Compartment Lines Forming Emergent Alternative Configurations of Vehicles on Weaving Sections

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Abstract. Drawing Compartment lines on weaving sections is planed for easing the traffic congestion through forming emergent alternative configurations of vehicles before weaving. The degree of the alternative configurations was quantified in weaving sections partitioned into identical cells. A new stochastic Cellular Automaton (CA) model was ruled for the dynamics along the line with the interactions between vehicles neighboring each other. The longitudinal formation of the zigzag pattern along the line was obtained for the first time through simulations and the four cluster approximation. Furthermore, both results fit to each other.

1 Introduction

These days, traffic dynamics has been studied more and more diligently [1] [2]. In particular, scientists have been interested in interactions among vehicles during merging and weaving. H.Kita modeled merging interactions with game theory [3]. P.Hidas investigated vehicle interactions in merging and weaving by using agent based simulations [4]. L. C. Davis introduced the cooperation in merging by adding interactions between pairs of vehicles in opposite lanes [5]. He showed that velocity of vehicles in cooperative merging was higher than that in no cooperation.

However, these previous works did not propose concrete plans to realize communications among vehicles in merging and weaving. One of the studies regarding plans to communications is the study of internal-vehicle communication device (IVC). Y.Ikemoto *et al.* proposed a self-control method of cooperative behavior in junctions with IVC [6]. His method used a local communication with a close-by vehicle without infrastructures. But IVC technology has serious problems in holding communication networks when a vehicle in a network crosses vehicles in other networks.

The purpose of this paper is to propose a concrete plan easily applicable to real weaving sections to ease the traffic congestion through communications among vehicles. This plan is drawing a compartment line forbidding changing lanes on

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the center of merging area of weaving sections. The line lets vehicles to communicate with the neighboring ones and arranges emergence of the alternative configurations of vehicles. These configurations lead to the zipper-like merging and lessen the disturbance caused by the lane-change.

This paper focuses on the emergent alternative configurations along the line before lane-changes by Cellular Automaton (CA). We made a new traffic CA model named Multi Lanes Stochastic Optimal Velocity (MLSOV) model. This is extended from Stochastic Optimal Velocity (SOV) model [7] with the interactions between a vehicle and the neighboring one. We quantified the degree of the zigzag pattern named *Geminity* (*Ge*) and obtained for the first time the longitudinal increase of *Ge* along the line by simulation and the four cluster approximation.

This paper is organized as follows. In Sec. II we explain our plan of the compartment line in detail. In Sec. III we make models of weaving sections along the line for simulations and the four cluster approximation. Sec. IV is the results of the simulations and the four cluster approximation and Sec. V is the conclusive discussion.

2 Compartment Lines on Weaving Sections

Weaving Sections are composed of two roads connected with each other as figure 1 (a). They have a merging area and a bifurcation area. They are used for *Kosuge Junction* of *Metropolitan Expressway Co., Ltd.* in Japan and other roads. Vehicles intending to move from one road to the other need to change lanes across the center line of them. Today, there is heavy traffic congestion in them. For instance, the average speed is less than 20 km/h at the upstream area of *Kosuge Junction* from 10 am to 12 am on weekdays in November, 2005 while it is more than 30 km/h at the downstream area. This congestion is caused by lane-changes of vehicles across the center line of it. Vehicles changing lanes disturb the movement of the following vehicles.

We propose a concrete plan to solve this problem. Our plan is drawing a compartment line forbidding lane-change between the center two lanes as figure 1 (b). The key factor of this plan is the emergent zigzag patterns of vehicles along the line. Vehicles on a lane and vehicles on the opposite lane see each other along the line. And they move away from each other to change lanes smoothly. This moving away of each vehicle accumulates and forms a zigzag pattern of vehicles. Vehicles in a zigzag pattern do not disturb following vehicles during changing lanes.

3 Modeling for Simulations and the Four Cluster Approximation

We prepared for simulations and the four cluster approximation to analyze the emergent alternative configurations along the compartment line as follows. (1) Modeling the center two lanes of weaving sections along a compartment line with CA. (2) Defining MLSOV model as the dynamics of vehicles along the line.

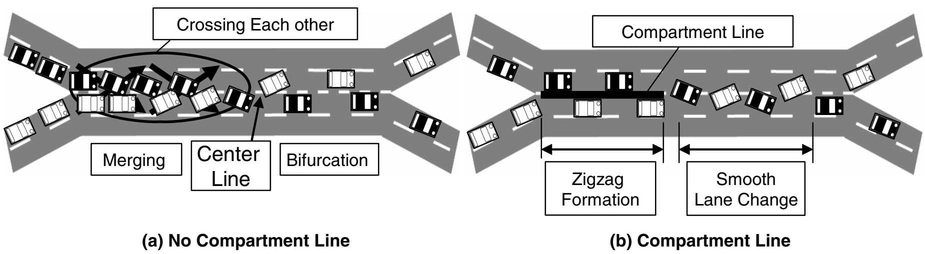

Fig. 1. (a) A weaving section without the compartment line. Vehicles disturb the movement of other vehicles by changing lanes across the center line. (b) A weaving section with the compartment line. The emergent zigzag pattern along the line leads to the smooth zipper-like lane-change.

(3) Defining the four cluster approximation. (4) Quantifying the degree of the alternative configurations in CA.

3.1 A CA Model of Weaving Sections along a Compartment Line

The center two lanes of weaving sections along a compartment line are partitioned into identical cells as figure 2. The cell size is fixed and each vehicle occupies a cell. The boundary condition of it is open and each vehicle is updated in parallel. Each Vehicle flows in the leftmost cell on lane 1 or lane 2, moves straight ahead along the line and flow out of the rightmost cell.

We set parameters of this model as figure 2. The length of lane 1, lane 2 and the line is d. The space coordinate x is set as figure 2. The leftmost cells are at $x = 0$ and the rightmost cells are at $x = d - 1$. Each vehicle flows in $x = 0$ side by side with the neighboring vehicle with the probability α as long as both cells at $x = 0$ are empty. This artificial flow-in condition helps to evaluate clearly the forming zigzag pattern of vehicles at $x \geq 0$. The probability of flowing out from the rightmost cell of lane i is β_i $(i = 1, 2)$.

3.2 MLSOV Model as the Dynamics along a Compartment Line

We made a multi-lanes stochastic CA model with interactions of vehicles along the line. We named this model as Multi Lanes Stochastic Optimal Velocity (MLSOV) model. It is extended from Stochastic Optimal Velocity (SOV) model [7] which is a kind of single-lane stochastic models. We chose to extend SOV model because the fundamental diagrams of SOV model have the meta-stationary state seen in real traffic data. In both SOV model and MLSOV model, i-th vehicle at time t moves straight forward one cell in one time step with probability v_i^t as long as the next cell is empty. This movement is described as

$$x_i^{t+1} = \begin{cases} x_i^t + 1, & \text{with probability } v_i^t \\ x_i^t, & \text{with probability } 1 - v_i^t, \end{cases} \tag{1}$$

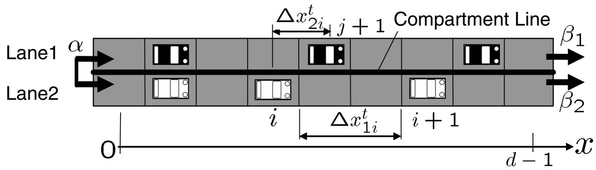

Fig. 2. CA model of a weaving section along the compartment line. Each vehicle flows in the cell on lane i ($i = 1, 2$) at $x = 0$ side by side with the neighboring vehicle with the probability α, moves straight ahead and flow out of the cell at $x = d-1$ with the probability β_i. $i+1$-th vehicle is the closest vehicle Δx^t_{1i} cells ahead of i-th vehicle on the same lane. $j+1$-th vehicle is the closest vehicle Δx^t_{2i} cells ahead on the neighboring lane.

where x^t_i is the position of i-th vehicle at time t as figure 2. $i+1$-th vehicle is the closest vehicle Δx^t_{1i} cells ahead of i-th vehicle on the same lane. v^t_i is called intension and normalized as $0 \le v^t_i \le 1$.

In MLSOV model, the time evolution of v^t_i is determined by not only $i+1$-th vehicle but also $j+1$-th vehicle which is the closest vehicle Δx^t_{2i} cells ahead on the neighboring lane as figure 2. v^t_i is given as

$$v^{t+1}_i - v^t_i = a \left\{ V(\Delta x^t_{1i}, \Delta x^t_{2i}) - v^t_i \right\}, \tag{2}$$

where $V(\Delta x^t_{1i}, \Delta x^t_{2i})$ is the two-lanes Optimal Velocity (OV) function [8]. v^t_i approaches $V(\Delta x^t_{1i}, \Delta x^t_{2i})$ with the response parameter a ($0 \le a \le 1$). Now, we set a simple OV function as

$$V(\Delta x^t_{1i}, \Delta x^t_{2i}) = \begin{cases} 0, & \Delta x^t_{1i} = 0 \\ r, & \Delta x^t_{1i} \ge 1 \text{ and } \Delta x^t_{2i} = 0 \\ q, & \Delta x^t_{1i} \ge 1 \text{ and } \Delta x^t_{2i} = 1 \\ p, & \Delta x^t_{1i} \ge 1 \text{ and } \Delta x^t_{2i} \ge 2. \end{cases} \tag{3}$$

This V set each vehicle see only one cell ahead. We pick up typical two cases of the time evolution of v^t_i with the initial condition of $v^0_i = p$; (a) $a = 0$ and (b) $a = 1$. In case (a), MLSOV model corresponds to the single-lane Asymmetric Simple Exclusion Process (ASEP). Each vehicle moves irrespective of neighboring ones and v^t_i is given as

$$v^t_i = \begin{cases} 0, & \Delta x^t_{1i} = 0 \\ p, & \Delta x^t_{1i} \ge 1. \end{cases} \tag{4}$$

In case (b), MLSOV model corresponds to the two-lanes Zero Range Process (ZRP) and v^t_i is given as

$$v^t_i = \begin{cases} 0, & \Delta x^t_{1i} = 0 \\ r, & \Delta x^t_{1i} \ge 1 \text{ and } \Delta x^t_{2i} = 0 \\ q, & \Delta x^t_{1i} \ge 1 \text{ and } \Delta x^t_{2i} = 1 \\ p, & \Delta x^t_{1i} \ge 1 \text{ and } \Delta x^t_{2i} \ge 2. \end{cases} \tag{5}$$

3.3 The Four Cluster Approximation

We prepared for analyzing the zigzag patterns along the line with MLSOV model through not only simulations but also the four cluster approximation. There are 10 kinds of state numbered $S(n)$ $(n = 1, 2, ...10)$ in the four cells at $x = \{k, k+1\}$ $(i \geq 0)$ as figure 3. The symmetry between lane 1 and lane 2 is considered. We defined $\Pi(n)_{kt}$ $(n = 1, 2, ...10)$ as the probability of the state $S(n)$ at time t. $\Pi(n)_{kt}$ is normalized as $\sum_1^{10} \Pi(n)_{kt} = 1$. The time evolution of $\Pi(n)_{k_t}$ is described as

$$\Pi_{kt+1} = P_k \Pi_{kt}, \tag{6}$$

where $\Pi_{kt} = \{\Pi(1)_{kt}, \Pi(2)_{kt}, \ldots, \Pi(10)_{kt}\}$ and P_k is the state transition matrix at $x = \{k, k+1\}$. $\Pi_{k\infty}$ is given as the solution of $\Pi_{k\infty} = P_k \Pi_{k\infty}$ with the normalized condition of $\sum_1^{10} \Pi(n)_{k\infty} = 1$.

The hopping probability of MLSOV model is updated in the positive x direction in making P_k. We assume the flow of vehicles along the line to be free flow and replace the time update of MLSOV model with the spatial update. We defined v_i^k as the hopping probability of vehicle i on the four cells at $x = (k, k+1)$. v_i^k is given as

$$\begin{aligned} v_i^k &= (1-a)\tilde{v}^k + aV(\Delta x_{1i}, \Delta x_{2i}) \\ &= \begin{cases} (1-a)\tilde{v}^k, & \Delta x_{1i} = 0 \\ (1-a)\tilde{v}^k + ar, & \Delta x_{1i} \geq 1 \text{ and } \Delta x_{2i} = 0 \\ (1-a)\tilde{v}^k + aq, & \Delta x_{1i} \geq 1 \text{ and } \Delta x_{2i} = 1 \\ (1-a)\tilde{v}^k + ap, & \Delta x_{1i} \geq 1 \text{ and } \Delta x_{2i} \geq 2, \end{cases} \end{aligned} \tag{7}$$

where $\tilde{v}^k$ is the intension common to the four cells at $x = (k, k+1)$. $\tilde{v}^k$ is updated in the positive x direction as

$$\tilde{v}^{k+1} = (1-a)\tilde{v}^k + a\bar{V}^k, \tag{8}$$

where $\bar{V}^k$ is the mean OV function at $x = k$ determined by the stationary state $\Pi_{k\infty}$. $\bar{V}^k$ is given as

$$\bar{V}^k = \frac{p\Pi(3)_{k\infty} + q\Pi(6)_{k\infty} + 2r\Pi(7)_{k\infty} + r\Pi(9)_{k\infty}}{\Pi(3)_{k\infty} + \Pi(6)_{k\infty} + 2\Pi(7)_{k\infty} + \Pi(9)_{k\infty}}. \tag{9}$$

P_k is determined by the left boundary condition at $x = \{k-1, k\}$ and the right boundary condition at $x = \{k+1, k+2\}$. The left boundary condition in determining P_k is given strictly. The left boundary condition in determining P_0 is given as a pair of vehicles flowing in both cells at $x = 0$ with the initial hopping probability $\tilde{v}^0$. They flow in with the probability α as long as both cells are empty. The left boundary condition in determining P_k $(k \geq 1)$ is the stationary state at $x = (k-1, k)$. This state is given strictly by $\Pi_{k-1\infty}$. However, the right boundary condition in determining P_k needs approximation. The right boundary condition in determining P_0 is approximated as a pair of vehicles existing on both cells at $x = 2$ with the probability $\alpha/(1+\alpha)$. The right boundary condition in determining P_k $(k \geq 1)$ is the stationary state at $x = (k+1, k+2)$. This state is not yet calculated and is approximated by $\Pi_{k-1\infty}$ which is the stationary state at $x = (k-1, k)$.

3.4 Quantifying the Degree of the Alternative Configurations in CA

We quantified the degree of the alternative configurations of vehicles named *Geminity* (Ge). Ge is a function of x and $Ge(k)$ denotes the degree of the zigzag pattern of vehicles at $x = k$.

In simulations, $Ge(k)$ is calculated by counting the states of the four cells at $x = \{k, k+1\}$ through M times of simulations with time steps of $0 \leq t \leq T$. We defined $c(n)_i$ $(n = 1, 2, ...10)$ as the total counted times of $S(n)$ at $x = \{k, k+1\}$. When there is at least one vehicle at $x = k$, the state of the four cells at $x = \{k, k+1\}$ can be $S(n)$ $(n = 3, 5, 6, 7, 8, 9, 10)$. Only $S(3)$ among them represents the perfect alternative state of vehicles obeying (1)-(3). Thus, $Ge(k)$ is given by $c(n)_k$ as

$$Ge(k) = c(3)_k/\big(c(3)_k + c(5)_k + c(6)_k + c(7)_k + c(8)_k + c(9)_k + c(10)_k\big). \quad (10)$$

In the four cluster approximations, $Ge(k)$ is also described as

$$\begin{aligned} Ge(k) = \Pi(3)_{k\infty}/(&\Pi(3)_{k\infty} + \Pi(5)_{k\infty} + \Pi(6)_{k\infty} \\ &+ \Pi(7)_{k\infty} + \Pi(8)_{k\infty} + \Pi(9)_{k\infty} + \Pi(10)_{k\infty}). \end{aligned} \quad (11)$$

Ge ranges from 0 to 1. The large value of $Ge(k)$ denotes that the zigzag pattern of vehicles at $x = k$ is highly achieved.

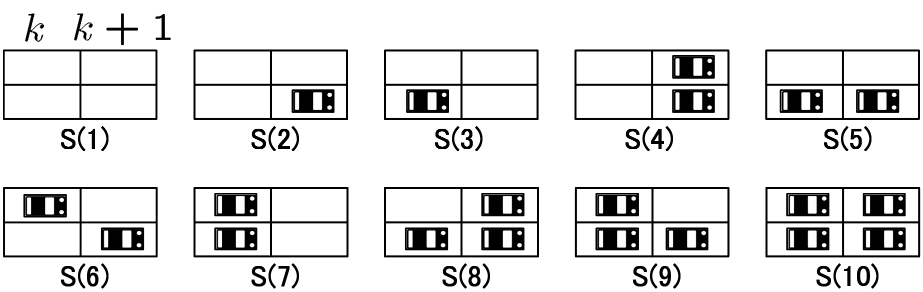

Fig. 3. 10 kinds of the state in the four cells at $x = \{k, k+1\}$. The symmetry between lane 1 and lane 2 is considered.

4 The Results of the Simulations and the Four Cluster Approximation

We measured the relationships between $Ge(x)$ and x with simulations of MLSOV model and with the four cluster approximation.

The conditions common to the simulations and the four cluster approximation are shown as follows. The length of the road is given as $d = 100$. The parameters of MLSOV model are given as $p = 0.999$, $q = 0.8$ and $r = 0.8$. Five kinds of the response parameter a are given as $a = \{0, 0.001, 0.01, 0.1, 1\}$. The probability of flowing in the cells at $x = 0$ is given as $\alpha = 0.05$. Vehicles flow in with the initial intension p. The conditions of the simulation are shown as follows. β_i $(i = 1, 2)$ is given as the v_i^t of each vehicle on the cell at $x = d - 1$. The times of simulations in the same condition is given as $M = 10$. The total time of a simulation is given as $T = 100000$ steps.

The results of the simulations and the four cluster approximation are shown as figure (4). $Ge(x)$ increased monotonically with x in all cases of simulation and the four cluster approximation. This relationships between $Ge(x)$ and x showed for the first time the achievement of zigzag pattern toward the space axis.

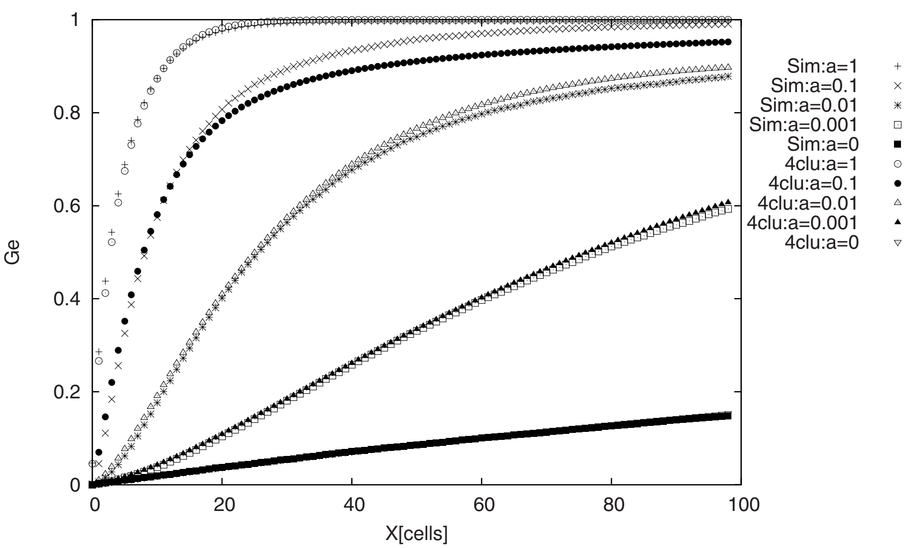

Fig. 4. $Ge(x)$ *vs.* x of the simulations and the four cluster approximation. The length of the road is given as $d = 100$. The probability of flowing in the cells at $x = 0$ is given as $\alpha = 0.05$. Vehicles flow in with the initial intension p. The parameters of MLSOV model are given as $p = 0.999$, $q = 0.8$ and $r = 0.8$. a is given as $a = \{0, 0.001, 0.01, 0.1, 1\}$. In simulations, β_i $(i = 1, 2)$ is given as the v_i^t of each vehicle at $x = d - 1$. The times of simulations in the same condition is given as $M = 10$. The total time of a simulation is given as $T = 100000$ steps.

The sharpness of the increase of $Ge(x)$ was positively correlated with a. a is the strength of the response of each vehicle toward the forward vehicle on the opposite lane. This positive correlation suggests that the more strong the response of each vehicle toward the forward vehicle on the opposite lane, the more strong the achievement of the alternative configurations becomes.

The increase of $Ge(x)$ in the four cluster approximation fit to the increase of $Ge(x)$ in the simulation. This correspondence suggests that the four cluster approximation is a good theoretical approximation to evaluate the forming of zigzag pattern toward the space axis.

5 Conclusive Discussions

We proposed a concrete plan to relieve the traffic congestion on weaving sections by drawing a compartment line. The key point of our plan is the emergence of alternative configurations of vehicles. We focused on the alternative configurations of vehicles along the line before lane-change. We make a CA model of a weaving section along the line and analyzed these configurations with MLSOV model through simulations and the four cluster approximation. We quantified the degree of the zigzag pattern at a cell named *Geminity* (Ge). We obtained for the first time the achievement of alternative configurations toward the space axis through the simulations and the four cluster approximation. And it is found that the sharpness of the increase of $Ge(x)$ became strong with the increase of a. The results of the four cluster approximation approximated closely to the results of simulation of MLSOV model.

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