

Occurrence of Gliders in an Infinite Class of Life-Like Cellular Automata

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Abstract. The Game of Life (GL), Larger Than Life (LtL), and the Kaleidoscope of Life (KL) are cellular automaton (CA) models with a rich palette of configurations, some of which facilitate universal computation. Common to all these models is that the transition rules by which they are governed are outer-totalistic. The KL distinguishes itself by the striking simplicity of its transition rule, which does not even take into account a cell's state itself for its update. This paper investigates an infinite class of CA, all of which are similar to KL except for their differently sized neighborhoods. Characterized by a discrete parameter d , a neighborhood in such a CA consists of the cells at Moore distances 1, 2, ..., or d of a cell. We show that signal-carrying configurations ("gliders") occur in infinitely many of these CA models. We also show that the probability of convergence of a random configuration toward a dead cellular space increases with the increase in parameter d . These seemingly contradictory results suggest that the presence of gliders are not necessarily a reliable benchmark for the sustainability of Life in cellular space.

1 Introduction

Life in CA refers to a class of outer-totalistic models able to sustain computational universality through autonomously behaving configurations in cellular space. Some of these configurations—usually called *gliders*—propagate in cellular space as part of a cyclical metamorphosis process and interact with other gliders or with other more or less stable configurations in well-defined and well-understood ways. The *Game of Life (GL)* [1,2] is at the root of a class of Life-like models, which has extended into the *Larger Than Life (LtL)* class of CA [3,4] and recently has also given birth to the *Kaleidoscope of Life (KL)* [5,3]. The KL is based on a transition rule that uses a very simple decision criterion on whether a cell will be alive in the next generation or not: if the number of living cells at Moore-distance 1 or 2 from a cell equals the number 4, the cell will be alive in

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the next generation. Remarkably, the state of the cell itself is irrelevant in this criterion. The independence of a cell's state from its previous state is shown in [5] to be closely connected to the properties of classical spin systems.

LtL covers a large class of models, part of which has been characterized in [4] by configurations called *bugs*. The KL, on the other hand, sports configurations more closely resembling those in the GL. While the dynamics of KL and GL are similar, the KL appears to be more “long-lived”: it takes longer to reach convergence to stable or periodic configurations, probably due to the larger neighborhood used in comparison with the GL. This invites the question of whether models can be constructed that have similar characteristics to the KL and differ only in the size of their cells' neighborhoods.

This paper explores an infinite class of such models, and shows that each member of this class contains gliders. Yet, in most of these models, Life is hard to sustain, since random initial configurations will rapidly converge to a dead cell space, as will be shown. This puts KL at the boundary of a class of CA, which are increasingly unlikely to contain configurations capable of computation as the size of the cell neighborhood increases.

This research may result in novel CA models that are characterized by an extremely simple transition rule, to the extent that physical implementations in terms of magnetic spin systems are within the realm of possibilities. Such implementations may be at the basis of nanocomputer architectures, which have attracted increasingly attention in recent years due to their promise to extend the life time of Moore's law for another couple of decades beyond the decade or so it is still expected to last.

This paper is organized as follows. In Section 2 we define the basic model, followed by some prominent cell configurations in the KL in Section 3. Section 4 describes variations of the KL that have larger neighborhood diameters, and shows the presence of gliders in such models. Simulation results on the convergence of some of these models as well as a probabilistic analysis of the models are shown in Section 5. The paper finishes with conclusions and a discussion.

2 An Infinite Class of Life-Like Cellular Automata

The model consists of a 2-dimensional square array of cells, each of which can be in either of the states, 0 (*dead*) and 1 (*alive*). We assume that each cell in the cell space is identified by a unique integer, and that $\sigma_i(t)$ is the state of cell i at time t and $N(i)$ is the neighborhood of cell i . This neighborhood consists of the cells at orthogonal or diagonal distances $1, 2, \dots, d$ from cell i (Moore neighborhood), giving a total of $4d^2 + 4d$ neighbors, with d denoting the radius of the neighborhood. Fig. 1 shows such a neighborhood for $d = 2$, which corresponds to the basic KL model in [5]. The transition rule of the model is defined in terms of the states of the cells in the neighborhood of cell i :

$$\sigma_i(t+1) = \begin{cases} 1 & \text{if } \sum_{j \in N(i)} \sigma_j(t) = k \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

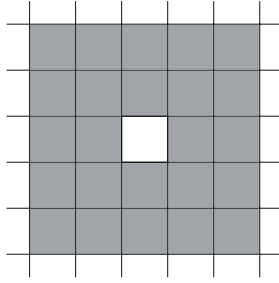


Fig. 1. Neighborhood of a cell (center) in the Kaleidoscope of Life (gray cells). The radius of the neighborhood is $d = 2$.

In other words, a cell becomes or stays alive if the number of living cells in its neighborhood is k ; otherwise the cell dies or remains dead. It is assumed that $k = 2d$. All cells in the cell space undergo transitions simultaneously. Transition rules in which a cell’s next state depends on the sum of the states in its neighborhood. are called *outer-totalistic*. If a cell’s next state does not depend on the state of the cell itself, the rule is called *inner-independent*. The rule proposed in this paper is thus inner-independent outer-totalistic. A member of the infinite class of these CA is denoted by $KL(d)$. The traditional Kaleidoscope of Life in [5] is $KL(2)$.

3 Useful Configurations in the Kaleidoscope of Life

The basic model of $KL(2)$ supports a wide variety of configurations, of which only a few are used for computation. Signals are encoded in this model by configurations (called gliders) that dynamically propagate along cellular space. Two types of gliders used in computation in $KL(2)$ are shown in Figs. 2 and 3, the first being only of peripheral use and the second type being the main mechanism to encode signals.

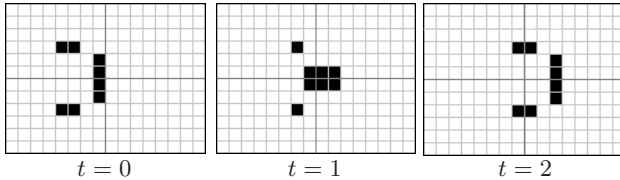


Fig. 2. A glider with period of 2 in $KL(2)$. This glider appears in some configurations as an intermediate form, but it is not actively used for computation.

The *Hanabi* (meaning ‘fireworks’ in Japanese) is a periodical configuration that finds wide use in $KL(2)$ to turn gliders to the left, to convert between the two types of gliders and to eliminate superfluous gliders (Fig. 4). It assumes a

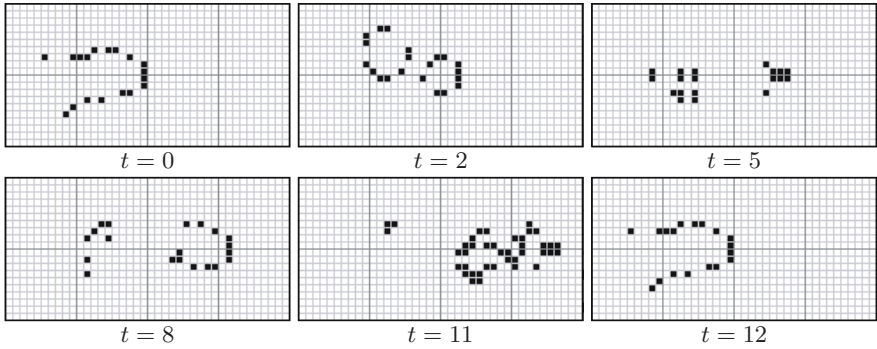


Fig. 3. A glider in $KL(2)$ with a period of 12. This glider is used to encode signals on the cellular space.

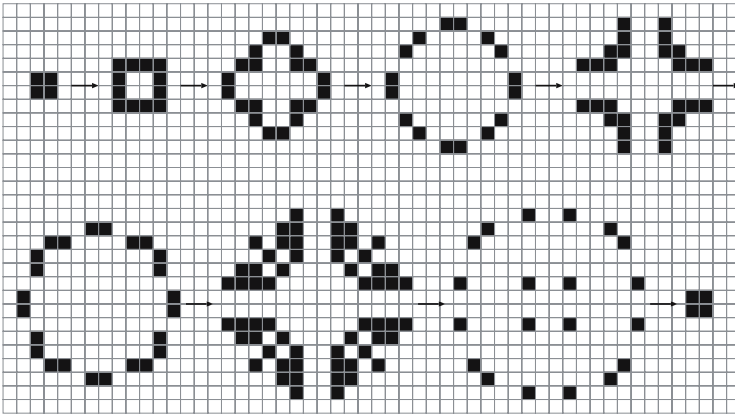


Fig. 4. The Hanabi pattern is used for a wide variety of tasks in $KL(2)$, such as turning gliders to the left or right, conversion between glider types, and the elimination of gliders. It has a period of 8.

period of eight generations, and this period as well as the phase of the period is left undisturbed in most cases when the Hanabi interacts with a glider.

A good impression can be obtained from the nature of a Life-like CA by starting it with a random configuration, and let it run for a few hundred generations. This will usually result in some of the standard configurations to emerge, such as gliders and the Hanabi in the case of $KL(2)$. Snapshots of the first 100 generations of $KL(2)$ initialized randomly are shown in Fig. 5.

4 Gliders in the Infinite Class of Cellular Automata

The abundance of patterns in the $KL(2)$ model invites the question whether the models in $KL(d)$ for different d can sustain similar patterns. This section

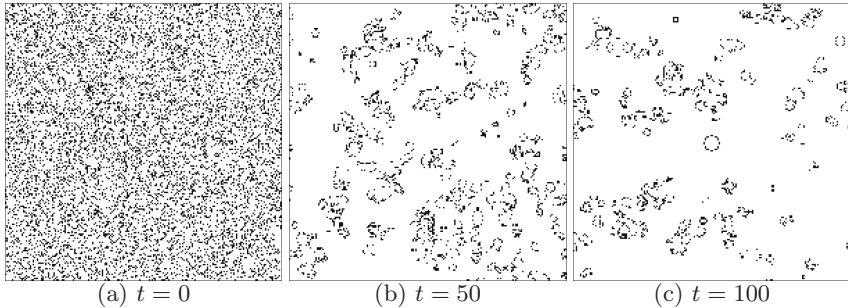


Fig. 5. Evolution from a random initial configuration in $KL(2)$ at times (a) $t = 0$, (b) $t = 50$, and (c) $t = 100$. The probability of a cell being alive in the initial configuration is 0.2.

gives a preliminary (positive) answer by showing that gliders are a very common phenomenon in $KL(d)$. In fact, we prove that infinitely many models in $KL(d)$ contain gliders. We focus on the model $KL(d)$ with the values $d = 4r$ for $r = 2, 3, 4, \dots$. It turns out that all these models contain gliders. Examples of gliders in the $KL(8)$ and $KL(16)$ models are given in Figs. 6 and 7. These gliders have period 2 and they have very similar shapes, at time $t = 0$ as well as at time $t = 1$.

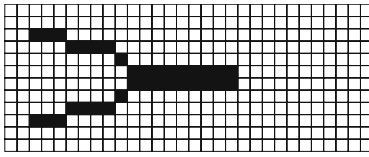
It turns out that this shape can be generalized for the case $d = 4r$ with $r = 2, 3, 4, \dots$ (Fig. 8). The basic principle is that the glider at $t = 0$ generates two vertical bars and six isolated living cells at $t = 1$. The bar at the front of the signal (right side of the signal) is just sufficient in size to generate a new head of the glider at the right of it at $t = 2$, but a similar structure will not be created at the left of the front bar because of the six isolated cells. Instead a tail is generated at $t = 2$ that slightly bifurcates toward the isolated cells. The vertical bar at the tail at $t = 1$ (left side of the signal) is sufficiently short to have no influence on successive generations, as it dies out. The result is a glider at $t = 2$ that has advanced by d cells in comparison with the glider at $t = 0$.

5 Probabilistic Analysis

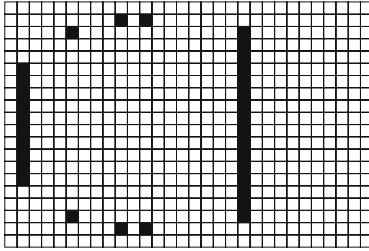
Since the presence of gliders in all models of the form $KL(4r)$ suggests the occurrence of Life, it makes sense to investigate the characteristics of these models in more detail. If we take a look at a simulation for the case $KL(1)$, we see that an initial random configuration results in configurations that appear just as random as the initial one, and that appear to have an increase in the density of living cells (Fig. 9). Indeed, this model fails to show convergence to a set of standard patterns, as simulations have shown.

The situation is quite different for the $KL(3)$ model. After less than 100 generations the cellular space is virtually dead in most cases (Fig. 10).

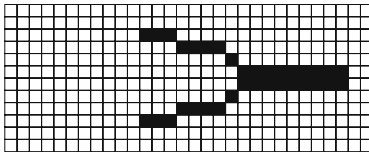
Simulations to measure the density of living cells in the cellular space show a very different behavior of $KL(d)$ in the cases $d = 1$, $d = 2$, and $d = 3$ (Fig. 11).



(a)

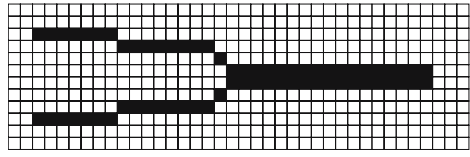


(b)

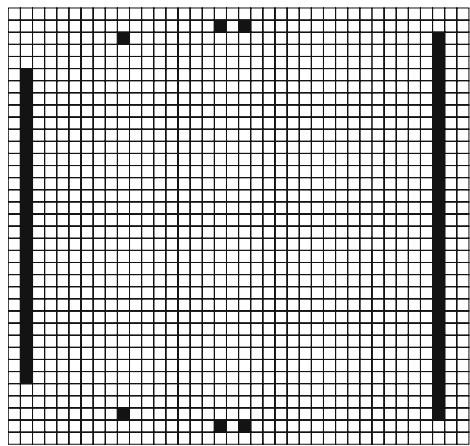


(c)

Fig. 6. Glider in $KL(8)$ with period 2. (a) $t = 0$, (b) $t = 1$, and (c) $t = 2$.

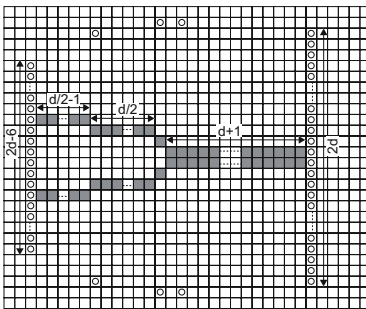


(a)

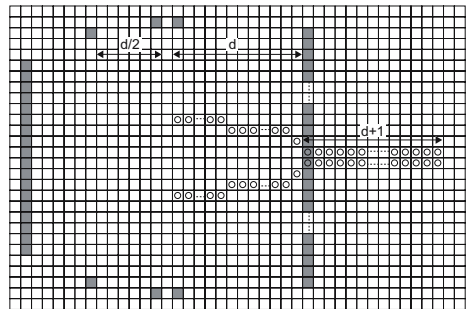


(b)

Fig. 7. Glider in $KL(16)$ with period 2. (a) $t = 0$ and (b) $t = 1$.



(a) $t = 0$



(b) $t = 1$

Fig. 8. Glider in $KL(d)$ with period 2 at (a) $t = 0$ and (b) $t = 1$. The gray cells denote the configuration at the indicated time t and the cells containing circles denote the configuration as it would appear at $t + 1$.

$KL(1)$ appears to have a density of living cells that stays at a high level over time, confirming the behavior observed in Fig. 9. $KL(2)$ appears to have a gradual decrease in density to an asymptotic positive value, which indicates a gradual

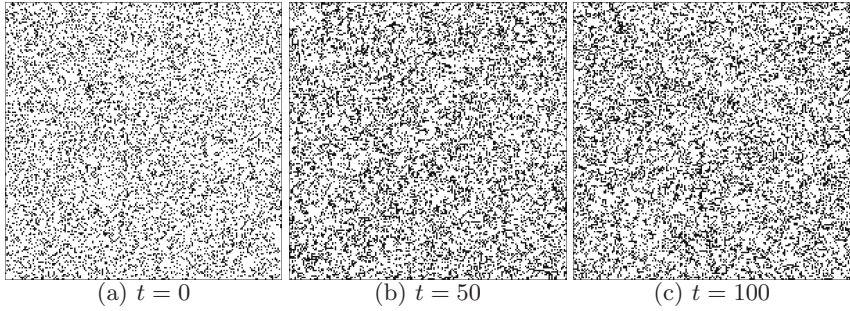


Fig. 9. Evolution from a random initial configuration in KL(1) at times (a) $t = 0$, (b) $t = 50$, and (c) $t = 100$. The probability of a cell being alive in the initial configuration is 0.2.

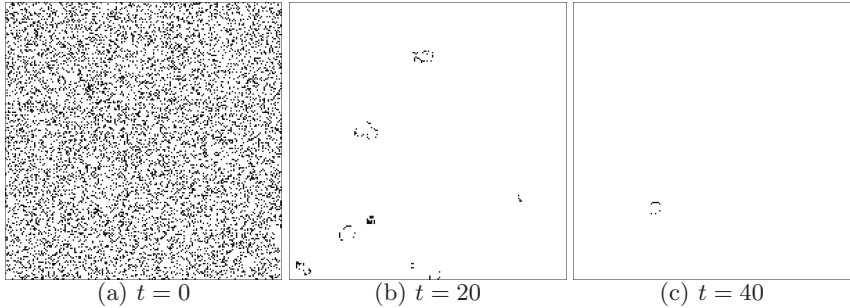


Fig. 10. Evolution from a random initial configuration in KL(3) at times (a) $t = 0$, (b) $t = 20$, and (c) $t = 40$. The probability of a cell being alive in the initial configuration is 0.2.

convergence to a certain set of patterns. KL(3) sees the density of living cells decrease rapidly to 0, which indicates infertile grounds to sustain Life.

How will the behavior of KL(d) be for higher values of d ? To obtain a rough estimate of this, we adopt the mean-field approach in [6] and calculate the probability that a cell is alive at time $t = x + 1$ given a certain probability of living cells at time $t = x$, which will be denoted by $p(x)$. We emphasize that this estimate has its limitations, since it assumes a random initial state of the cellular space as well as negligible correlations between neighboring cells. Especially the latter assumption fails to hold for CA in general, but the mean-field approximation still gives a useful first impression when comparing the CA models KL(d) for different values of d . The probability $p(x)$ can be expressed as:

$$p(x + 1) = f(p(x)),$$

with

$$f(p) = \binom{n}{m} p^m (1 - p)^{n-m},$$

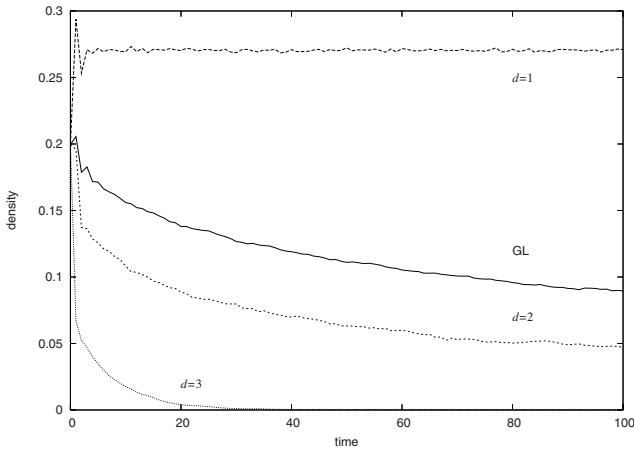


Fig. 11. Density of living cells in $KL(d)$ for $d = 1$, $d = 2$, and $d = 3$ over time. In the $KL(1)$ model the density of living cells remains at a quite high level, which suggests either Life-like behavior or a lack of convergence. For the $KL(2)$ model the density converges to a positive value, confirming the Life-like behavior of the model. In the $KL(3)$ model the density converges rapidly to zero, suggesting that few configurations survive over time. As reference, the density curve of the Game of Life (GL) is also given.

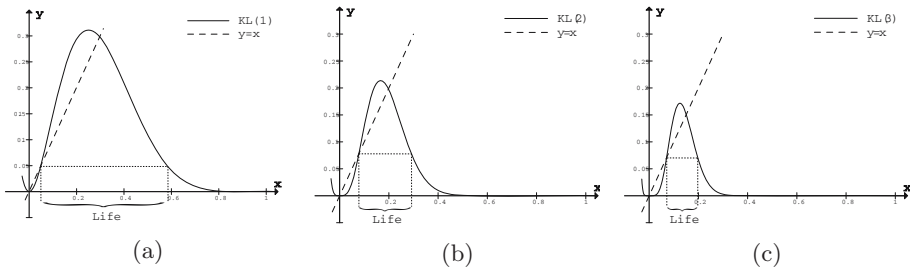


Fig. 12. Mean-field approximations of (a) $KL(1)$, (b) $KL(2)$, and (c) $KL(3)$. The horizontal axis denotes $p(x)$ and the vertical axis $p(x + 1)$. The area indicated by the text *Life* corresponds to the probability interval between which living cells are unlikely to die.

and $n = 4d^2 + 4d$ and $m = 2d$. Fig. 12 gives the graphs of $p(x + 1)$ (vertical) against $p(x)$ (horizontal) for the values $d = 1$, $d = 2$, and $d = 3$. The probability interval in which cells are unlikely to die—indicated by the phrase “Life”—is quite large for $d = 1$, smaller for $d = 2$, and much smaller for $d = 3$. This indicates that cells have a smaller probability of being alive for the higher values of d than for the smaller values. To show that this trend holds in general, we investigate function f in more detail. Its derivative is:

$$f'(p) = \binom{n}{m} p^{m-1} (1-p)^{n-m-1} (m-np)$$

This function has an $(m-1)$ -fold root in $p=0$, an $(n-m-1)$ -fold root in $p=1$, and a single root in $p=m/n=1/(2d+2)$. This implies that the local maximum of the function lies always between 0 and 1, and that this maximum moves to the left as d increases, and will eventually converge to 0 as d goes to infinity.

What is the length of the interval in which cells are likely to be alive? It is hard to derive an analytical solution of this length, so we give an approximation. The two points between 0 and 1 at which $f''(p)=0$ serve as a reasonable estimate for the start- and end-points of this interval, so to give an approximation of the interval's length we calculate the difference of their x -coordinates. It is easy to derive that

$$f''(p) = \binom{n}{m} p^{m-2} (1-p)^{n-m-2} ((n^2-n)p^2 - 2m(n-1)p + m^2 - m)$$

so the roots of this function are, apart from the ones at $p=0$ and $p=1$:

$$p_{12} = \frac{m(n-1) \pm \sqrt{m(n-1)(n-m)}}{n(n-1)}$$

It is easily verified that these roots lie between 0 and 1. The difference of these two roots, expressed in terms of d , is:

$$p_2 - p_1 = \sqrt{\frac{2d+1}{(d+1)^2(4d^2+4d-1)}}$$

Obviously, this value decreases monotonously with increasing d , converging to 0 as d goes to infinity. Though the mean-field approximation differs from the actual behavior of a CA, we can still draw meaningful conclusions from its asymptotic behavior. The convergence to 0 of the probability interval that sustains life, makes it extremely unlikely that the cellular space as a whole will be a fertile ground for patterns to emerge from initial random configurations. This is of course a probabilistic analysis. In reality, there can be patterns that survive, even in the models with higher values of d . Indeed, the previous section shows that gliders are among such patterns. These gliders grow in size, however, as a linear function of d , making their emergence from random configurations increasingly unlikely for high values of d .

6 Conclusions and Discussion

This paper presents an infinite class of inner-dependent outer-totalistic CA that all have gliders among their possible configurations. The discrete-valued parameter $d=4r$ for $r=2,3,4,\dots$, which corresponds to the cell neighborhood's radius, serves as an index for the members $\text{KL}(d)$ of this class. Gliders in $\text{KL}(d)$ have a length of $O(d)$, which makes their occurrence extremely unlikely in randomly initialized configurations or configurations emerging from them. The proven presence of gliders in $\text{KL}(d)$, however, shows that this probability will never be 0.

In the light of the convergence of the probability of living cells in $KL(d)$ to 0 as d approaches infinity, we conclude that interesting phenomena may occur in a cellular space that is at first sight—when doing trial-and-error computer simulations—virtually dead. A next logical step in this research would be to identify other living configurations in the infinite class of CA, and to find ways to make these configurations interact in possibly useful ways (such as to support computation).

Our results seem to confirm an observation on the web site [7] that the presence of gliders in CA is more common than expected, and that it appears to be hardly correlated with the classification of CA dynamics in the four classes proposed by Wolfram in [8]. In other words, Life may be more common than expected, yet it may be in unexpected places and spaces!

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