Remarks on Population Ethics

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1 Introduction

Population ethics is about principles for social evaluation of alternatives with different population sizes. Different environmental policies lead to different population sizes as well as different quality of lives involved. Therefore, as a necessary step towards laying foundations for such policy recommendations, discussing relevant issues on population principles is of critical importance.

One of the most important issues in population ethics has been the repugnant conclusion introduced by Derek Parfit (1976, 1982, 1984). He criticized classical utilitarianism as it implies the following conclusion:

> *The Repugnant Conclusion*: For any possible population of at least ten billion people, all with a very high quality of life, there must be some much larger imaginable population whose existence, if other things are being equal, would be better, even though its members have lives that are barely worth living (see Parfit (1984, p. 388)).

Since then, avoiding the repugnant conclusion has been one of the most important axioms in population ethics. And, this is well-documented by two facts. First, Blackorby, Bossert and Donaldson, leading figures in population ethics, survey the literature concerning the repugnant conclusion in a handbook chapter on social choice and welfare (Blackorby, Bossert, and Donaldson, 2002). Second, there is a book that is entirely devoted to the issues of the repugnant conclusion (Ryberg and Tännsjö, 2004). Despite these, a number of theorists have argued that the repugnant conclusion may not be so repugnant and thus avoiding the conclusion is not that compelling (see Arrhenius (2003, p. 168)). This motivates Arrhenius (2003) to modify the concept of the repugnant conclusion in such a way that even those theorists critical of the original version would find the modified version very hard to accept:

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P.K. Pattanaik et al. (eds.) *Rational Choice and Social Welfare: Theory and Applications*, Studies in Choice and Welfare.

The Very Repugnant Conclusion: For any perfectly equal population A with very high positive welfare, and for any number of lives with very negative welfare, there is a population B consisting of the lives with negative welfare and lives with very low positive welfare which is better than population A, other things being equal (Arrhenius (2003, p. 167)).

Arrhenius (2003), then, proceeds to formalize this idea and shows that a version of the mere addition paradox (Parfit, 1984) still holds even if one replaces avoidance of the repugnant conclusion with avoidance of the very repugnant conclusion. In this paper, we investigate what happens to the results on generalized utilitarianism in population ethics established by Blackorby, Bossert, and Donaldson (2004, 2006) when we replace avoidance of the repugnant conclusion with avoidance of the very repugnant conclusion. Arrhenius' own version of the very repugnant conclusion is stated in a model that has considerably different structures than ours. Therefore, we reformulate the very repugnant conclusion in our framework. Consequently, Arrhenius' own version of the very repugnant conclusion and ours are non-comparable.

Arrhenius (2000) introduces two versions of the sadistic conclusion and argues that it should be avoided, too. If a population principle implies that adding people with negative utilities can make a society better off, the conclusion is sadistic. Blackorby, Bossert, and Donaldson (2004) explore logical relations between avoidance of sadistic conclusion and critical-level generalized utilitarian principles. In this paper, we reexamine one of their results.

In Section 2, we introduce the model and state avoidance of the repugnant conclusion and avoidance of the very repugnant conclusion. We show that the incompatibility between Pareto plus and avoidance of the repugnant conclusion is rather robust in the sense that replacing the latter with avoidance of the very repugnant conclusion does not upset the result. In Section 3, we state avoidance of sadistic conclusion. The last section concludes with some remarks.

2 The Model

We work with the model set up by Blackorby, Bossert, Donaldson, and Fleurbaey (1998).

Let $\mathbb{N}$ be the set of natural numbers and let $\mathbb{R}(\mathbb{R}_{++}, \mathbb{R}_{--})$ be the set of all (positive, negative) real numbers. $\mathbb{R}^{\mathbb{N}}$ be the set of all maps from $\mathbb{N}$ into $\mathbb{R}$. Let $\mathcal{N}$ be the set of all non-empty and finite subsets of $\mathbb{N}$. Typical elements of $\mathcal{N}$ are denoted by L, M, N and so on. For each $N \in \mathcal{N}$, $\mathbb{R}^N(\mathbb{R}^N_+)$ is the set of all maps from N into $\mathbb{R}(\mathbb{R}_+)$. Typical elements of $\mathbb{R}^N(\mathbb{R}^N_+)$ are denoted by $u = (u_i)_{i \in N}$, $v = (v_i)_{i \in N}$, $w = (w_i)_{i \in N}$ and so on. For each $N \in \mathcal{N}$, 1_N is the element in $\mathbb{R}^N$ defined by $(1_N)_i = 1$ for each $i \in N$. For all disjoint sets $N, M \in \mathcal{N}$, for all $u = (u_i)_{i \in N}$, $v = (v_i)_{i \in N}$, (u, v) is the element of $\mathbb{R}^{N \cup M}$ defined by $(u, v)_i = u_i$ for $i \in N$ and $(u, v)_j = v_j$ for $j \in M$.

We take a welfarist approach to population ethics: To discuss evaluations of social states, all we need to know is information about population and utility allocations.[1] We employ a comprehensive notion of utilities as indicators of lifetime well-being to avoid counter-intuitive results on the termination of lives.[2] Let $\mathcal{D} = \{(N;u) | N \in \mathcal{N} \text{ and } u \in \mathbb{R}^N\}$. A typical element $(N;u) \in \mathcal{D}$ consists of population N and utility allocation u for N. A social-evaluation ordering is a complete and transitive binary relation R on $\mathcal{D}$.[3] For $(N;u)$, $(M;v) \in \mathcal{D}$, $(N;u)R(M;v)$ means $(N;u)$ is socially at least as good as $(M;v)$. The asymmetric part of R is denoted by P and the symmetric part by I.

An individual considers her or his life neutral if it is as good as the one without any experiences. We employ the convention in population ethics that utilities are normalized so that the zero level of utility represents neutrality.[4]

Repugnant conclusion: For all $N \in \mathcal{N}$, for all $\xi \in \mathbb{R}_{++}$, for all $\varepsilon \in (0, \xi)$, there exists $M \in \mathcal{N}$ such that $M \supset N$ and $(M; \varepsilon 1_M)P(N; \xi 1_N)$.

Avoidance of the repugnant conclusion is the negation of repugnant conclusion.

Avoidance of the repugnant conclusion: There exist $N \in \mathcal{N}$, $\xi \in \mathbb{R}_{++}$, $\varepsilon \in (0, \xi)$ such that $(N; \xi 1_N)R(M; \varepsilon 1_M)$ for all $M \in \mathcal{N}$ such that $M \supset N$.

We formalize the idea of the very repugnant conclusion introduced by Arrhenius (2003) in our framework as follows.

Very repugnant conclusion: For all $N \in \mathcal{N}$, for all $\xi \in \mathbb{R}_{++}$, for all $M \in \mathcal{N}$, for all $\eta \in \mathbb{R}_{--}$, for all $\varepsilon \in (0, \xi)$, there exists $L \in \mathcal{N}$ such that $L \cap M = \emptyset$, and $(L \cup M; \varepsilon 1_L, \eta 1_M)P(N; \xi 1_N)$.

Avoidance of the very repugnant conclusion is the negation of very repugnant conclusion.

Avoidance of the very repugnant conclusion: There exist $N, M \in \mathcal{N}$, $\xi \in \mathbb{R}_{++}$, $\eta \in \mathbb{R}_{--}$ and $\varepsilon \in (0, \xi)$ such that $(N; \xi 1_N)R(L \cup M; \varepsilon 1_L, \eta 1_M)$ for all $L \in \mathcal{N}$ with $L \cap M = \emptyset$.

Let us recall the standard axiom of strong Pareto.

Strong Pareto: For all $N \in \mathcal{N}$ and $u, v \in \mathbb{R}^N$, if $u_i \geq v_i$ for every $i \in N$ and $u_i > v_i$ for some $i \in N$, then $(N;u)P(N;v)$.

Avoidance of the repugnant conclusion together with strong Pareto implies avoidance of the very repugnant conclusion.

Lemma 1. *If a population principle satisfies strong Pareto and the very repugnant conclusion, then it satisfies the repugnant conclusion.*

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[1] For and against welfarism, see Blackorby, Bossert, and Donaldson (2002).

[2] See, for example, Blackorby et al. (2002) for an account of lifetime well-being.

[3] R is complete if for all $(N;u), (M;v) \in \mathcal{D}$, $(N;u)R(M;v)$ or $(M;v)R(N;u)$. R is transitive if for all $(N;u), (M;v), (L;w) \in \mathcal{D}$, $(N;u)R(M;v)$ and $(M;v)R(L;w)$ imply $(N;u)R(L;w)$.

[4] See Broome (1993, 2004) for a discussion of neutrality and its normalization to zero.

Proof. Let $N \in \mathcal{N}$, $\xi \in \mathbb{R}_{++}$, $M \in \mathcal{N}$, $\eta \in \mathbb{R}_{--}$ and let $\varepsilon \in (0, \xi)$. By the very repugnant conclusion, there exists $L \in \mathcal{N}$ such that $L \cap M = \emptyset$, $L \cap N = \emptyset$, and $(L \cup M; \varepsilon 1_L, \eta 1_M) P(N; \xi 1_N)$. By strong Pareto, $(L \cup M; \varepsilon 1_L, \varepsilon 1_M) P(L \cup M; \varepsilon 1_L, \eta 1_M)$. By transitivity, $(L \cup M; \varepsilon 1_L, \varepsilon 1_M) P(N; \xi 1_N)$. □

Lemma 1 says that avoidance of the repugnant conclusion along with strong Pareto imply avoidance of the very repugnant conclusion.

A population principle R satisfies generalized utilitarianism if there exists a continuous and increasing transformation $g : \mathbb{R} \to \mathbb{R}$ of utilities with $g(0) = 0$ such that for all $NM \in \mathcal{N}$, for all $u = (u_i)_{i \in N} \in \mathbb{R}^N$, for all $v = (v_i)_{i \in M} \in \mathbb{R}^M$, $(N; u) R(M; v)$ if and only if

$$\sum_{i \in N} g(u_i) \geq \sum_{i \in NM} g(v_i).$$

Sikora (1978) introduces an axiom consisting of strong Pareto and the requirement that adding an individual with a utility level above neutrality should be a social improvement. He calls this axiom Pareto plus. Following Blackorby, Bossert, and Donaldson (2006), we retain strong Pareto as a separate axiom and state Pareto plus as follows.

Pareto plus: For all $N \in \mathcal{N}$, for all $u = (u_i)_{i \in N} \in \mathbb{R}^N$, for all $j \in \mathbb{N} \setminus N$, for all $a \in \mathbb{R}_{++}$, $(N \cup \{j\}; u, a) P(N, u)$.

The following impossibility result provides a yet another criticism against Pareto plus.

Theorem 1. *There exists no population principle that satisfies generalized utilitarianism, Pareto plus and the avoidance of the very repugnant conclusion.*

Proof. Suppose that there exists a generalized-utilitarian population principle satisfying Pareto plus. Let $M, N \in \mathcal{N}$ be sets of cardinalities m and n, respectively, and let $\varepsilon \in (0, \xi)$. Since $g(\varepsilon) > 0$, one can pick $l \in \mathbb{N}$ large enough to have $lg(\varepsilon) + mg(\eta) > (n+m) g(\xi)$. Let $L \in \mathcal{N}$ be a finite set with cardinality l satisfying $L \cap M = \emptyset$.

Thus, by generalized utilitarianism, $(L \cup M; \varepsilon 1_L, \eta 1_M) P(N \cup M; \xi 1_N, \xi 1_M)$. By repeated application of Pareto plus and transitivity, $(N \cup M; \xi 1_N, \xi 1_M) P(N; \xi 1_N)$. By transitivity, $(L \cup M; \varepsilon 1_L, \eta 1_M) P(N; \xi 1_N)$. Thus, the very repugnant conclusion holds. This completes the proof. □

3 Critical-Level Generalized Utilitarianism

A population principle R satisfies critical-level generalized utilitarianism if there exist a critical-level of utility $\alpha \in \mathbb{R}$ and a continuous and increasing transformation $g : \mathbb{R} \to \mathbb{R}$ of utilities with $g(0) = 0$ such that for all $N, M \in \mathcal{N}$, for all $u = (u_i)_{i \in N} \in \mathbb{R}^N$ for all $v = (v_i)_{i \in M} \in \mathbb{R}^M$, $(N; u) R(M; v)$ if and only if

$$\sum_{i\in N}[g(u_i)-g(\alpha)] \geq \sum_{i\in M}[g(v_i)-g(\alpha)].$$

The following theorem is essentially a strengthening of Theorem 3 (i) in Blackorby, Bossert, and Donaldson (2004).

Theorem 2. *A critical-level generalized utilitarian principle satisfies avoidance of the very repugnant conclusion if and only if the critical level α is positive.*

Proof. Let R be a critical-level generalized utilitarian population principle with a continuous and increasing transformation g of utilities and a critical level α. Then, R satisfies avoidance of very repugnant conclusion if and only if there exist $n,m \in \mathbb{N}$, $\xi \in \mathbb{R}_{++}$, $\eta \in \mathbb{R}_{--}$ and $\varepsilon \in (0,\xi)$ such that $l[g(\varepsilon)-g(\alpha)]+m[g(\eta)-g(\alpha)] \leq n[g(\xi)-g(\alpha)]$ for all $l \in \mathbb{N}$.

Suppose $\alpha > 0$. Let $n = 1$, $\xi = 2\alpha$, $\varepsilon = \alpha/2$. Pick any $\eta \in \mathbb{R}_{--}$. Clearly, $g(\varepsilon)-g(\alpha) = g(\alpha/2)-g(\alpha) < 0$, $g(\eta)-g(\alpha) < 0$ and $0 < g(2\alpha)-g(\alpha) = g(\xi)-g(\alpha)$.

Hence, $l[g(\varepsilon)-g(\alpha)]+m[g(\eta)-g(\alpha)] < n[g(\xi)-g(\alpha)]$ for all $l \in \mathbb{N}$. Thus R satisfies avoidance of very repugnant conclusion.

Suppose $\alpha \leq 0$. Let $n,m \in \mathbb{N}$, $\xi \in \mathbb{R}_{++}$, $\eta \in \mathbb{R}_{--}$ and $\varepsilon \in (0,\xi)$. Take $l \in \mathbb{N}$ such that $l > [ng(\xi) - mg(\eta) + (m-n)g(\alpha)]/[g(\varepsilon) - g(\alpha)]$. Clearly, $l[g(\varepsilon) - g(\alpha)]+m[g(\eta)-g(\alpha)] > n[g(\xi)-g(\alpha)]$.

Let $L,M,N \in \mathcal{N}$ be finite sets with cardinality l,m and n, respectively. Clearly, $(L\cup M;\varepsilon 1_L,\eta 1_M)P(N;\xi 1_N)$. Thus, the very repugnant conclusion holds. This completes the proof. □

A social evaluation ordering implies the sadistic conclusion if adding people with negative utilities can be better than adding people with positive utilities. The idea is expressed formally as follows.

Sadistic conclusion: There exist $N \in \mathcal{N}$, $M \in \mathcal{N}$, $L \in \mathcal{N}$, $u \in \mathbb{R}^N$, $v \in \mathbb{R}^M_{--}$ and $w \in \mathbb{R}^L_{++}$ such that $(N\cup M;u,v)P(N\cup L;u,w)$.

The following statement is due to Theorem 3 (ii) in Blackorby, Bossert, and Donaldson (2004). Their argument is designed for critical-level utilitarianism but it does not work for its generalized counterpart. So, we shall provide a proof which invokes continuity of utility transformations.

Theorem 3. *A critical-level generalized utilitarian principle satisfies the sadistic conclusion if and only if the critical level a is non-zero.*

Proof. Suppose $\alpha > 0$. Since $g(0) = 0$ and g is increasing, $g(\alpha) > 0$. Since g is continuous at 0, there exist $v_1 \in \mathbb{R}_{--}$ and $w_1 \in \mathbb{R}_{++}$ such that $g(\alpha) > 2g(w_1) - g(v_1)$.

This inequality is equivalent to the following.

$$[g(v_1)-g(\alpha)] > [g(w_1)-g(\alpha)]+[g(w_1)-g(\alpha)].$$

Let i,j,k,l be distinct natural numbers and let $N = \{i\}$, $M = \{j\}$, $L=\{k,l\}$, $u_i=\alpha, v_j = v_1$ and let $w_k = w_l = w_1$.

Then, $v \in \mathbb{R}^M_{--}$, $w \in \mathbb{R}^L_{++}$ and $(N \cup M; u, v) P (N \cup L; u, w)$.

Suppose $\alpha < 0$. Since $g(0) = 0$ and g is increasing, $g(\alpha) < 0$. Since g is continuous at 0, there exist $v_1 \in \mathbb{R}_{--}$ and $w_1 \in \mathbb{R}_{++}$ such that $-g(\alpha) > g(w_1) - 2g(v_1)$. This inequality is equivalent to the following.

$$[g(v_1) - g(\alpha)] + [g(v_1) - g(\alpha)] > [g(w_1) - g(\alpha)].$$

Let i, j, k, l be distinct natural numbers and let $N = \{i\}$, $M = \{j,k\}$, $L = \{l\}$, $u_i = \alpha, v_j = v_k = v_1$ and let $w_l = w_1$. Then, $v \in \mathbb{R}^M_{--}$ and $w \in \mathbb{R}^L_{++}$ but $(N \cup M; u, v)$ $P(N \cup L; u, w)$.

For the case $\alpha = 0$, the proof is identical to that of Theorem 3 (ii) in Blackorby, Bossert, and Donaldson (2004). □

4 Concluding Remarks

Though we have established a few results on generalized utilitarianism in this paper, the issues of investigating the robustness of impossibility theorems involving avoidance of repugnant conclusions are still wide open. For instance, Blackorby, Bossert, and Donaldson (2006) establish that there exists no anonymous population principle that satisfies minimal increasingness, weak inequality aversion, Pareto plus and avoidance of the repugnant conclusion. What happens to this impossibility result when we replace avoidance of the repugnant conclusion with avoidance of the very repugnant conclusion? Similar questions can be asked for the impossibility results in Blackorby, Bossert, Donaldson, and Fleurbaey (1998).

Acknowledgments The author is grateful to Walter Bossert and Marc Fleurbaey for helpful suggestions on an earlier version of the paper. The current version benefitted considerably from the detailed comments by a referee.

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