# **Cellular Automata Simulation of Traffic Jam in Sag Section**

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**Abstract.** A traffic congestion often occurs near a sag section on a freeway. Since a road gradient changes gradually in a sag section, a driver dose not notice it and therefore, a traffic congestion occurs.

This paper describes the cellular automata simulation of the traffic flow through a sag section. The simulation is performed by the stochastic velocity model[\[4\]](#page-5-0). The results show that the effect of a sag zone to the traffic congestion becomes strong according to the increase of car density and that average velocity tends to decrease not at the sag section but at a little ahead point of sag section.

# **1 Introduction**

Since a traffic congestion strongly affects not only a traffic system but also environment, many researchers have been studying for solving the problems[\[1](#page-5-1)[,2\]](#page-5-2). A traffic congestion is occurred by several reasons. A sag section, which is a bottleneck of road structures, is also one of the obvious and important reasons[\[3](#page-5-3)[,5](#page-5-4)[,6\]](#page-5-5). A road gradient changes slightly and gradually in a sag section. Since drivers often do not notice the slight change, car velocity decreases gradually and therefore, a traffic congestion occurs.

The effect of a sag section for the traffic congestion is discussed in this paper. For this purpose, one uses the cellular automata simulation. The simulation model is based on the stochastic velocity model[\[4\]](#page-5-0). In this model, the car velocity and the movement is defined with a stochastic variable. Since the maximum movable distance of vehicles is restricted to one cell alone, the local rules to control the car behavior can be simplified. In the numerical examples, one discusses the dynamic property of the traffic congestion near the sag section.

# **2 Simulation Model**

#### **2.1 Object Under Consideration**

A road including a sag zone is illustrated in Fig[.1.](#page-1-0) The cell representation of the road is shown in Fig[.2.](#page-1-1) Since the cell size is  $3m \times 3m$ , the road length is 3333 cells  $(9999m)$ . The sag zone is specified from a 2668th cell from left end to a rightmost cell. A open condition is specified at the right end. Cars go through the domain from the left end and go out from the right end. When cars go through

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<span id="page-1-0"></span>

<span id="page-1-1"></span>**Fig. 2.** One-lane road with sag zone

the sag zone, negative acceleration is applied to them. The negative acceleration  $a_{saa} = 0.3(m/s^2)$  is defined as the average value of the them calculated from the gradients of the sag zones in the Tomei highway in Japan.

## **2.2 Local Rules**

Cars are controlled according to the local rules. Cars go straight ahead or change a driving lane and therefore, the car behaviour local rules are only for going straight ahead and changing lanes. Addition to them, the velocity local rule is employed.

**Safety Car Distance.** Assume that cars move so that their distance from a forehand car is kept to be safety. According to Japanese automobile inspection certification manual, a safety car distance  $G_{safe}$  is defined as the function of the car velocity  $v$  as

$$
G_{safe} = 0.15 \times v + 0.0097 \times v^2.
$$
 (1)

**Velocity Local Rule.** The velocity local rule change a car velocity according to a car distance. The parameter  $v, v_{\text{max}}$ ,  $G_{safe}$  and a denote the present car velocity, the feasible maximum velocity, the safety car distance and the acceleration of a car, respectively.

Acceleration If  $v < v_{\text{max}}$  and  $G > G_{safe}$  then,  $v \leftarrow v + a$ . Deceleration If  $v > 0$  and  $G < G_{safe}$  then,  $v \leftarrow v - b$ 

where the acceleration rate  $a$  and the deceleration rate  $b$  are estimated from the actual traffic data as  $0.6 < a < 2.4$  and  $1.2 < b < 3.0$ , respectively.

### **Local rule for going straight ahead**

- 1. A distance from a forehand car G is estimated.
- 2. If  $G < G_{safe}$ , a car moves to a forehand cell at the next time-step.
- 3. If not so, a car stops at the present cell.

#### **Local rule for changing lanes**

- 1. A distance from a forehand car is estimated at the present lane;  $G_1$ .
- 2. A distance from a forehand car is estimated at the other lane;  $G_2$ .
- 3. If  $G_1 < G_2$ , a car changes to the other lane.

# **3 Numerical Results**

#### **3.1 Traffic Jam in Sag Zone**

It is pointed out that the traffic congestion starts not just from a sag zone but from the slightly far point from the sag zone. When a cluster of cars reaches at a sag zone, a forehand car of the cluster is decelerated firstly. The velocity reduction propagates from the forehand car to the following cars, just like a shock wave. If the car density of the cluster is high enough, the traffic congestion occurs there. The frequency of the traffic congestion depends on the car density in the cluster.

First, the effect of the car density  $\alpha$  to a car position every five minutes is discussed in the numerical example for observing the development of a traffic congestion. The value  $\alpha$  is defined as the rate of cars going into the object domain from the left end. The car density is defined as the inverse number of the car distance  $\Delta x$ . The traffic flows at  $T = 5000$ ,  $T = 8000$  and  $T = 11000$  for different car density  $\alpha$  are compared in Fig[.3.](#page-3-0) The abscissa and the ordinate denote the car position and the car density, respectively.

We notice from the results at  $T = 5000$  and  $T = 8000$  that there is high car density between  $8001m$  and  $9999m$  from the left end. Next, we notice from results from  $T = 8000$  to  $T = 11000$  that there is also high car density between  $6001m$  and  $8000m$ . These mean a traffic congestion occurs in the sag zone first and then, develops from the sag zone to the upstream of the sag zone. When a cluster of cars reaches a sag zone, a forehand car is decelerated and therefore, the car density is increased. If the density is higher, the traffic congestion occurs there.

We will compare the traffic flows at difference values of  $\alpha$ . When focusing on the traffic congestion occurred between  $6001m$  and  $8000m$  at  $T = 8000$  to  $T = 11000$ , we notice that car density increases according to the increase of the car density  $\alpha$ . This means that the traffic congestion strongly depends on the car density  $\alpha$ .

## **3.2 Behavior of a Car**

We will focus on the behavior of a 300th car going from the left end. The reason why the 300th car is taken is that it goes through all traffic congestions after traffic congestions develop enough. The rate of cars going from the left end is fixed as  $\alpha = 10\%$ .



<span id="page-3-0"></span>**Fig. 3.** Development of traffic congestion

The results are shown in Fig[.4.](#page-4-0) The abscissa and the ordinate denote the carhead distance from the left end  $\Delta x$  and the velocity v, respectively. We notice from Fig[.4](#page-4-0) that the curves are so-called hysteresis loops and that cars move between the



<span id="page-4-0"></span>**Fig. 4.** Behavior of a car

high car density at the low velocity and the low car density at the high velocity. Besides, the profile of the hysteresis loop dose not depend on the value  $\alpha$ .

Next, we estimate the variation of the hysteresis loop at different traffic zones. The zones are taken as follows:

- Zone A: 0m to  $4000m$  (at upstream which is far from sag zone),
- Zone B:  $4001m$  to  $6000m$  (at upstream which is near sag zone),
- Zone C:  $6001m$  to  $8000m$  (at upstream which is adjacent to sag zone), and
- Zone D: 8001m to 9999m (in sag zone).

The results are shown in Tables [1](#page-4-1) to [3.](#page-5-7) The average velocity and the average absolute deviation of a car velocity v are referred to as  $v_{ave}$  and  $v_{a\nu edef}$ , respectively. Besides, the average value and the average absolute deviation of a car-head distance  $\Delta x$  are as  $\Delta x_{ave}$  and  $\Delta x_{a\nu edef}$ , respectively.

We notice the following points from Tables [1](#page-4-1) to [3.](#page-5-7) In cases of  $\alpha = 10\%$  and 30%, the values  $v_{ave}$ ,  $v_{avedef}$ ,  $\Delta x_{ave}$ , and  $\Delta x_{avedef}$  in zone A are smaller than them in zone B. This means that the hysteresis loop in zone A is bigger than that in zone B. Although the traffic flow in zone A is instable, the flow is getting stable to the zone B and C if the value of  $\alpha$  is small. However, also in zone B and C, the value  $\Delta x_{a \nu edef}$  in case of  $\alpha = 50\%$  is bigger than that in the lower value of  $\alpha$ . This means that the traffic flow is instable during whole zone if the value of  $\alpha$  is big enough.

<span id="page-4-1"></span>**Table 1.** Variation of car velocity and car-head distance  $(\alpha = 10\%)$ 

Position		$v_{ave}$ $v_{avedef}$ $\Delta x_{ave}$ $\Delta x_{avedef}$
$\begin{tabular}{cccc} 0-4000m & 76.27 & 5.61 & 70.954 & 15.40 \\ 4001-6000m & 77.78 & 3.11 & 68.52 & 14.80 \\ 6001-8000m & 78.48 & 0.00 & 74.70 & 13.97 \\ \end{tabular}$		
$8001\text{-}9999m\ 49.30\quad 15.07\quad 51.30$		26.84

Position		$v_{ave}$ $v_{avedef}$ head <sub>ave</sub> head <sub>avedef</sub>
		16.77
		14.73
		16.74
$\begin{tabular}{ccc} 0-4000m & 50.12 & 15.80 & 40.32 \\ 4001-6000m & 54.82 & 11.53 & 42.96 \\ 6001-8000m & 55.75 & 12.79 & 42.27 \\ 8001-9999m & 40.98 & 9.02 & 34.19 \\ \end{tabular}$		14.82

<span id="page-5-6"></span>**Table 2.** Variation of car velocity and car-head distance ( $\alpha = 30\%$ )

<span id="page-5-7"></span>**Table 3.** Variation of car velocity and car-head distance ( $\alpha = 50\%$ )

Position			$v_{ave}$ $v_{avedef}$ head <sub>ave</sub> head <sub>avedef</sub>
		29.20	13.09
		45.17	15.79
		51.21	31.13
$\begin{tabular}{c} 0-4000m & 36.88 & 16.35 \\ 4001-6000m & 53.54 & 14.26 \\ 6001-8000m & 51.39 & 16.41 \\ 8001-9999m & 51.95 & 18.81 \\ \end{tabular}$		54.83	26.77

## **4 Conclusions**

In this study, the traffic flow simulation is performed in order to estimate the effect of the sag zone to the development of a traffic congestion.

When there exists a sag zone in the road, a traffic congestion occurs in the sag zone and the congestion develops to the upstream of the sag zone. The sag zone affects the traffic congestion more strongly according to the increase of cars. The curve of the car-head distance and the velocity is hysteresis loop. In case of low traffic density, the traffic flow is instable at the far upstream point from the sag zone and then, become stable to the point near the sag zone. However, in case of high traffic density, the traffic flow is instable during whole road.

In the future, we would like to estimate the effect of the sag zone to traffic flows on the cruising and the passing lanes of multi-lane road.

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