

A CA Model of Spontaneous Formation of Concentration Gradients

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Abstract. It is shown that a two-compartment isolated fluid system, where a chemical reaction takes place close to the surfaces of the semipermeable separating membrane, can spontaneously develop a transient concentration difference across the membrane. If the system is open to the flow of chemicals, the difference can persist in the steady state. This allows concentrating chemicals in a single compartment, which may be useful for chemical engineering purposes, and which is particularly interesting in the study of the dynamics of vesicles and protocells. The phenomenon is investigated and demonstrated here with a CA model: it is also shown that, in the limiting case of infinitely fast diffusion, the results are coherent with those of a homogeneous model.

Keywords: self-organization, entropy, dissipative structure, cellular automaton.

1 Introduction

It is generally believed that the approach to thermodynamic equilibrium in isolated systems leads to a monotonic decrease of the inhomogeneities which may be present in the initial conditions of a system, at least when gravitational effects can be neglected. However, it has recently been shown, both by theoretical analysis and experimental test, that a transient onset or increase of concentration differences can be achieved in a two-compartment system when the pores of the separating membrane are asymmetric [1].

We will describe below a different kind of two-compartment isolated system where concentration differences may transiently increase in time without violating the second law of thermodynamics. Moreover, when this system is allowed to exchange matter with the environment, some chemicals are concentrated in the smaller compartment. The effect is related to the presence of chemical reactions which can take place at a significant rate only in a region very close to the membrane surface [2] and to the different permeabilities of the various chemicals involved, as it will be described below.

Applications of this effect to chemical reactors are quite straightforward, but we will briefly comment here on a possible non obvious application to the problem

of the origin of life and to the development of a new technology. Vesicles in water are composed by a lipid bilayer which surrounds an aqueous interior: they display interesting physical phenomena [3,4] and, since they can spontaneously fission, they are believed to be important for the study of the origin of life as well as for possible applications of protocells [3,5,6,7].

It would be important to be able to concentrate chemicals in vesicles with respect to the external phase: indeed, the presence of a concentration difference across the membrane corresponds to a high energy state, which may be coupled to other chemical reactions (like it happens e.g. in photosynthesis [8]). If the concentration of some chemicals is higher within the vesicle, then one may achieve very effective chemical processing (synthesis, degradation, removal). Moreover, a more speculative possibility is that the high inner concentration of chemicals might allow the formation of large autocatalytic reaction sets. There are indeed theoretical arguments in favour of the spontaneous development of such cycles [9], but they are very difficult to achieve experimentally: a possible explanation for this difference between theory and experiments is that the concentrations of some key components in the bulk are too low, but this problem might be mitigated in a vesicle using the approach introduced in this work.

In order to describe the phenomenon of interest, let us consider a system composed by two compartments with different volumes separated by a semipermeable membrane (Fig. 1). There are chemicals on both sides, and the permeabilities of the membrane to the various chemicals may be very different. We will suppose that the volumes of the two compartments do not change, and that chemical reactions may take place in both compartments, in a region very close to the membrane surface: for example, the reactions may be catalyzed by some molecules which are bound to the membrane [2]. In section 2 we present a specific CA model of a simple system of this kind, which will be used to investigate its possible behaviours. However, as it will be apparent from the analysis, the phenomenon described below is robust and largely independent from the details of the model. The main results of the analysis are the following.

Let us consider first a closed system and an initial state with equal densities of each chemical on both sides of the membrane. If the densities are exactly those corresponding to the values at chemical equilibrium, they of course remain unchanged. But if they differ from those values (while being equal for each chemical on both sides) and if the volumes of the two compartments are different, then an interesting phenomenon takes place, i.e. a transient density difference is established across the membrane. According to the second law, this difference eventually vanishes but, depending upon the values of the parameters, it may last for a relatively long time.

Moreover, if the larger compartment is not isolated, but there is a mass flow, then the density difference does not vanish but it persists in the steady state. The density of non-permeating chemicals is higher within the smaller compartment, therefore providing a mean to steadily concentrate them, which, as observed above, may lead to relevant applications.

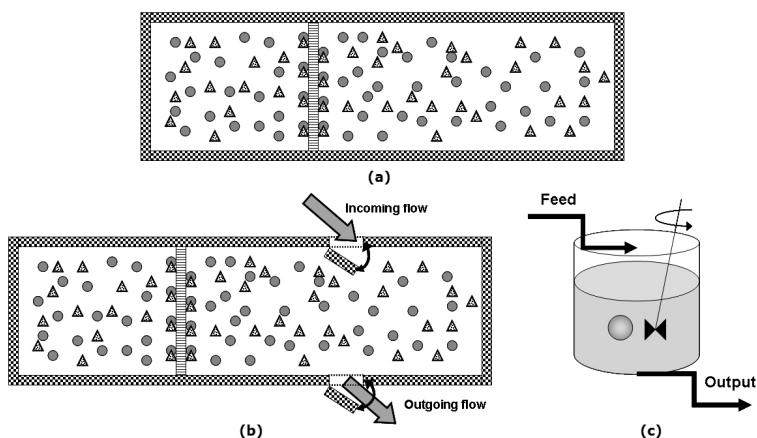


Fig. 1. (a) A two-compartment isolated system, with two different chemicals interacting near the semipermeable membrane; (b) The case of an open system, where one of the chemicals enters the reactor and both can leave it; (c) A particularly interesting two-compartment open system: a vesicle inside a continuously stirred tank reactor

In section 2 we introduce the model; in section 3 we present a simplified theoretical treatment of the system, and in the final section 4 we comment on the generality and relevance of the phenomenon.

2 The Model and Its Behaviour

In order to study the system described in Fig. 1 let us introduce a 2 dimensional CA model, with a square grid and von Neumann neighbourhood (Fig. 2). The model belongs to the class of macroscopic CA [10] so the state of each cell in each of the two compartments is defined by a vector of real numbers, i.e. the concentrations of the various chemical species. This class of models should not be confused with the more common lattice gases, and it has already proven able to deal effectively with several real world phenomena, including soil bioremediation, landslides and lava flows (for a discussion, see [10]). As it is often the case in macroscopic CAs, also this model is not homogeneous, in that there are some special cells which model the boundaries and the membrane; in the case of an open system, “source” and “sink” cells are used to model drive the flow. The updating is synchronous for all the cells.

Referring to Fig. 2a, the diffusion of chemical C_j among neighbouring cells is modelled as a flow ruled by Fick’s law, resulting in the transition rule $\Delta C_0^j = \sum_{i=1}^4 w_i^j (C_0^j - C_i^j)$, where the weight w_i^j is proportional to the transport coefficient between the two interested cells. In such a way the presence of a membrane among two cells (whose permeability can be different for different chemicals) can be modelled by using lower transport coefficients.

For the sake of simplicity, we consider a simple unimolecular reaction $A \longleftrightarrow X$ which takes place only in the cells which are neighbours of the separating

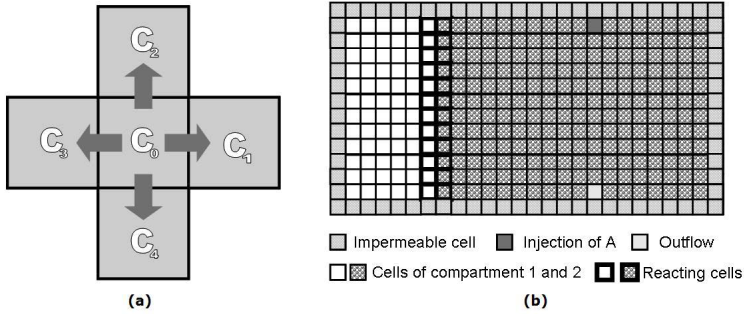


Fig. 2. Schematic representation of the CA model of the two-compartment system

membrane (on both sides, represented with thick edges in Fig. 2b). The reaction is supposed to be reversible, the equilibrium being shifted in the region which favours the formation of X, which will therefore sometimes be called the product (and A the reactant). The corresponding transition rules are therefore $\Delta A_0 = -p_f A_0 + p_r X_0$ and $\Delta X_0 = p_f A_0 - p_r X_0$, where p_f and p_r are proportional to the reaction kinetic constants. Other more complicated reaction schemes have been investigated, leading however to similar phenomena, so we will limit here to describe the simplest case. The system boundaries are modelled as not

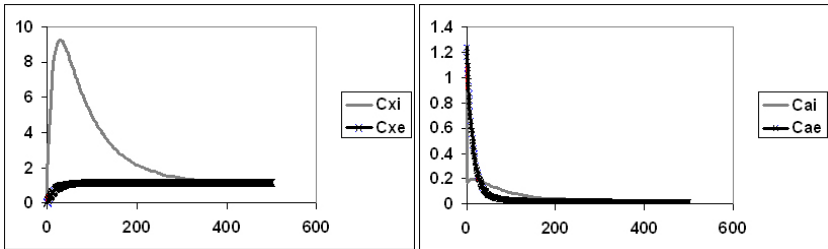


Fig. 3. CA model of an isolated system. Average concentration vs. time of X (left) and A (right) in the smaller (thin line) and larger (thick line) compartment.

permeable walls ($w_i^j = 0$). When A reaches the semipermeable membrane which divides the two compartments, it can cross it while X cannot cross the membrane.

We consider two cases, namely that of a closed system and that of an open system. In this latter case, a source and sink cell are introduced: in the source cell the value of the concentration of reactant A is kept fixed (simulating inflow), while in the sink the value of both A and X are instantaneously set to 0 (simulating outflow).

The behaviour of the isolated system is simulated starting from a condition where the concentrations of both A and X are equal on both sides. If they are equal to each other, but different from their equilibrium values, a transient concentration gradient sets in (see Fig. 3). Of course, in agreement with the

second law, the difference eventually dies out, but this may require a long lasting transient. On the other hand, if the system is open the concentration difference can be steady, as shown in Fig. 4.

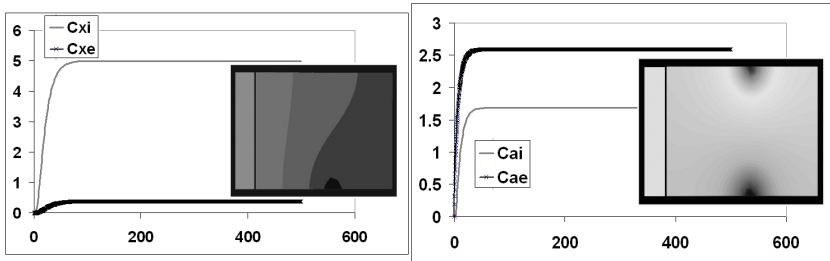


Fig. 4. CA model of an open system. Average concentration vs. time of X (left) and A (right) in the smaller (thin line) and larger (thick line) compartment. Also the instantaneous values of the concentration are shown.

3 The Limiting Case of Two Homogeneous Compartments

In order to interpret the results of the above simulations we will compare them with those of a simpler model of the same process, where we suppose that the concentrations are homogeneous in each compartment (this corresponds to assume that diffusion in the bulk is very fast).

Let us suppose that a chemical reaction takes place on both sides of the membrane surface, both inside and outside: therefore, all the reactions take place in a small “effective volume” $V_r \approx S\delta$ near the surface (S being its area). The direct and reverse kinetic constants are such that formation of X is favoured. Reactant A can cross the membrane, φ being its flow rate and D its diffusion coefficient across the membrane, while X cannot (it would be easy to introduce a small diffusion coefficient of X as well). In order to ease the exposition we will often refer to the smaller and larger volumes as the internal and external one respectively (this is reminiscent of the vesicle case but, at an abstract level, “internal” just means “smaller”).

Let ρ and Q denote respectively the densities (moles/litre) and the quantities (moles) of A and X, using a superscript to indicate the chemical species and a subscript the compartment (“i” denoting the smaller one, with volume V' , and “e” the larger one, with volume V), so for example Q_i^A is the number of moles of A in the smaller compartment. Diffusion time both in V and V' is supposed to be negligible, so the densities simply equal the ratios of the quantities to the corresponding volumes.

As in section 2, we may let an inflow of A (not of X) in the external compartment: if F denotes a constant volume flow, the quantity of A which enters the external compartment is $F\rho_e^A$, while the quantities of A and X which leave it are respectively $F\rho_e^A$ and $F\rho_e^X$. If the system is closed of course $F=0$.

Using the above notations, the flow of A across the separating membrane is:

$$\varphi = DS(\rho_i^A - \rho_e^A) \tag{1}$$

where D is the diffusion coefficient of chemical A across the membrane.

Assuming that the kinetics of the reaction is first order, by imposing mass conservation one obtains the following rate equations for the internal and external quantities of A and X:

$$\begin{cases} \frac{dQ_e^A}{dt} = -(kV_r + F)\rho_e^A + k'V_r\rho_e^X + \varphi + F\rho_{ext}^A \\ \frac{dQ_e^X}{dt} = kV_r\rho_e^A - (k'V_r + F)\rho_e^X \\ \frac{dQ_i^A}{dt} = -kV_r\rho_i^A + k'V_r\rho_i^X - \varphi \\ \frac{dQ_i^X}{dt} = kV_r\rho_i^A - k'V_r\rho_i^X \end{cases} \tag{2}$$

In the case of a closed system (F=0) one can compute the equilibrium values and, if the initial conditions coincide with those values, no change is of course observed. Suppose now that the initial densities of A and X are the same in the two compartments but that they do not coincide with their equilibrium values.

In this case one observes (Fig.5) an increase in the inner density of X which may be, depending upon the parameters, very high, and which may last long (although, according to the second law of thermodynamics, it asymptotically vanishes). Note that this behaviour is similar to the one shown in Fig.3 of section 2.

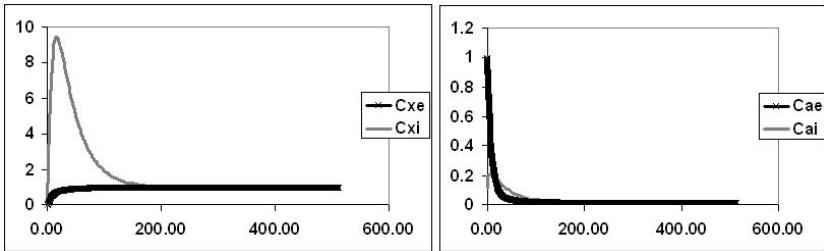


Fig. 5. Numerical integration of equation (2) (isolated system); average concentration of X (left) and A (right) in the smaller (thin line) and larger (thick line) compartment

Simulations show that the value of the ratio $\frac{\rho_i^X}{\rho_e^X}$ at the peak of ρ_i^X is inversely proportional to V'/V : if the “small” compartment were a vesicle, this would allow a possible independent estimate of its volume. If we assume spherical vesicles also the surface S is thus determined. It has also been observed in simulations that the duration of the transient (defined e.g. as the time elapsed from the beginning of the simulation till the moment when the two curves showing the values of ρ_i^X and ρ_e^X vs. time embrace 95% of the total area between them) depends upon the value of DS. This remark, combined with the previous one, provides a way to estimate the value of the diffusion coefficient D.

Let us now consider the case of a non isolated system, where $F \neq 0$. In this case it can be analytically proven that at steady state the internal concentration of X is larger than the external one, and in particular that:

$$\bar{\rho}_e^X = \frac{\bar{\rho}_i^A = \bar{\rho}_e^A}{(k'V_r + F)} \bar{\rho}_e^A < \bar{\rho}_i^X = \frac{k}{k'} \bar{\rho}_e^A \quad (3)$$

where a bar denotes asymptotic (steady-state) values. This estimate is in agreement with the results of the CA simulation.

4 Conclusions

The fact that a membrane is permeable to one chemical but not to the other might be due to different physical or chemical properties. Since it might be questionable to consider only a unimolecular reaction, we have also analyzed (in the limiting case of fast diffusion) the case of a bimolecular reaction $A + A \longleftrightarrow X$ [11]. The behaviours observed in this case are closely similar to those of the previous one, described in Sections 2 and 3.

We have also analyzed in a similar way the case of a reaction $A + B \longleftrightarrow X + Y$, considering different hypotheses about the permeability of reactants and products, again confirming the concentration effects observed in the simpler model. It is worth to note that the concentration effect at steady state ($F \neq 0$) of non-permeable products may be indeed very large.

A perhaps surprising remark is that the model described here is formally linear: therefore the onset of a concentration difference cannot be considered a nonlinear effect (although it is observed also when nonlinear kinetic equations are considered). One might also observe that the inhomogeneity of the system provides itself a kind of nonlinearity, even if this is not apparent from the transition functions of section 2 nor from the equations of section 3.

From a methodological viewpoint it is worth emphasizing that the comparison of the results of the CA model with the simpler mathematical model of section 3 provides interesting cues to the interpretation of the results of the former, as well as indications concerning the robustness of the major results with respect to changes in the modelling level and approach.

While the phenomenon has been discussed here in a model, it is likely to be real and experimentally verifiable. Indeed, the mechanism which leads to the breaking of the initial equality of internal and external concentrations is that, since reactions take place on the vesicle surface, the same quantities of chemicals per unit time react on both sides of the membrane. But the internal and external volumes are different, and therefore the internal and external concentrations become different. This is essentially independent from the details of the specific kinetic model used.

In a multiphase system, increased concentration of a chemical in a particular phase according to its partition coefficient is of course a well-known phenomenon, but here it arises between two phases with the same physico-chemical properties. It should also be noticed that it is not due to active transport. Therefore the

appearance of concentrations gradients seems surprising, although, as we have seen, in a closed system it is a transient phenomenon which does not violate the second law. In these systems concentration gradients may be coupled to chemical reactions and therefore lead to (transient) interesting effects, while in a flow reactor the gradients are stable and therefore they can provide an energy source driving chemical reactions which might have been otherwise impossible.

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