Possibility Measures in Probabilistic Inference

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Abstract. By means of a logical condition between two partitions $\mathscr{L}$ and $\mathscr{L}'$ ("weak logical independence"), we find connections between probabilities and possibilities. We show that the upper envelope of the extensions of a probability on $\mathscr{L}$ is a possibility on the algebra generated by $\mathscr{L}'$. Moreover we characterize the set of possibilities obtained as extensions of a coherent probability on an arbitrary set: in particular, we find the two "extreme" (i.e., dominated and dominating) possibilities.

Keywords: Probabilistic inference, Weakly logical independence, Uncertainty measures, Coherence.

1 Introduction

The classic approaches to knowledge acquisition or decision processes start from a knowledge–base able to settle once for all the set of objects ruling the inferential process (states of nature, events, "rules", functions measuring uncertainty, etc.), requiring also further conditions (such as closure of the family of events with respect to Boolean operations). In particular, for semantic reasons a framework of reference (probability theory, Dempster-Shafer theory, possibility theory, default logic, fuzzy set theory and so on) is usually chosen once for all. Actually, often we need to manage uncertainty relative to a set of events while having information only for a *different* family of events. In fact, making inference essentially means extending a structured information (carried, for example, by a particular measure of uncertainty) to "new" events, and this is done by taking into account only the logical relations among the events of the two given families.

In general, in the extension processes, the enlargements can lead to uncertainty measures different from the initial ones. For instance, in [2] it has been proved that, if we start from a (coherent) assessment P on a set $\mathscr{L}$ of pairwise incompatible events, and consider any algebra of events $\mathscr{A}$, then the lower [upper] envelope of the class of coherent probabilities extending P to $\mathscr{L} \cup \mathscr{A}$ is a belief [plausibility] function. Vice versa, for any belief function Bel on an algebra $\mathscr{A}$, there exists a partition $\mathscr{L}$ and a relevant probability P such that the lower bound of the class of probability extending P on $\mathscr{A}$ coincides with Bel (similarly for a plausibility function, and referring to the upper bound). This result is independent of any logical relation between the partition $\mathscr{L}$ and that $\mathscr{L}'$

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of atoms of $\mathscr{A}$. Obviously, any logical constraint between the two partitions rules the numerical values of the belief (or plausibility) function.

In [3] we proved that under suitable logical conditions between the partitions, the upper envelope (i.e., plausibility) is a possibility and the lower envelope is a necessity. Moreover, any possibility measure on an algebra $\mathscr{A}$, can be obtained as an enlargement of a probability distribution on a partition satisfying the same logical condition. This logical condition between the partitions is a suitable weakening of logical independence (see Sect. 3). A particular case is that corresponding to the logical independence of the two aforementioned partitions, in which we get a plausibility equal to 1 on $\mathscr{A} \setminus \emptyset$ for any P (which is also a noninformative possibility). These results are based on the assumption that the initial information consists of a probability distribution on the elements of a *partition* of Ω. But this is not realistic in real problems, so we study what happens starting from a (coherent) probability on an *arbitrary set of events* $\mathscr{E}$ and enlarging this assessment to an other finite set $\mathscr{E}'$: we need to handle a class of probability $\mathbf{P}_0$ (all those consistent with the coherent assessment) on the partition $\mathscr{C}$ constituted by the set of atoms generated by $\mathscr{E}$. Clearly, for every distribution on $\mathbf{P}_0$, we obtain (as lower and upper envelope of the relevant extension on $\mathscr{E}'$) a coherent belief function and a plausibility respectively, and when $\mathscr{C}$ and $\mathscr{C}'$ are weakly logically independent ($\mathscr{C}'$ is the set of atoms generated by $\mathscr{E}'$) we obtain a coherent necessity and a possibility on $\mathscr{E}'$, respectively. Obviously, it is interesting to characterize the class of these measures and in particular to study whether there is a minimum and a maximum element: in general this characterization is not possible, since the upper [lower] envelope of plausibilities [belief function] is not a plausibility [belief]. On the contrary, we prove that a characterization is possible when $\mathscr{C}$ and $\mathscr{C}'$ are weakly logically independent, obtaining a class of possibilities such that both its upper and lower envelopes Π^* and Π_* are (respectively, the dominating and dominated) possibilities. This class contains all the possibilities weakly comonotone with Π_* and Π^* (equivalent results hold for necessities).

These results contribute to the deepening of hybrid models involving probability, plausibility and possibility, which have been studied in many papers, e.g. [7, 8, 9, 10, 11, 13]: our approach is essentially syntactic and emphasizes an inferential point of view.

2 Coherent Assessments and Their Enlargements

The axioms defining an uncertainty measure strictly refer to the assumption that its domain is a Boolean algebra. Then dealing with an *arbitrary set of events* requires to characterize assessments which are coherent (or consistent) with a specific measure on a Boolean algebra containing this set.

In probability theory it is well known the concept of coherence introduced by de Finetti [6] through a betting scheme, or its dual version based on the solvability of a linear system. An analogous notion of coherence for possibilities has been introduced in [4].

Definition 1. *Let $\mathscr{E} = \{E_1, ..., E_n\}$ be a finite set of events and denote by $\mathscr{A}$ the algebra generated by $\mathscr{E}$. An assessment φ on $\mathscr{E}$ is a coherent possibility [probability] if there exists a possibility [probability] Φ defined on $\mathscr{A}$ extending φ (i.e. $\Phi_{|\mathscr{E}} = \varphi$).*

The so–called *fundamental theorem of probability* assures that, given a *coherent* assessment P on an *arbitrary* finite family $\mathscr{E}$, it can be extended (possibly not in a unique way) to any set $\mathscr{E}' \supset \mathscr{E}$; moreover, for each event $E \in \mathscr{E}' \setminus \mathscr{E}$ there exist two events E_* and E^* (possibly $E_* = \emptyset$ and $E^* = \Omega$) that are, respectively, the "maximum" and the "minimum" union of atoms A_r (generated by the initial family $\mathscr{E}$) such that $E_* \subseteq E \subseteq E^*$. If E is logical dependent on $\mathscr{E}$, then $E_* = E = E^*$. Then, given the set $\{\tilde{P}\}$ of all possible extensions of P, coherent assessments of $\tilde{P}(E)$ are all real numbers of a closed interval $[p_*, p^*]$, with

$$p_* = \inf \tilde{P}(E_*) = \inf \sum_{A_r \subseteq E_*} \tilde{P}(A_r) \,, \qquad p^* = \sup \tilde{P}(E^*) = \sup \sum_{A_r \subseteq E^*} \tilde{P}(A_r) \,. \tag{1}$$

We proved in [4] for a possibility Π a similar result: coherence of a possibility assessment assures its extendibility to new events, and for any new event the coherent possibility values belong to an interval $[\pi_*, \pi^*]$ with

$$\pi_* = \min \left(\max_{A_r \subseteq E_*} \Pi'(A_r) \right) \,, \qquad \pi^* = \max \left(\max_{A_r \subseteq E^*} \Pi'(A_r) \right) \,,$$

where $\{\Pi'\}$ is the set of all possible extensions of Π.

It is well known that by computing for some "new" events the relevant coherence probability [possibility] intervals, not all the choices of values in these intervals lead to "an overall" coherent probability [possibility]. In the probabilistic framework, if we choose for any event the minimum [the maximum] value (which correspond essentially to natural extension, see [14]), we obtain a lower [upper] probability. Furthermore, in the possibilistic setting we get different results: in fact, the upper envelope of possibilities is still a possibility [4], while the lower envelope of possibilities is not necessarily a possibility.

3 Weakly Logically Independent Partitions

We recall that two partitions $\mathscr{L}, \mathscr{L}'$ of Ω are logically independent if for every $E_i \in \mathscr{L}$ and $E'_j \in \mathscr{L}'$ one has $E_i \wedge E'_j \neq \emptyset$ (or, equivalently, $\Omega = \bigvee_{E_i \wedge E'_j \neq \emptyset} E_i$ for any $E'_j \in \mathscr{L}'$). In [3] we introduced the following "weaker" condition: for any $E'_j \in \mathscr{L}'$, denote by A_j the minimal (with respect to the inclusion) event logically dependent on $\mathscr{L}$ containing E'_j, that is

$$A_j = \bigvee_{E_i \wedge E'_j \neq \emptyset} E_i \,.$$

(Obviously, A_j is an element of the algebra $\mathscr{A}$ spanned by $\mathscr{L}$). Given $\mathscr{L}, \mathscr{L}'$, for any $E'_j \in \mathscr{L}'$ we consider the corresponding $A_j \in \mathscr{A}$.

Definition 2. *The partition $\mathscr{L}'$ is weakly logically independent of the partition $\mathscr{L}$ (in symbols, $\mathscr{L}' \perp_w \mathscr{L}$) if, for any given $E'_i \in \mathscr{L}'$, every other $E'_k \in \mathscr{L}'$ ($k \neq i$) satisfies al leat one of the following conditions*

- $E'_k \subseteq A_i$
- $E'_k \wedge E_j \neq \emptyset$ *for any* $E_j \subseteq A_i$.

Clearly, if $\mathscr{L}, \mathscr{L}'$ are logically independent, then $\mathscr{L}' \perp_w \mathscr{L}$, but the vice versa does not hold: let $\mathscr{L} = \{E, E^c\}$, $\mathscr{L}' = \{F, F^c\}$ with $F \subset E$, then $\mathscr{L}' \perp_w \mathscr{L}$, but $\mathscr{L}'$ and $\mathscr{L}$ are not logically independent. As proved in [3] the notion of weakly logically independent partitions is symmetric (i.e. $\mathscr{L}' \perp_w \mathscr{L} \Longrightarrow \mathscr{L} \perp_w \mathscr{L}'$).

We recall now some properties of weakly logically independent partitions.

Proposition 1. *Let $\mathscr{L}, \mathscr{L}'$ be two partitions of Ω. If $\mathscr{L}' \perp_w \mathscr{L}$, then the following statements hold:*

1. *for every $E'_i, E'_j \in \mathscr{L}'$, $A_j \subseteq A_i$ or $A_i \subseteq A_j$;*
2. *there exists $E'_i \in \mathscr{L}'$ such that $E'_i \wedge E_j \neq \emptyset$ for any $E_j \in \mathscr{L}$;*
3. *if there exist $E'_i \in \mathscr{L}'$ and $E_j \in \mathscr{L}$ such that $E'_i \subseteq E_j$, then, for every $E'_r \in \mathscr{L}'$, we have $E'_r \wedge E_j \neq \emptyset$.*
4. *there exists at most one $E_k \in \mathscr{L}$ such that $E'_i \subseteq E_k$ for some $E'_i \in \mathscr{L}'$.*

Proposition 1 easily implies that if $\mathscr{L}$ is a refinement of $\mathscr{L}'$, then $\mathscr{L}' \not\perp_w \mathscr{L}$.

Theorem 1. *Let $\mathscr{L} = \{E_1, ..., E_i, ..., E_n\}$ and $\mathscr{L}' = \{E'_1, ..., E'_j, ..., E'_m\}$ be two partitions of Ω. The following two conditions are equivalent:*

1. *$\mathscr{L}' \perp_w \mathscr{L}$;*
2. *there exists a permutation of the indices $1, ..., m$ such that the corresponding events $A_1, ..., A_j, ... A_m$ are completely ordered by inclusion.*

4 Possibility as Enlargement of a Coherent Probability

In [2, 5] it has been proved that, if $\mathscr{L}, \mathscr{L}'$ are two partitions of Ω and $\mathscr{A}'$ the algebra spanned by $\mathscr{L}'$, and P a probability distribution on $\mathscr{L}$, then, considering the family $\mathbf{P}$ of probabilities P_i extending P on $\mathscr{L} \cup \mathscr{A}'$, the lower bound of $\mathbf{P}$ on $\mathscr{A}'$ is a belief function (and the upper bound a plausibility function). Vice versa, for any belief function *Bel* on an algebra $\mathscr{A}'$ there exists a partition of Ω and a relevant probability distribution such that the lower bound of the class of probability extending P on $\mathscr{A}'$ coincides with *Bel* [2] (similarly for a plausibility function). This result is independent of any logical relation between the partition $\mathscr{L}$ and that of atoms of $\mathscr{A}'$. Obviously, the logical constraints rule the numerical values of the belief (or plausibility) function.

In [3] we proved that if two partitions are weakly logically independent, then the plausibility obtained as upper envelope of the class $\mathbf{P}$ is a possibility:

Theorem 2. *Let $\mathscr{L}, \mathscr{L}'$ be two partitions of Ω and $\mathscr{A}'$ the algebra spanned by $\mathscr{L}'$. Let P be a probability distribution on $\mathscr{L}$ and $\overline{P}$ the upper envelope of the class $\mathbf{P} = \{P'\}$ of all the probabilities extending P onto $\mathscr{L} \cup \mathscr{A}'$. If $\mathscr{L}' \perp_w \mathscr{L}$, then $\overline{P}$ is a possibility measure on $\mathscr{A}'$.*

This result is related to that given in [9]: any set of lower bounds on a nested class $A_1, ..., A_m$ induces an upper probability, that is a possibility. As shown in [3] a possibility can be obtained also when $\mathscr{L}' \not\perp_w \mathscr{L}$ (but not if the probability distribution is strictly positive).

Theorem 3 shows how weakly logically independent partitions rule the transition from probability to possibility and also the other way round.

Theorem 3. *Consider a possibility measure Π on an algebra $\mathscr{A}$ and let $\mathscr{L}$ be the set of atoms of $\mathscr{A}$. Then, there exists a partition $\mathscr{L}'$ and a probability distribution on $\mathscr{L}'$ such that:*

1. $\mathscr{L}' \perp_w \mathscr{L}$,
2. *the upper envelope $\overline{P}$ of the class $\mathbf{P} = \{P'\}$ of all the probabilities extending P on $\mathscr{L}' \cup \mathscr{A}$ coincides on $\mathscr{A}$ with the possibility measure Π.*

Remark 1. In [3] we proved that, given two logically independent partitions $\mathscr{L}$ and $\mathscr{L}'$, the upper envelope of the extensions on $\mathscr{L} \cup \mathscr{A}'$ of a probability P on $\mathscr{L}$ is a possibility on $\mathscr{A}'$ and, for any $A \in \mathscr{A}' \setminus \emptyset$, $\overline{P}(A) = 1$. Thus, we get in this case the non informative possibility independently of the initial probability distribution.

5 From a Coherent Probability to the Upper Possibility

All the results of the previous Section are based on the assumption that the initial information is handled by a probability distribution on the elements of a partition of Ω. Now we start instead from a coherent probability on an arbitrary set of events $\mathscr{E}$. Then, we need to consider all the extensions on any other finite set $\mathscr{E}'$. Since coherence implies the existence of a class $\mathbf{P} = \{P_i\}$ of probabilities on the set $\mathscr{C}$ of atoms generated by $\mathscr{E}$, for any such probability distributions $P_i \in \mathbf{P}$ we have a plausibility [belief] as an upper [lower] bound of the probabilities extending P_i in $\mathscr{E}'$; moreover if $\mathscr{C} \perp_w \mathscr{C}'$ (with $\mathscr{C}'$ the set of atoms generated by $\mathscr{E}'$) for each $P_i \in \mathbf{P}$ we obtain a possibility.

In general it is not possible to characterize the set of plausibilities, since the upper envelope of plausibilities is not a plausibility. In this Section we prove instead that, when $\mathscr{C} \perp_w \mathscr{C}'$, we obtain a class of possibilities such that both their upper and lower envelopes are possibilities (i.e., that dominating and that dominated by all other possibilities, respectively).

Theorem 4. *Let $\mathscr{E}, \mathscr{E}'$ be two finite sets of events and $\mathscr{C}, \mathscr{C}'$ the corresponding sets of atoms generated by $\mathscr{E}$ and $\mathscr{E}'$. Moreover, let P be a coherent probability on $\mathscr{E}$, and $\mathbf{P}$ the set of coherent probability extensions of P on $\mathscr{E} \cup \mathscr{E}'$. If $\mathscr{C} \perp_w \mathscr{C}'$, then the upper envelope of $\mathbf{P}$ on $\mathscr{E}'$ is a coherent possibility.*

Proof. The coherent probability P on $\mathscr{E}$ can be extended on $\mathscr{E} \cup \mathscr{C}$ and let $\mathbf{P} = \{P'\}$ be the set of all the coherent probability extensions of P on $\mathscr{E} \cup \mathscr{C}$. Since $\mathscr{C}$ is finite [12] there exists a finite subset $\mathbf{P}_m$ of $\mathbf{P}$ such that

$$\overline{P}(C) = \sup_{P' \in \mathbf{P}} P'(C) = \sup_{P' \in \mathbf{P}_m} P'(C)$$

for any $C \in \mathscr{C}$. Since $\mathscr{C} \perp_w \mathscr{C}'$, the upper envelope of the extensions of a probability $P' \in \mathbf{P}_m$ is a possibility distribution on the algebra $\mathscr{A}'$ generated by $\mathscr{C}'$ by Theorem 2.

Then, we can consider the finite set $\{\Pi\}$ of possibilities on $\mathscr{A}'$ associated to $\mathbf{P}_m$. The upper envelope Π^* of $\{\Pi\}$ is a possibility and then the restriction of Π^* on $\mathscr{E}' \subseteq \mathscr{A}'$ is a coherent possibility. The coherent possibility Π^* on $\mathscr{E}'$ coincides with the upper envelope of P on $\mathscr{E}'$, in fact for any $E \in \mathscr{E}'$

$$\Pi^*(E) = \sup \Pi(E) = \sup_{P' \in \mathbf{P}_m} \sum_{C_r \wedge E \neq \emptyset} P'(C_r) = \sup_{P' \in \mathbf{P}} \sum_{C_r \wedge E \neq \emptyset} P'(C_r) = \overline{P}(E). \qquad \square$$

The coherent possibility Π^* of the above result is the less informative, in the sense that it dominates any possibility arising in the enlargement procedure. Now, we are interested also to look for the most informative one, in the sense that is dominated by any other one.

Theorem 5. *Let $\mathscr{E}, \mathscr{E}'$ be two finite sets of events, $\mathscr{C}, \mathscr{C}'$ the corresponding sets of atoms generated by $\mathscr{E}$ and $\mathscr{E}'$ and $\mathscr{A}, \mathscr{A}'$ the algebras spanned by $\mathscr{E}, \mathscr{E}'$, respectively. Given a coherent probability P on $\mathscr{E}$, consider the lower envelope $\underline{P}$ of the set $\mathbf{P} = \{P'\}$ of extensions of P on $\mathscr{A}$ and the function Π_* defined on $\mathscr{A}'$ as follows: for any $B \in \mathscr{A}'$*

$$\Pi_*(B) = \inf_{A \in \mathscr{A} : A \supseteq B} \underline{P}(A).$$

If $\mathscr{C} \perp_w \mathscr{C}'$, then Π_* is a coherent possibility on $\mathscr{E}'$. Moreover, the upper envelope Π_1 on $\mathscr{A}'$ of the extensions of any $P' \in \mathbf{P}$ dominates Π_*.

Proof. If $\mathscr{C}' \perp_w \mathscr{C}$, then by Theorem 1 there exists an ordering on the elements of $\mathscr{C}' = \{E'_1, ..., E'_m\}$ such that $A_i \subseteq A_{i+1}$ for $i = 1, ..., m-1$. Hence, for any $E'_i \in \mathscr{C}'$ one has

$$\Pi_*(E'_i) = \inf_{A \in \mathscr{A} : A \supseteq E'_i} \underline{P}(A) = \underline{P}(A_i).$$

In particular, since $A_m = \Omega$, it follows $\Pi_*(E'_m) = 1$. Consider any $F = \bigvee_{j \in J} E'_j$: there exists $\overline{j} \in J$ such that $j < \overline{j}$ for any $j \in J$ (with $j \neq \overline{j}$), then $A_j \subseteq A_{\overline{j}}$ and so

$$\Pi_*(F) = \inf_{A \in \mathscr{A} : A \supseteq F} \underline{P}(A) = \underline{P}(A_{\overline{j}}) = \max_{j \in J}\{\underline{P}(A_j)\} = \max_{E'_i \subseteq F} \Pi_*(E'_i)$$

then Π_* is a possibility on $\mathscr{A}'$ and so Π_* on $\mathscr{E}'$ is a coherent possibility. Now, given $P' \in \mathbf{P}$, since $\mathscr{C}' \perp_w \mathscr{C}$, by Theorem 2 the upper envelope Π_1 of the extensions of P' on $\mathscr{A}'$ is a possibility and for any $F \in \mathscr{A}'$, $F = \bigvee_{j \in J} E'_j$, there exists $\overline{j} \in J$ such that $A_j \subseteq A_{\overline{j}}$ for any $j \in J$, $F \subseteq A_{\overline{j}}$ and

$$\Pi_*(F) = \inf_{A \in \mathscr{A} : A \supseteq F} \underline{P}(A) = \underline{P}(A_{\overline{j}}) \leq P'(A_{\overline{j}}) = \Pi_1(F). \qquad \square$$

By the previous result we obtain a possibility Π_* that is dominated by any possibility obtained as the upper envelope of the extensions of a coherent probability (on the assumption that the two sets of atoms are weakly logically independent). Note that in general the minimum of a set of possibilities is not a possibility, while in the case that the possibilities are obtained through the inferential procedure shown in Section 4 their infimum, that coincides with Π_*, is still a possibility. Then, for any $F \in \mathscr{A}'$ we get two (possibly coincident) values $\Pi_*(F)$ and $\Pi^*(F)$. The following Theorem 6 shows that any possibility Π weakly comonotone with (Π_*, Π^*) can be obtained as the upper envelope of the extensions of a coherent probability, where weakly monotonicity is defined as follows:

Definition 3. *A possibility Π on $\mathscr{A}'$ is weakly comonotone with (Π_*, Π^*) if $\Pi_*(F) \leq \Pi(F) \leq \Pi^*(F)$ for any $F \in \mathscr{A}'$ and for any pair of atoms $E'_i, E'_j \in \mathscr{A}'$ the following conditions hold:*

- *if $\Pi_*(E'_i) < \Pi_*(E'_j)$ or $\Pi^*(E'_i) < \Pi^*(E'_j)$, then $\Pi(E'_i) \leq \Pi(E'_j)$,*
- *if $\Pi_*(E'_i) = \Pi_*(E'_j)$ and $\Pi^*(E'_i) = \Pi^*(E'_j)$, then $\Pi(E'_i) = \Pi(E'_j)$.*

Theorem 6. *Let $\mathscr{E}, \mathscr{E}', \mathscr{C}, \mathscr{C}', \mathscr{A}, \mathscr{A}'$,* **P** *and Π_* as in Theorem 5 and consider the upper envelope Π^* on $\mathscr{A}'$ of the coherent extensions of P. If $\mathscr{C} \perp_w \mathscr{C}'$, then Π_* and Π^* are possibilities on $\mathscr{A}'$. Moreover, Π_* is dominated by Π^* and, for any coherent possibility Π on $\mathscr{A}'$ weakly comonotone with (Π_*, Π^*), there exists a coherent probability P on $\mathscr{E}$ such that the upper envelope of the extensions of P on $\mathscr{A}'$ coincides with Π.*

Proof. By Theorem 5 and by Theorem 4 the functions Π^* and Π^* are possibilities and $\Pi_*(F) \leq \Pi^*(F)$ for any $F \in \mathscr{A}'$. Now, consider any possibility Π on $\mathscr{A}'$ satisfying the conditions in the hypothesis. Let us assume (without loss of generality, see Proposition 1) that the partition $\mathscr{C}' = \{E_1, ..., E_k\}$ is ordered in a way that for any $i < j$ one has $\Pi(E_i) \leq \Pi(E_j)$ for any $E_i, E_j \in \mathscr{C}'$. This order is compatible with that built starting from Π_* or Π^*, then the partition $\mathscr{C}' = \{F_1, ..., F_m\}$ is such that $\mathscr{C}' \perp_w \mathscr{L}$ and considering $E'_i, E'_j \in \mathscr{C}'$ if $i < j$, then $\Pi(E'_i) \leq \Pi(E'_j)$, since the associated A_i and A_j are such that $A_i \subseteq A_j$. Hence, there exists a probability on $\mathscr{A}$ such that $P(A_i) \leq P(A_j)$ for any $i < j$ and $P(A_j) = \Pi(E'_j) - \Pi(E'_{j-1})$ for any $j = 1, ..., k$ by putting $\Pi(E'_0) = \Pi(\emptyset)$. This probability on $\mathscr{A}$ generates Π through the inferential process. □

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