On a Membrane Formation in a Spatio-temporally Generalized Prisoner's Dilemma

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Abstract. A spatio-temporal generalization involves not only conventional temporal strategies that determine the player's action based on the opponent past actions but also spatial strategies based on the neighbor players' current actions and configurations. This framework allows the model to be dealt as a second order cellular automaton. With this involvement of the spatial strategies, we have observed a membrane formation which protects the cooperating clusters from being corroded by defecting intruders.

Keywords: spatial prisoner's dilemma, generalized prisoner's dilemma, second order cellular automata, membrane formation, maintenance of cooperating clusters.

1 Introduction

It has been long debated and discussed that the core mechanism that allows cooperation to evolve in social, biological, or ecological systems in spite of seemingly more advantageous strategy of the defection [1,2,3,4]. After a genius work of spatial framework by Nowak and May [5,6], the maintenance and protection of cooperators' cluster can be regarded as a problem of cellular automaton. Many possible mechanisms for the maintenance and protection of cooperators' cluster have been proposed [7,8,9]. In a spatio-temporal generalization of Prisoner's Dilemma (PD) [10], we observed a membranous phenomenon where a membrane in a perimeter of cooperators' cluster and protect the cluster from the invasion by defectors where the cooperators' cluster would be invaded otherwise.

Prisoner's Dilemma has been providing motivations in many fields not only international politics but evolutionary biology since a seminal work by Axelrod. Spatial prisoner's dilemma invented by Nowak and May also provides another dimension that these originally game theoretic studies can be related to the field of cellular automata. In the spatio-temporal generalization of dilemmaDcomes even more obvious that the generalized model is a sort of CA (cellular automata): a second order CA where the strategy first determines the rule based on the neighbors' configuration and the then rule in turn determines the next action.

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One could point out that the involvement of player's benefit (expressed by a payoff matrix as in Table 1) is crucial but it is still mathematically (with a language of Mappings) regarded as a second order CA.

Section 2 states definitions and notations used in this note. Section 3 presents the main results of conditions for the membrane formation. Conditions are stated without proof. Formal discussions will be presented elsewhere.

2 Definitions and Notations of Generalized Prisoner's Dilemma

We have studied a spatial version of PD, and proposed a generalized TFT (Tit-for-Tat; it would defect only when the adversary defects, and would cooperate otherwise) such as *k1C, k2D* and their combination *k1C-k2D*, where *k1* is a parameter indicating generosity and *k2* contrariness. Dynamics of these spatial strategies in a two-dimensional lattice has been also studied in a noisy environment.

The PD is a game played just once by two players with two actions (cooperation, C, or defect, D). Each player receives a payoff (R, T, S, P) where $T > R > P > S$.

In IPD (Iterated PD), each player (and hence the strategy) is evaluated with further constraint: $2R > T + S$. In Spatial Prisoner's Dilemma (SPD), each site in a two-dimensional lattice corresponds to a player. Each player plays PD with the neighbors (8 adjacent players as in Fig.1), and changes its action by the total score it received.

Our model generalized SPD by introducing spatial strategy. Each player placed at each lattice of the two-dimensional lattice. Each player has an action and a strategy, and receives a score. Spatial strategy determines the next action dependent upon the spatial pattern of actions in the neighbors. Each player plays PD with the neighbors, and changes its strategy to the strategy that earns the highest total score among the neighbors. Table 1 is the Payoff matrix of PD. In our simulations, $R, S, T,$ and P are respectively set to $1, 0, b$ ($1 < b < 2$, a bias for defectors) and 0 in simulations below following the Nowak-May's simulations [6].

Table 1. The Payoff Matrix of the Prisoner's Dilemma Game. R, S, T, P are payoff to the player 1. ($1 < b < 2$)

		Other C	Other D
Player	C	$R(1)$	$S(0)$
	D	$T(b)$	$P(0)$

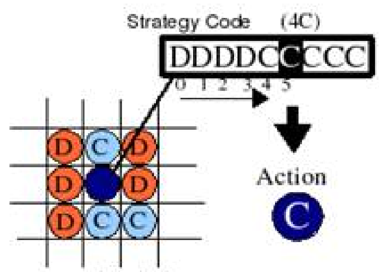

Fig. 1. A Strategy Code for Spatial Strategies

Our SPD is done with spatial strategies: the next action will be determined based on the pattern of neighbors' actions. Score is calculated by summing up all the scores received from PD with 8 neighbor players and itself. After q (strategy update cycle) steps of interactions with neighbors and self, the strategy will be chosen from the strategy with the highest score among the neighbors including the self. Thus, the strategy will be updated at every q (set to be 1 throughout this paper) steps. In an evolutionary framework, strategy will be also changed by a mutation rate (set to be 0 throughout this paper) where mutation is operated on the string of the strategy code below.

To specify a spatial strategy, actions of the eight neighbors in the neighborhood radius $r = 1$ (i.e., the *Moore* neighbors as in Fig.1. When $r = 2$, it would be 24 neighbors.) and the player itself must be specified (Fig.1), hence 2^9 rules are required. For simplicity, we restrict ourselves on a "totalistic spatial strategy" that depend on the number of D (defect) action of the neighbor, not on their positions.

This k-D can be regarded as a spatial version of TFT where k indicates the spatial version of the generosity [11] (how many D actions in the neighbor are tolerated.).

3 Membrane Formation

In studying a mechanism that allows cooperators clusters to be preserved, we are studying a spatial version of generosity: how many defections in the neighborhood are tolerated rather than how many previous defections of the opponent in the spatio-temporal generalized context. In interactions between All-D v.s. k-D instead of All-D v.s. All-C (as in Nowak-May's SPD), we found that clusters of k-D form membrane of action D protecting the inner cluster of action C (Note that k-D can take both C or D depending on the number of Ds in the neighborhood). We observed that this membrane formation occurs as in Fig. 2 when a certain parameter scope of k (spatial generosity), r (neighborhood radius) and b (bias for defectors). We will focus on the condition of the membrane formation with respect to these parameters. Throughout this note, simulations are conducted in a square lattice with periodic boundary condition with the following parameters listed in Table 2.

Table 2. List of Parameters for Simulations

Name	Description	Value
$L \times L$	Size of the space	1,500 × 1,500
N	Number of the players	2,250,000
T	Number of steps	2,500
$N_{k-D(0)}$	Initial number of the k-D with C state	All-D and k-D are randomly assigned with equal probability. Similarly to C/D.
r	Neighborhood radius	1, 2, 3

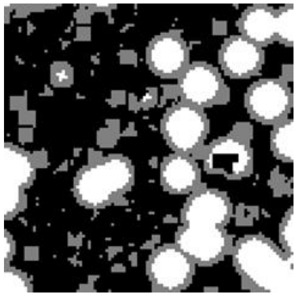

Fig. 2. Membrane formation in a generalized PD. Black cells are All-D; white and gray cells are C and D state of k-D, respectively. In this snapshot, $k = 6$ and 6-D strategy players are allocated in a random positions initially.

The parameter b is set to be a minimal value that allows All-D to expand. Fig. 3 shows a part of the square lattice where C and D players are indicated white and black cells respectively. For All-D in the corner (indicated by a red circle) to gain the profit higher than the cooperators, b must satisfy $5b > 9$ since the highest profit of the cooperators is 9 when $r = 1$ (similarly, $16b > 25$ when $r = 2$, and $33b > 49$ when $r = 3$).

After the membranes are formed, the following three phenomena are observed depending on the k value.

1. k is too small: The membrane will grow toward the center of k-D cluster and will corrode the C state of k-D.
2. k is small: The k-D cluster covered by the membrane will stay stable.
3. k is large: The k-D cluster covered by the connected membrane will expand.
4. k is too large: The k-D cluster covered by the broken membrane will expand and the cluster will eventually collapse.

Since we are interested in conditions for cooperators to be preserved, we focus on the conditions on the cases 2 and 3 above. The spatial generosity k increases as the case proceeds downward from 1 to 4.

For the membrane formation, the spatial generosity k must exceed a certain value formulated by the neighborhood radius r:

$$k \geq (2r + 1)(r + 1) - r.$$

Otherwise, the membrane will grow inside toward the center of the k-D cluster as in case 1.

For the cluster protected by the membrane to expand, the spatial generosity k must further exceed a larger threshold:

$$k \geq (2r + 1)(r + 1).$$

Otherwise, the cluster does not expand although it is indeed protected by the membrane.

In the case 4, membrane is broken if k exceeds a threshold (Fig. 4)

$$k \geq (2r+1)^2 - \sum_{i=0}^{i<\frac{-1+\sqrt{8r+17}}{2}} P(\{-\frac{i(i+1)}{2} + (2+r)\}),$$

where $P(x) = x$ when $x > 0$ and $P(x) = 0$ otherwise, and the k-D clusters with the broken membrane will collapse when they contact with each other in the expansion.

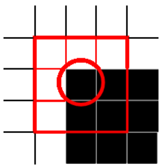

Fig. 3. C and D players are indicated white and black cells respectively. For All-D in the corner (indicated by the circle) to gain the profit higher than the cooperators, b must satisfy $5b > 9$ since the highest profit of the cooperators is 9 when $r = 1$ (similarly, $16b > 25$ when $r = 2$, and $33b > 49$ when $r = 3$).

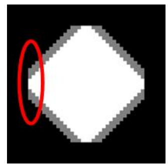

Fig. 4. Broken membrane indicated by the oval. The minimal cluster of 7-D is set in the center of the sea of All-D. All-D is indicated by black cell, while C state and D state of 7-D is indicated by white and gray cell, respectively. The figure shows a snapshot after 16 steps starting from 3×3 square of k-D in the center.

Fig. 5 plots the time evolution the fraction of k-D when the simulation starts from a random configuration stated with parameters as stated in Table 2. The fraction of 6-D is highest among other k-Ds, since the membrane protects the expanding 6-D clusters. The fractions of 7-D and 8-D are lower than 6-D because the membranes are broken in these k-Ds. The fraction of 5-D is the lowest, since the 5-D cluster does not expand although the cluster is protected by the membrane.

4 Discussions

We proposed yet another mechanism for preserving and protecting the cluster of cooperation in the spatio-temporally generalized Prisoner's Dilemma. After the membrane is formed, it can protect the cluster of cooperators from being invaded by the defectors. The condition for the membrane formation can be formulated by the parameter indicating spatial generosity. If the spatial generosity is too large, the membrane will be broken, while the membrane will develop into inside the cluster eradicating the cooperators if the spatial generosity is too small.

We also observed that several different polygons will form depending on the parameters and lattice topology. This phenomenon will be related to crystal

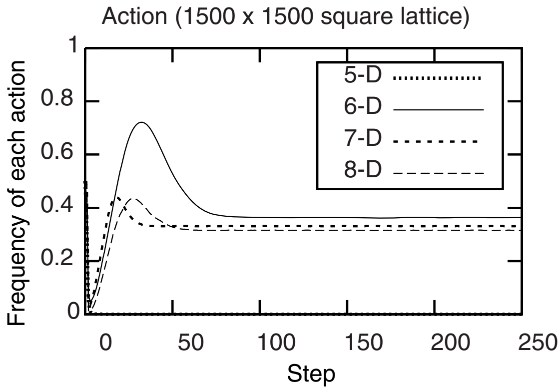

Fig. 5. Time evolution of the fraction of k-D when $r = 1$ and $b = 1.800010$. The fraction of 6-D is highest among other k-Ds, since the membrane protects the expanding 6-D clusters. The fractions of 7-D and 8-D are lower than 6-D because the membranes are broken in these k-Ds. The fraction of 5-D is the lowest, since the 5-D cluster does not expand although the cluster is protected by the membrane.

formation of different shapes in physical phenomena, while the membrane formation is related to biological phenomena where clusters of cells and chemical substances must be preserved for a certain amount of time and space.

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