Modelling of Transport and Traffic Problems

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Abstract. The Asymmetric Simple Exclusion Process (ASEP) is the simplest cellular automaton which captures the essential aspects of most transport and traffic phenomena. It describes the directed motion of particles obeying an exclusion principle. For specific applications, however, various generalizations of the ASEP are necessary. These are discussed for the case of highway traffic, ant trails, pedestrian dynamics and intracellular transport.

Keywords: traffic flow, jam, ASEP, ant trails, pedestrian dynamics, intracellular transport.

1 Introduction

Cellular automata (CA) have become an important tool for the investigation of traffic systems from both the theoretical as well as practical point of view [1,2,3,4]. The discreteness of all relevant variables (space, time, state) makes them ideally suited for high-performance computer simulations. However, with increasing computer power this advantage will become less important in the future. Instead the fact that the dynamics of CA models is usually based on intuitive rules is an important advantage. Especially in interdisciplinary applications, where the interactions between agents are not based on the fundamental physical forces, rule-based models allow to take into account e.g. psychological aspects in a natural and efficient way.

Here we first discuss the most fundamental CA model which describes traffic and transport problems, the ASEP (Sec. 2). We will see that already this extremely simple model is able to capture the basic properties of such system, but not all. Specific system require specific modifications to improve the realism of the model. This will be discussed for highway traffic (Sec. 3), traffic on ant trails (Sec. 4), pedestrian dynamics (Sec. 5) and intracellular transport (Sec. 6).

2 Asymmetric Simple Exclusion Process (ASEP)

Generically all traffic and transport systems belong to the class of nonequilibrium systems in which many fascinating effects can be observed [2,5,6,7]. The simplest model which captures the main features of such systems is the so-called *Asymmetric Simple Exclusion Process (ASEP)*. It is not only the "mother of all traffic models", but also a paradigmatic example of *driven diffusive systems*.

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The ASEP describes the motion of particles which obey an exclusion principle, i.e. the space occupied by a particle is not available for others, on a discrete lattice. The dynamics is rather simple: A particle (•) moves to its empty right neighbour site (∘) with probability q (... • ∘... $\xrightarrow{q}$ ... ∘ •...). In all other cases the particle will not move (... • ∘... $\xrightarrow{1-q}$ ... • ∘... and ... • •... $\xrightarrow{1}$ ... • •...). In physics usually a random-sequential update is used corresponding to continuous time dynamics. Then the hopping probability becomes a hopping rate. For most applications, discrete updates like synchronous (parallel) or sequential ones are more realistic because they provide a timescale for calibration. However, the qualitative behaviour does not change for the different updates [8].

For periodic boundary conditions the exact solution for the stationary state can be derived using various methods (see [1] and references therein). For random-sequential dynamics a site-oriented mean-field theory becomes exact, i.e. the occupations of neighbouring sites are uncorrelated. For parallel dynamics correlations are generated by Garden-of-Eden states that can not be reached by the dynamics [9]. In this case the interparticle distribution function factorizes and the stationary state is described exactly by a car-oriented mean-field theory.

The most important quantitative characterization of traffic systems is the *fundamental diagram* defined as the density-dependence of the flow, $J(\rho)$. For the ASEP the fundamental diagram can be given explicitly (Fig. 1) Due to the particle-hole symmetry of the rules it is symmetric around $\rho = 1/2$.

From a physics as well as a practical point of view open boundary conditions are more interesting. These are usually realized by "reservoirs" at both ends of the chain where particles can enter and exit with probabilities (or rates) α and β, respectively. This case has been studied extensively in recent years and is now well understood (see e.g. [6,7]). As in the periodic case, the stationary state can be obtained exactly [10,11], e.g. using the matrix-product Ansatz [7].

Fig. 1 shows the phase diagram obtained by varying the boundary rates α and β. The origin of the three phases can easily be understood. In the low-density phase (A) the current depends only on the input rate α. The input is less efficient than the transport in the bulk or the output and therefore dominates the behaviour of the whole system. In the high-density phase (B) the output is the least efficient part. Therefore the current depends only on β. In the maximal current phase (C), input and output are more efficient than the transport in the bulk. Here the current has reached the largest possible value corresponding to the maximum of the fundamental diagram of the periodic system.

Quantitative predictions for the phase diagram and the boundary-induced phase transitions can be made using a coarse-grained description [12] which remains correct for the more sophisticated models discussed later. It relates the phase boundaries to properties of the periodic system which can be derived from the fundamental diagram, namely the so-called shock velocity v_s and the collective velocity v_c. v_s is the velocity of a 'domain-wall' which in nonequilibrium systems denotes an object connecting two possible stationary states. The collective velocity v_c describes the velocity of the center-of-mass of a local perturbation in a homogeneous, stationary background of density ρ.

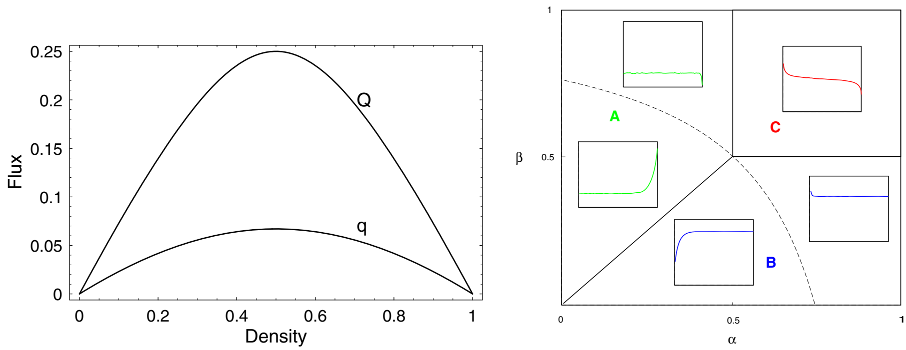

Fig. 1. Left: Fundamental diagram of the ASEP for two different hopping probabilities $Q > q$. Right: Phase diagram with low-density (A), high-density (B) and maximal-current phase (C). The insets show the typical shape of the density profiles.

The results for the ASEP show that boundary conditions play an important role in nonequilibrium systems. For most traffic applications periodic boundary conditions are not very realistic, e.g. since the number of vehicles fluctuates strongly due to on- and off-ramps.

3 Highway Traffic

3.1 Empirical Results

Theoretical results have to be compared with empirical observations on a qualitative or quantitative level. *Qualitative* results are usually related to the occurrence of spatio-temporal structures among like *jams*. *Quantitative results* like fundamental diagrams can be used for calibration of model parameters.

Two types of jams can be distinguished. *Bottleneck-induced jams* occur at locations of reduced capacity (*bottlenecks*) when the inflow is larger than this capacity. For *spontaneous jams* or *phantom jams* this is not true, at least not in an obvious way. Both empirical observations [13] as well as controlled experiments [14] indicate that growing instabilities can lead to jams even in the absence of bottlenecks. At intermediate densities the imperfect driving of human drivers can create a chain reaction where drivers overreact in braking manoevers which become necessary to avoid accidents when approaching the preceding car with large velocity.

In [15] it is argued that *all* jams are created by bottlenecks which are just sometimes not easy to identify. Often jams occur at the same location every day, especially close to ramps, sharp bends etc. However, these jams are not necessarily bottleneck-induced and might occur even though the local capacity has not yet been reached. Probably both mechanisms are relevant in the sense that inhomogeneities increase the probability of spontaneous jamming.

Quantitative results are obtained at many highway locations where empirical data are collected automatically by stationary inductive loops. Flow (current)

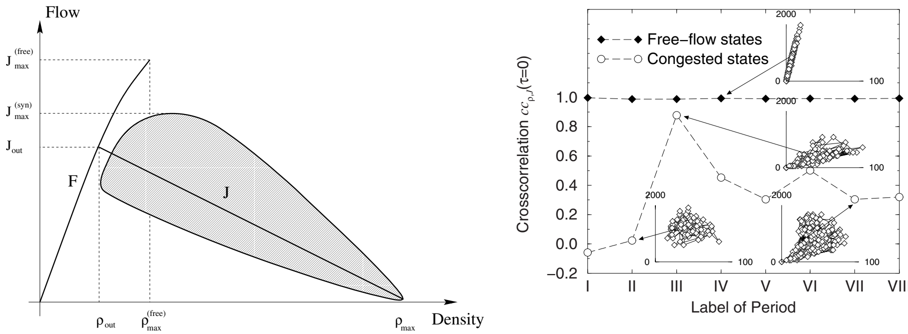

Fig. 2. Left: Schematic form of the fundamental diagram. F denotes the free flow branch and J the jam line. Right: Empirical cross-correlation function. Different periods of free-flow and congested traffic are labeled by I through VIII.

J and velocity v can be derived easily but density can not be measured locally and is usually determined using the hydrodynamic relation $J = \rho v$.

Nowadays three different phases of traffic flow are distinguished [16,17]: In *free flow* interactions between vehicles are rare. Cars move with their desired velocity and flow increases linearly with density (Fig. 2). States with flows larger than J_{out} form the *metastable branch*. All states not of free flow type are called *congested states*. They are characterized by an average velocity smaller than the desired velocity of the drivers. Two congested phases can be distinguished. *Wide jams* are regions of high density and negligible average velocity and flow. The jam front moves upstream (opposite to the driving direction) at typical velocity $v_{\text{Jam}} \approx 15$ km/h. In *synchronized flow* [16] the average velocity is significantly lower than in free flow, but the flow can be much larger than in wide jams. Characteristic is the absence of a functional flow-density relation and data points are spread irregularly over a 2d area (Fig. 2). This leads to a vanishing cross-correlation function [18] between density ρ and flow J (Fig. 2).

Modern detectors provide detailed information about the microscopic structure of traffic flow, e.g. through the distribution of time-headways, optimal-velocity functions, correlation functions, cluster distributions etc. [18,19].

3.2 NaSch Model

The ASEP does neither reproduce spontaneous jam formation, which requires a mechanism that creates chain reactions of braking manoevers, nor the observed asymmetry of the fundamental diagram. To obtain a more realistic model an extension of the ASEP to higher velocities is necessary.

The Nagel-Schreckenberg (NaSch) model [20,21] is a probabilistic CA. The state of each car n is characterized by its velocity $v_n = 0, 1, ..., v_{\text{max}}$. The position of the n-th vehicle is denoted by x_n. Then $d_n = x_{n+1} - x_n - 1$ is its headway, i.e. the number of empty cells in front of it. At each time step $t \to t+1$, all cars are updated *in parallel* according to the following "rules":

Step 1: Acceleration.
If $v_n < v_{max}$, velocity is increased by 1, i.e. $v_n^{(1)} = \min(v_n + 1, v_{\max})$.

Step 2: Deceleration (due to other cars).
If $d_n < v_n^{(1)}$, velocity is reduced to d_n, i.e. $v_n^{(2)} = \min(v_n^{(1)}, d_n)$.

Step 3: Randomization.
If $v_n^{(2)} > 0$, velocity is decreased randomly by 1 with probability p, i.e.

$$v_n^{(3)} = \begin{cases} \max(v_n^{(2)} - 1, 0) & \text{with probability } p, \\ v_n^{(2)} & \text{with probability } 1 - p. \end{cases}$$

Step 4: Vehicle movement.
Each car is moved forward according to its new velocity $v_n = v_n^{(3)}$ determined in Steps 1–3, i.e. $x_n \to x_n + v_n$.

Step 1 expresses the desire of the drivers to move as fast as possible (or allowed). *Step 2* reflects the interactions between vehicles and guarantees the absence of collisions in the model. *Step 3* incorporates many effects, e.g. natural fluctuations in the driving behaviour. It is responsible for spontaneous jam formation since it can lead to the chain reactions described above. Finally, in *Step 4* all cars will move with their new velocity as determined in the first three steps. This set of rules is minimal in the sense that every subset or change in the order will no longer produce realistic behaviour, e.g. spontaneous jams.

The timescale corresponding to one update step can be estimated in different ways [20]. Typical parameter values lead to timesteps which correspond to approximately 1 sec in real time which is of the same order of magnitude as the smallest relevant timescale in real traffic, the reaction time of the drivers.

The fundamental diagram of the NaSch model consists of a free flow and a congested branch (lines F and J in Fig. 2). However, it does not reproduce neither metastable states nor the synchronized phase.

3.3 Extensions of the NaSch Model

A simple modification of the NaSch model which reproduce the metastable states of high flow is the so-called *Velocity-Dependent-Randomization (VDR) model* [22] where the randomization parameter depends on the velocity of the car, $p = p(v)$:

Step 0: Determination of the randomization parameter.
The randomization parameter for the n-th car is given by $p = p(v_n(t))$.

This step is carried out before the acceleration *Step 1*. Metastable states occur for *slow-to-start rules* [22] where

$$p(v) = \begin{cases} p_0 & \text{for } v = 0, \\ p & \text{for } v > 0, \end{cases} \tag{1}$$

with $p_0 > p$, i.e. cars which have been standing in the previous timestep brake with higher probability p_0 than moving cars. This leads to fundamental diagrams

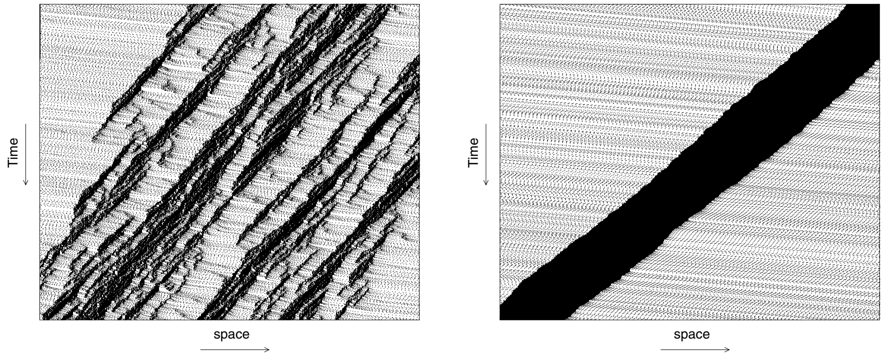

Fig. 3. Typical space-time diagrams of the NaSch model (left) and the VDR model for $p \ll p_0$ (right). One can clearly see the spontaneous jam formation in the NaSch model and different structure of the jams in both models.

which consist of the two branches F and J in Fig. 2, including the states with $J > J_{\text{out}}$. However, no synchronized traffic is found in this simple modification.

The macroscopic structure of the congested states is very different from that of the NaSch model [22]. It exhibits phase separation into a free flow region and a large jam which is almost compact for $p \ll 1$ (see Fig. 3). In contrast, in the NaSch model stop-and-go waves are found (Fig. 3). The structure of the free flow branch is very similar to that of the NaSch model. However, for $J > J_{\text{out}}$ the homogeneous free flow states are not stable, but can decay to a congested state through fluctuations or small perturbations.

The dynamics of the NaSch and VDR model is mainly based on the avoidance of accidents. This does not reproduce the synchronized phase and also the agreement with empirical data on a microscopic level is not very satisfactory [23]. Therefore it has been suggested that the desire of the drivers for smooth and comfortable driving is responsible for the occurrence of synchronized traffic [24]. This is realized through “anticipation” of the behaviour of preceding cars which reduces the risk of braking abruptly and allows for smoother driving. Thus the three observed traffic phases correspond to different driving strategies. In free flow, drivers drive as fast as possible and interactions are rare. In the jammed phase, the avoidence of accidents determines the behaviour and in synchronized traffic it is the desire to drive in a smooth and comfortable way.

These aspects are incorporated in the *brake light model* [25] which is able to reproduce all three phases and shows good agreement with detailed empirical single-vehicle data [23]. Anticipation is realized through brake lights which indicate (within an interaction horizon) velocity changes of the preceding car.

Similar ideas are used in the KKW model [26]. Drivers change their behaviour within a synchronization distance to the preceding vehicle where they try to move at the same velocity as the preceding car instead of maximizing their speed. Another approach [27] emphasizes the conflict between human overreaction and limited acceleration and deceleration capabilities as possible origin of congested traffic states. However, this model is no longer intrinsically accident free.

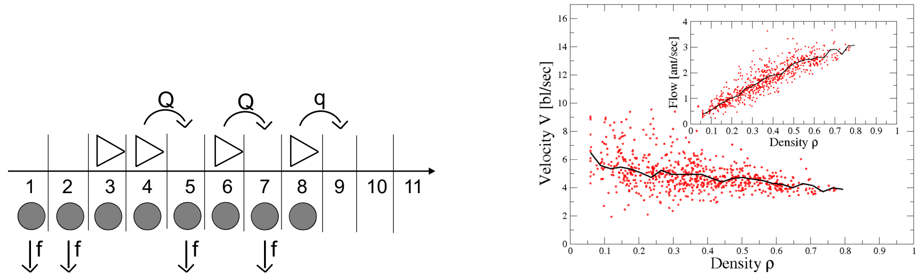

Fig. 4. Definition of the ant trail model (left): The symbols correspond to ants (▷) and pheromone marks (•). Fundamental diagrams from empirical observations (right).

4 Ant Trails

Ants form transport networks that have many similarities with human highway systems [28]. Essential for the formation and maintainance of these trails is a special form of chemical communication called *chemotaxis*. Ants mark their path by a chemical called pheromone that can be "smelled" by other ants which follow the trace to find food sources etc.

Chemotaxis can be incorporated into an ASEP-based model [29,30,31,32]. Now particles, corresponding to ants, move with two different hopping probabilities Q and q depending whether or not a pheromone is present at the target cell (Fig. 4). To model the trail following, Q in the presence of pheromone should be larger than q in the absence of pheromones. In order to model evaporation of the trace free pheromones (at sites without ant) are removed with probability f.

Surprisingly for small evaporation rates f the fundamental diagram is qualitatively different from that of highway traffic. The average velocity is no longer a monotonically decreasing function of the density. Instead in the presence of chemotaxis [29,30,31,32] it can exhibit a maximum at a finite value of the density. This is related to the formation of platoons of ants.

Empirical observations show the absence of a jammed regime in natural trails of the species *Leptogenys processionalis* [33]. The average velocity is almost independent of the density and flow always increases with increasing density (Fig. 4). Also no overtaking was observed. Qualitative observations confirm the platoon formation predicted by the ASEP-based ant trail model [33].

5 Pedestrian Dynamics

Due to its generically 2d nature pedestrian motion is more difficult to describe in terms of simple models, but it exhibits many interesting collective effects and self-organization phenomena [34,17]: At large densities various kinds of *jamming* phenomena occur, e.g. when many people try to leave a large room at the same time. In counterflow, when two groups of people move in opposite directions, *lane formation* can occur. Pedestrians self-organize such that (dynamically varying) lanes of unidirectional flow are formed. This reduces interactions with oncoming pedestrians and allows higher walking speeds. At bottlenecks, e.g. doors, counterflow can lead to *oscillations* of the flow direction. At intersections various collective patterns of motion like short-lived roundabouts can be formed which make the motion more efficient.

Several models for the description of pedestrian dynamics have been suggested [34,17]. The *social force model* [35,17] treats pedestrians as particles subject to long-ranged forces induced by the social behaviour of the individuals. This leads to (coupled) equations of motion similar to Newtonian mechanics.

In [36,37,38] a CA model has been introduced which takes its inspiration from the process of chemotaxis. It is in many respects a two-dimensional variant of the ant trail model of Sec. 4. Moving pedestrians create a "trace" which is, in contrast to chemotaxis, only virtual. Its main purpose is to transform effects of long-ranged interactions (e.g. following people walking some distance ahead) into a local interaction (with the "trace"). This allows for efficient simulations on a computer, especially in complex geometries.

This basic idea is realized through so-called *floor fields*. In one time step each pedestrian can move to one of the nine neighbouring cells in direction (i, j) of a 2d lattice. The transition probabilities now depend on the strength of the floor field in the target cell. A motion in the direction of large fields is preferred.

In fact two different floor fields are used. The *static floor field* S is constant and takes into account the geometry of the system (building). In order to model people leaving a room one uses a static floor which increases with decreasing distance to the exit. The second field, called *dynamic floor field* D, is just the virtual trace left by the pedestrians. In contrast to the ant trail model, where only the presence or absence was distinguished, the dynamic floor field can have different strengths. This allows to incorporate diffusion to neighbouring cells which corresponds to the broadening and dilution of the trace.

Finally the transition probabilities depend on the preferred walking direction and speed of each individual. This is encoded in the *matrix of preferences* M. Its entries are related to the preferred velocity vector and its longitudinal and transversal standard deviations [36].

The transition probability p_{ij} in direction (i, j) is then determined by

$$p_{ij} = N M_{ij} D_{ij} S_{ij} (1 - n_{ij}). \tag{2}$$

N is a normalization factor to ensure $\sum_{(i,j)} p_{ij} = 1$ where the sum is over the nine possible target cells. The factor $1 - n_{ij}$, where n_{ij} is the occupation number of cell (i, j), takes into account that transitions to occupied cells are forbidden.

The details of the update rules can be found in [36,37,38]. There it is also shown that the floor-field model - despite its simplicity - allows to reproduce the empirically observed phenomena.

6 Molecular Motors

Intracellular transport is carried by *molecular motors* which are proteins that can directly convert chemical into mechanical energy required for their movement along filaments constituting what is known as the *cytoskeleton* [39]. The cytoskeleton has many similarities with human-built road networks. *Microtubules* which are the tracks for long-range transport can be considered the analogues of highways. But in contrast to the latter, motors can enter and leave the microtubule at any location, not just at the ends or at on- or off-ramps. This so-called *Langmuir kinetics* can be incorporated into the ASEP by allowing particle creation and annihilation at *any* site of the lattice [40,41] not only the first an last one. This leads to the existence of novel phases, e.g. the coexistence of low and high density regimes, separated from each other by domain walls. Empirical evidence for such a phase has been found in [42].

For a more detailed discussion of intracellular transport we refer to [43].

7 Discussion

The ASEP is the most basic CA model for the description of transport and traffic problems. It captures only two vary basic features, namely the directionality of motion and the exclusion principle. This is already sufficient to reproduce many aspects at least qualitatively.

ASEP-based approaches are flexible enough to allow for simple and intuitive extensions that are able to provide a quantitive description in many situations. This has lead to interesting applications, e.g. traffic forecasting [44].

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