4	A
5.	86.5	432.5	h	4	A
6.	24.4	463.6	h	4	A
7.	49.92	5840.64	h	2	A
8.	71.2	854.4	h	3	A
9.	49.5	594	m	6	A
10.	33.2	398.4	h	3	A
11.	40.82	163.28	h	3	B
12.	110.4	883.2	m	5	B
13.	160.5	2407.5	m	4	B
14.	72	216	m	5	B
15.	53.02	212.08	h	2	B
16.	47.5	570	m	5	B
17.	5.12	1075.2	h	2	B
18.	20.87	1043.5	m	5	B
19.	134.34	268.68	l	7	B

Fig. 6. The Case Set Input

- Rule 7. (A3 $\leq m$) & (A4 $\leq$ 4) $\Rightarrow$ (Class at most **B**);
- Rule 8. (A1 $\leq$ 20.87) $\Rightarrow$ (Class at most **B**);
- Rule 9. (A2 $\leq$ 883.2) & (A3 $\leq m$) & (A4 $\leq$ 5) $\Rightarrow$ (Class at most **B**);
- Rule 10. (A2 $\geq$ 5037.12) $\Rightarrow$ (Class at least **A**);
- Rule 11. (A2 $\geq$ 398.4) & (A3 $\geq h$) & (A4 $\geq$ 3) $\Rightarrow$ (Class at least **A**);
- Rule 12. (A3 $\geq m$) & (A4 $\geq$ 6) $\Rightarrow$ (Class at least **A**);
- Rule 13. (A3 $\geq h$) $\Rightarrow$ (Class at least **B**);
- Rule 14. (A2 $\geq$ 455) & (A3 $\geq m$) & (A4 $\geq$ 4) $\Rightarrow$ (Class at least **B**);
- Rule 15. (A1 $\geq$ 34.4) & (A4 $\geq$ 7) $\Rightarrow$ (Class at least **B**);
- Rule 16. (A1 $\geq$ 57.98) & (A2 $\geq$ 3478.8) $\Rightarrow$ (Class at least **B**);
- Rule 17. (A1 $\geq$ 72) & (A3 $\geq m$) $\Rightarrow$ (Class at least **B**);

(5) Classification precision

All items in the case set are then reclassified using the generated rules. The reclassification results are used to assess classification precision. The generated rules successfully reclassified all items in the case study into corresponding "correct" groups. Therefore, the generated decision rules can accurately capture the DM's preferences, as represented in the classification results by using the AHP-based approach [9].

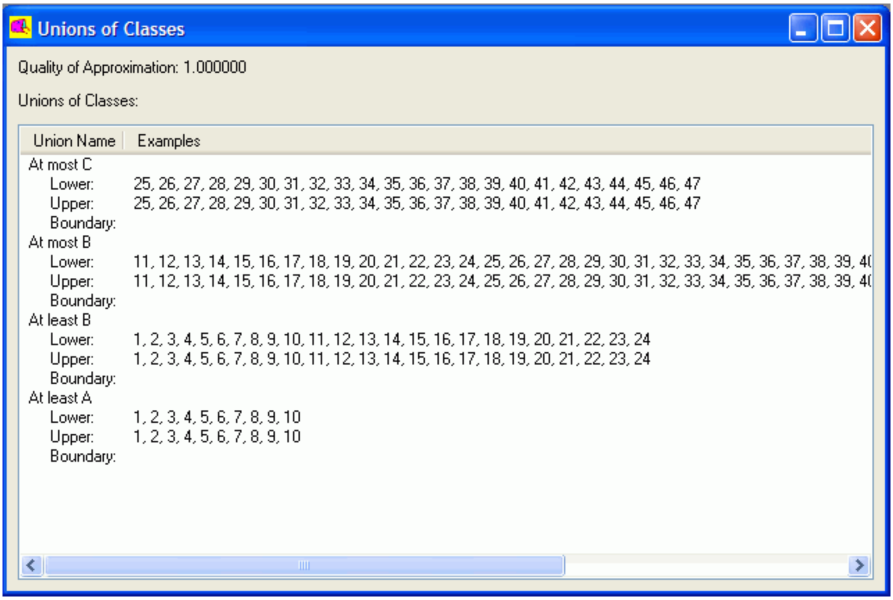


Fig. 7. The Unions in the Case Set

Comparison Summary. Similar procedures are employed to generate decision rules based on the classification information provided by other approaches including Ramanathan's [27], Ng's [17] (Note that the criterion, critical factor, is dropped in Ng's paper and hence, DRSA only analyzes the data set without the condition attribute of critical factor.), and Zhou and Fan's [33]. The detailed analytical steps are skipped here and the compatibility of DRSA with other methods is summarized in Table 2.

Table 2. Summary of Decision Rule Generation Comparisons

Approach Name	Reclassification Results		
	Correct Answers	Incorrect Decisions	Ambiguous Decisions
Flores et al. (AHP)	47	0	0
Ramanathan	17	0	30
Ng	47	0	0
Zhou and Fan	47	0	0

Conclusions : With the approaches of Flores et al. [9], Ng [17], and Zhou and Fan [33], DRSA successfully reclassified all SKUs into the relevant "correct" groups, and there are no incorrect or ambiguous decisions. However, the precision of reclassification using Ramanathan's approach [27] is not so promising, since there are 17 correct, but 30 ambiguous decisions. This large number of ambiguous

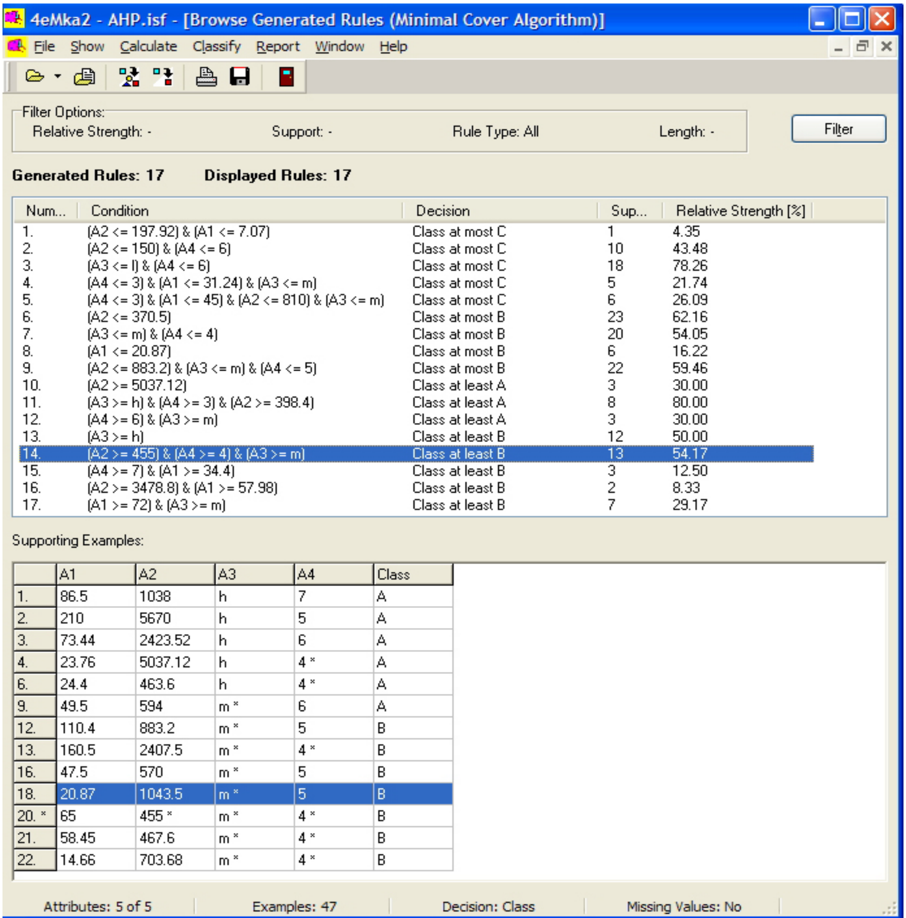


Fig. 8. The Rules Generated by 4eMka2

decisions seemingly resonates with Zhou and Fan's [33] call for an improvement of Ramanathan's model [27] as the classification results may be skewed by extreme values in less important criteria [33]. In short, these comparisons demonstrate that DRSA can successfully generate decision rules that reclassify SKUs into corresponding "correct" groups. This experiment confirms the main conclusion as drawn by Slowinski et al. [31] and Greco et al. [13] that DRSA is the most general MCDA methodology and other MCDA approaches can be represented in terms of decision rules.

4.3 Comparison of Classification Results

Now, a sample of 15 SKUs, consisting of three **A**, five **B**, and seven **C** items, is randomly drawn from the classification results of the AHP-based approach. This sample is input into the DRSA procedure as a training set to generate a list of decision rules. These rules are then applied to all 47 SKUs for classifying them into appropriate groups. This sampling process is repeated 20 times in order to draw statistically significant inferences. The summary of the experiment is given in Table 3. The first column from the left identifies the random training set (the actual lists of T_i, $i = 1, 2, ..., 20$ are not elaborated for the sake of space, but they are available upon request). The second, third, and fourth columns summarize the results from the output of 4eMka2 [16], specifying the number of correct, incorrect, and ambiguous classification results out of the 47 SKUs. The last column indicates the number of "extreme errors", where original **A** items in [9] are classified into **C**, or **C** items in [9] are classified into group **A** as per the generated rules. From a managerial point of view, this kind of "errors" is severe and should always be avoided.

Table 3. Summary of Comparison of Classification Results

Test	Reclassification Results			
Set	Correct Answers	Incorrect Decisions	Ambiguous Decisions	Extreme Errors
T_1	36	9	2	3
T_2	29	13	5	0
T_3	33	3	11	0
T_4	39	6	2	0
T_5	35	6	6	3
T_6	35	10	2	0
T_7	30	15	2	0
T_8	33	10	4	2
T_9	39	7	1	0
T_{10}	41	6	0	0
T_{11}	31	7	9	0
T_{12}	36	5	6	2
T_{13}	35	9	3	0
T_{14}	41	6	0	0
T_{15}	32	8	7	7
T_{16}	37	10	0	0
T_{17}	39	6	2	0
T_{18}	36	4	7	0
T_{19}	28	11	8	0
T_{20}	37	7	3	0

Table 3 demonstrates the applicability of our proposed DRSA approach for MCABC. Firstly, it is rare to have extreme errors of classifying **A** items into group **C**, or **C** items into group **A**: our sample gives a 95% confidence interval of 0.850 ± 0.835. Secondly, most of the 47 SKUs can be categorized into corresponding "correct" groups (the 95% confidence interval is 35.100 ± 1.777). The remaining ambiguity is largely due to the lack of effective mechanisms in DRSA to prioritize different criteria in classifying SKUs while the AHP-based approach allows a DM to determine the weights of criteria. If some conflicting rules generated from the training set are examined and properly removed by the experts who conducted the AHP-based analysis [9], one can expect an increasing number of "correct decisions" and a lower number of "ambiguous decisions".

5 Conclusions

Classical ABC analysis is a straightforward technique to achieve cost-effective inventory management by categorizing SKUs into three groups according to annual dollar usage and then applying similar inventory management procedures throughout each group. However, management can often be made more effective by classifying SKUs under additional criteria, such as lead time and criticality. MCABC furnishes an inventory manager with the flexibility of accounting for more factors when an SKU is categorized.

This paper proposes a dominance-based rough set approach to solve MCABC problems under the umbrella of MCDA theory. Two comparison experiments are conducted based upon a case study. The first experiment, *decision rule generation comparison*, examines whether the DRSA can reproduce the results obtained by other decision models. It is shown that, in most situations, the results are comparable with those obtained using other decision analysis methods such as the AHP-based approach, thereby confirming the applicability of this approach.

In the second experiment, *comparisons of classification results*, the classification result obtained by using the AHP-based approach is adopted as a benchmark and is compared with the one generated by the DRSA. It demonstrates that the decision rules obtained by the DRSA can provide a good approximation of the decision analysis conducted by the AHP method. Future research is needed to compare the classification abilities of this method in various situations with other case-based classification methods, such as methods described by Doumpos and Zopounidis [6].

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