A Family of Fuzzy Description Logics with Comparison Expressions*

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Abstract. The fuzzy knowledge plays an important role in many applications on the semantic web which faces imprecise and vague information. The current ontology languages on the semantic web use description logics as their logic foundation, which are insufficient to deal with fuzzy knowledge. Comparisons expressions between fuzzy membership degrees are frequently used in fuzzy knowledge systems. However, the current fuzzy extensions of description logics are not support the expression of such comparisons. This paper defines fuzzy comparison cuts to represent comparison expressions, extends fuzzy description logics by importing fuzzy comparison cuts and introducing new constructors. Furthermore, the reasoning algorithm is proposed. It enables representation and reasoning for fuzzy knowledge on the semantic web.

Keywords: Ontology, Distributed, Fuzzy, Description logic.

1 Introduction

Description logics (DLs) [1] are a family of knowledge representation languages widely used in the semantic web as a logic foundation for knowledge representation and reasoning. It is often necessary to represent fuzzy knowledge in real-life applications [2]. The fuzzy knowledge plays an important role in many domains which faces a huge amount of imprecise and vague knowledge and information, such as text mining, multimedia information system, medical informatics, machine learning, human natural language processing. However, classical DLs are insufficient to representing fuzzy knowledge [3]. Fuzzy DLs import the fuzzy set theory to enable the capability of dealing with fuzzy knowledge.

Many research work on fuzzy DLs have been carried out. Yen provided a fuzzy extension of DL $\mathcal{FL}^-$ [4]. Tresp presents a fuzzy extension of $\mathcal{ALC}$, $\mathcal{ALC}_{F_M}$ [3]. Straccia presented fuzzy $\mathcal{ALC}$ and an algorithm for assertional reasoning [8]. There are many extensions of fuzzy $\mathcal{ALC}$. Höldobler introduced the membership manipulator constructor to present $\mathcal{ALC}_{FH}$ [9]. Sanchez generalized the quantification in fuzzy $\mathcal{ALCQ}$ [10]. Stoilos provided pure ABoxes reasoning algorithms

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for the fuzzy extensions of $\mathcal{SHIN}$ [11]. There are several works using the idea of fuzzy cuts. Straccia transformed fuzzy $\mathcal{ALC}$ into classical $\mathcal{ALCH}$ [5]. Li presented a family of extended fuzzy DLs ($\mathcal{EF}$DLs) [6]. Calegari showed the fuzzy OWL [12] and Straccia presented a fuzzy $\mathcal{SHOIN}(D)$[13].

It is a familiar description that "Tom is taller than Mike," which can be seemed as a comparison between two fuzzy membership degrees. We call such descriptions *comparison expressions* on fuzzy membership degrees. However, the current fuzzy DLs do not support the expression of comparisons between fuzzy membership degrees. So it is necessary to extend fuzzy DLs with the ability of expressing comparison expressions.

This paper defines *fuzzy comparison cuts* (*cuts* for short) to represent comparison expressions on fuzzy membership degrees. The reasoning algorithm is proposed. It enables representation and reasoning for expressive fuzzy knowledge on the semantic web.

2 Fuzzy DLs with Comparison Expressions

2.1 Represent Comparison Expressions

For an individual a and a fuzzy concept C, let $a : C$ be the degree to which a is an instance of C. Similarly, $(a, b) : R$ is the degree to which two individuals a and b has a role R. In fuzzy DLs, the degrees can have their values in $[0, 1]$. We can show ranges of degrees in a set of *fuzzy assertions* of the form $\langle \alpha \bowtie n \rangle$, where α is a degree, $n \in [0, 1]$ is a constant and $\bowtie \in \{=, \neq, <, \leq, \geq, >\}$.

It is often necessary to compare the fuzzy membership degrees. There can be different kinds of comparisons between fuzzy membership degrees:

- A *numerical comparison* compares a degree to a constant. $Tom : Tall > 0.8$ means "Tom is quite tall." $Mike : Tall \leq 0.9$ means "Mike is not very tall."
- An *abstract comparison* compares degrees of the same individual. $Tom : Absolutist < Tom : Liberalist$ means "Tom prefers liberalism to absolutism."
- A *relative comparison* compares degrees between different individuals. $Tom : Tall < Mike : Tall$ means "Tom is taller than Mike."
- A *complex comparison* is constructed from the above kinds of simple comparisons. If for any person x such that $(Tom, x) : hasFriend > 0.9$, it holds $Tom : Tall > x : Tall$ or $Tom : Strong > x : Strong$, then we can say "No close friend (the degree of friendship is greater than 0.9) of Tom is taller and stronger than him."

Our idea is to define new elements to express the above kinds of comparisons, and integrate them into the current fuzzy DLs. We call the new elements *fuzzy comparison cuts*. In the fuzzy set theory[2], the cut sets are indeed classical sets, but facilitate a normative theory for formalizing fuzzy set theory. The idea of fuzzy cuts can also be used for fuzzy DLs. [5] use the idea of cut sets of fuzzy concepts to transform fuzzy DL $\mathcal{ALC}$ to classical DL $\mathcal{ALCH}$. [6] defined cuts of fuzzy concepts for more expressive ability.

2.2 Syntax and Semantics

The new languages with comparison cuts is called FCDLs. The syntax of FCDLs starts from three disjoint sets: N_I, N_C and N_R: N_I is a set of individual names, N_C is a set of fuzzy concept names, and N_R is a set of fuzzy role names. Complex fuzzy descriptions can be built from them inductively with fuzzy concept constructors and fuzzy role constructors.

Definition 1. *The set of* fuzzy role descriptions *(or* fuzzy roles *for short) is defined as: every fuzzy role name $R \in N_R$ is a fuzzy role; and for any fuzzy role R, R^- is also a fuzzy role (let $(R^-)^- := R$). For two fuzzy role R, S, $R \sqsubseteq S$ is called a* fuzzy role inclusion axiom*. A finite set of role inclusions is called a* role hierarchy*. For individual names $a, b \in N_I$ and a constant $n \in [0, 1]$, $\langle (a, b) : R \bowtie n \rangle$ is called a* fuzzy role assertion*.*

If for any a, b, $(a, b) : R = n$ iff $(b, a) : S = n$, then S is an inverse role of R, written R^-. The role inclusion is transitive and $R \sqsubseteq S$ implies $R^- \sqsubseteq S^-$. For a role hierarchy $\mathcal{R}$, let $\sqsubseteq_{\mathcal{R}}$ be the transitive reflexive closure of $\sqsubseteq$ on $\mathcal{R} \cup \{R^- \sqsubseteq S^- | R \sqsubseteq S \in \mathcal{R}\}$. Beside the definition, N_R consists both transitive and normal fuzzy role names $N_R = N_{R^+} \cup N_{R^P}$, where $N_{R^+} \cap N_{R^P} = \emptyset$. For a transitive fuzzy role R, if $(a, b) : R \geq n$ and $(b, c) : R \geq n$, then it must have $(a, c) : R \geq n$. Let trans$(S, \mathcal{R})$ be true if for some R with $R = S$ or $R \equiv_{\mathcal{R}} S$ such that $R \in N_{R^+}$ or $R^- \in N_{R^+}$, where $R \equiv_{\mathcal{R}} S$ is an abbreviation for $R \sqsubseteq_{\mathcal{R}} S$ and $S \sqsubseteq_{\mathcal{R}} R$. A role R is *simple* w.r.t. $\mathcal{R}$ iff trans$(S, \mathcal{R})$ is not true for all $S \sqsubseteq_{\mathcal{R}} R$. Simple roles is required in order to avoid undecidable logics [1].

Definition 2. *The set of* fuzzy concepts *is that*

1. *every fuzzy concept name $A \in N_C$ is a fuzzy concept, $\top$ and $\bot$ are fuzzy concepts,*
2. *if C, D are fuzzy concepts, $o \in N_I$ is an individual name, R is a fuzzy role, S is a simple fuzzy role, and $q \in \mathbb{N}$, then $\neg C, C \sqcap D, C \sqcup D, \forall R.C, \exists R.C, \geq qS.C, \leq qS.C, \{o\}$ are also fuzzy concepts,*
3. *if R is a fuzzy role, S is a simple fuzzy role, P is a cut, and $q \in \mathbb{N}$, then $\exists R.P, \forall R.P, \geq qS.P, \leq qS.P$ are also fuzzy concepts.*

For two fuzzy concept C, D, $C \sqsubseteq D$ is called a fuzzy concept inclusion*. For $a \in N_I$ and $n \in [0, 1]$, $\langle a : C \bowtie n \rangle$ is called a* fuzzy concept assertion*.*

Definition 3. *The set of* fuzzy comparison cuts *(or* cuts *for short) is defined as: if C, D are fuzzy concepts, $n \in [0, 1]$ and $\bowtie \in \{=, \neq, >, \geq, <, \leq\}$, then $[C \bowtie n]$, $[C \bowtie D]$ and $[C \bowtie D^\uparrow]$ are cuts (and $[C \bowtie]$ is an abbreviation of $[C \bowtie C^\uparrow]$); if P, Q are cuts, then $\neg P$, $P \sqcap Q$ and $P \sqcup Q$ are also cuts. For any cut P and $a \in N_I$, $P(a)$ is called an* absolute cut*. If a cut P contains no $\uparrow$, then P itself is an absolute cut, and we do not distinguish P and $P(a)$ for any a. For two absolute cuts P, Q, $P \sqsubseteq Q$ is called a* cut inclusion*. For an absolute cut $P(b)$ and $a \in N_I$, $\langle a : P(b) \rangle$ is called a* cut assertion*.*

Definition 4. *A* fuzzy interpretation $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ *consists a nonempty set* $\Delta^{\mathcal{I}}$ *as its domain, and a function* $\cdot^{\mathcal{I}}$ *maps every individual* $a \in N_I$ *to an element* $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$*, maps every fuzzy concept name* $A \in N_C$ *to a function* $A^{\mathcal{I}} : \Delta^{\mathcal{I}} \to [0,1]$*, and maps every fuzzy role name* $R \in N_R$ *to a function* $R^{\mathcal{I}} : \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \to [0,1]$*.*

The interpretation function is also extended to complex descriptions. It maps every fuzzy concept C *to a function* $C^{\mathcal{I}} : \Delta^{\mathcal{I}} \to [0,1]$*, maps every fuzzy role* R *to a function* $R^{\mathcal{I}} : \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \to [0,1]$*, maps every cut* P *to a function* $P^{\mathcal{I}} : \Delta^{\mathcal{I}} \to 2^{\Delta^{\mathcal{I}}}$*, and maps every absolute cut* $P(a)$ *to a set* $P^{\mathcal{I}}(a^{\mathcal{I}}) \subseteq \Delta^{\mathcal{I}}$*.*

The syntax and semantics of FCDLs are showed in Table 1. Table 1 does not list all available constructors, but only selects the most common ones.

From the semantics, it is clear that the interpretation of $[C \bowtie n]^{\mathcal{I}}(s)$ and $[C \bowtie D]^{\mathcal{I}}(s)$ do not depend on s. For any cut P and individual name a, $P(a)$ is an absolute cut, and $(P(a))^{\mathcal{I}} = P^{\mathcal{I}}(a^{\mathcal{I}})$. If a cut P contains no $\uparrow$, then $P^{\mathcal{I}}(s)$ is independent of s. So P itself is an absolute cut.

With the cuts and new constructors, FCDLs are more expressive than the current fuzzy DLs. FCDLs support all kinds of comparison expressions. They enable representation of expressive fuzzy knowledge on the semantic web.

2.3 Knowledge Bases and Reasoning

Definition 5. *A* knowledge base (KB) *of FCDLs is consists of an ABox, a TBox and a RBox:*

An ABox *is a finite set of concept assertions of the form* $\langle a : C \bowtie n \rangle$*, role assertions of the form* $\langle (a,b) : R \bowtie n \rangle$*, and cut asserions of the form* $\langle a : P(b) \rangle$*. An interpretation* $\mathcal{I}$ *satisfies an ABox* $\mathcal{A}$ *iff* $\mathcal{I}$ *satisfies each assertion in* $\mathcal{A}$*; such* $\mathcal{I}$ *is called a model of* $\mathcal{A}$*.* $\mathcal{I}$ *satisfies* $\langle a : C \bowtie n \rangle$ *iff* $C^{\mathcal{I}}(a^{\mathcal{I}}) \bowtie n$*,* $\mathcal{I}$ *satisfies* $\langle (a,b) : R \bowtie n \rangle$ *iff* $R^{\mathcal{I}}(a^{\mathcal{I}}, b^{\mathcal{I}}) \bowtie n$*,* $\mathcal{I}$ *satisfies* $\langle a : P(b) \rangle$ *iff* $a^{\mathcal{I}} \in P^{\mathcal{I}}(b^{\mathcal{I}})$*.*

An TBox *is a finite set of concept inclusions of the form* $C \sqsubseteq D$*, and cut inclusions of the form* $P \sqsubseteq Q$*.* $\mathcal{I}$ *satisfies a TBox* $\mathcal{T}$ *iff* $\mathcal{I}$ *satisfies each inclusion in* $\mathcal{T}$*; such* $\mathcal{I}$ *is called a model of* $\mathcal{T}$*.* $\mathcal{I}$ *satisfies* $C \sqsubseteq D$ *iff for any* $s \in \Delta^{\mathcal{I}}$*,* $C^{\mathcal{I}}(s) \leq D^{\mathcal{I}}(s)$*,* $\mathcal{I}$ *satisfies* $P \sqsubseteq Q$ *iff* $P^{\mathcal{I}} \subseteq Q^{\mathcal{I}}(s)$*.*

An RBox *(or called role hierarchy) is a finite set of role inclusions of the form* $R \sqsubseteq S$*.* $\mathcal{I}$ *satisfies an RBox* $\mathcal{R}$ *iff* $R^{\mathcal{I}}(s,t) \leq S^{\mathcal{I}}(s,t)$ *for each* $R \sqsubseteq S \in \mathcal{R}$*; such* $\mathcal{I}$ *is called a model of* $\mathcal{R}$*.*

For a knowledge base $\mathcal{K} = \langle \mathcal{A}, \mathcal{T}, \mathcal{R} \rangle$*, if* $\mathcal{I}$ *is a model of* $\mathcal{T}$*,* $\mathcal{R}$ *and* $\mathcal{A}$*, then* $\mathcal{I}$ *is called a model of* $\mathcal{K}$

Definition 6. *The basic inference problems of FCDLs include*

- Satisfiability of concepts*: a concept* C *is satisfiable w.r.t. a TBox* $\mathcal{T}$ *and a RBox* $\mathcal{R}$ *to a given degree* n*, iff there exists a model* $\mathcal{I}$ *of* $\mathcal{T}$ *and* $\mathcal{R}$ *with* $\exists s \in \Delta^{\mathcal{I}}, C^{\mathcal{I}}(s) \geq n$*.*
- Consistency of ABoxes*: an ABox* $\mathcal{A}$ *is consistent w.r.t.* $\mathcal{T}$ *and* $\mathcal{R}$*, iff there exists a model of* $\mathcal{T}$*,* $\mathcal{R}$ *and* $\mathcal{A}$*.*

$\mathcal{ALC}_{fc}$ is the most basic FCDL. For any $\mathcal{ALC}_{fc}$-role R, $R \in N_{R^P}$; and $\mathcal{ALC}_{fc}$-concepts are $C, D ::= A|\top|\bot|\neg C|C \sqcap D|C \sqcup D|\exists R.C|\forall R.C|\exists R.P|\forall R.P$. The $\mathcal{ALC}_{fc}$-cuts are $P, Q ::= [C \bowtie n]|[C \bowtie D]|[C \bowtie D^{\uparrow}]|\neg P|P \sqcap Q|P \sqcup Q$.

Table 1. Syntax and semantics of FCDLs

Syntax	Semantics	Symbol
R	$R^{\mathcal{I}}(s,t) \in [0,1]$	
R^-	$(R^-)^{\mathcal{I}}(s,t) = R^{\mathcal{I}}(t,s)$	$\mathcal{I}$
$R \in N_{R+}$	$\sup_{t_i \in \Delta^{\mathcal{I}}}\{R^{\mathcal{I}}(s_1,t_i) \wedge R^{\mathcal{I}}(t_i,s_2) \leq R^{\mathcal{I}}(s_1,s_2)$	$\mathcal{R}^+(\mathcal{S})$
A	$A^{\mathcal{I}}(s) \in [0,1]$	
$\top$	$\top^{\mathcal{I}}(s) = 1$	
$\bot$	$\bot^{\mathcal{I}}(s) = 0$	
$\neg A$	$(\neg A)^{\mathcal{I}}(s) = 1 - A^{\mathcal{I}}(s)$	$\mathcal{AL}(\mathcal{S})$
$C \sqcap D$	$(C \sqcap D)^{\mathcal{I}}(s) = \min(C^{\mathcal{I}}(s), D^{\mathcal{I}}(s))$	$\mathcal{AL}(\mathcal{S})$
$C \sqcup D$	$(C \sqcup D)^{\mathcal{I}}(s) = \max(C^{\mathcal{I}}(s), D^{\mathcal{I}}(s))$	$\mathcal{U}(\mathcal{S})$
$\neg C$	$(\neg C)^{\mathcal{I}}(s) = 1 - C^{\mathcal{I}}(s)$	$\mathcal{C}(\mathcal{S})$
$\forall R.C$	$(\forall R.C)^{\mathcal{I}}(s) = \inf_{t \in \Delta^{\mathcal{I}}}\{\max(1 - R^{\mathcal{I}}(s,t), C^{\mathcal{I}}(t))\}$	$\mathcal{AL}(\mathcal{S})$
$\exists R.C$	$(\exists R.C)^{\mathcal{I}}(s) = \sup_{t \in \Delta^{\mathcal{I}}}\{\min(R^{\mathcal{I}}(s,t), C^{\mathcal{I}}(t))\}$	$\mathcal{E}(\mathcal{S})$
$\geq qR$	$(\geq qR)^{\mathcal{I}}(s) = \sup\limits_{t_1,\ldots,t_q \in \Delta^{\mathcal{I}}} \min_{i=1}^{q}\{R^{\mathcal{I}}(s,t_i)\}$	$\mathcal{N}$
$\leq qR$	$(\leq qR)^{\mathcal{I}}(s) = \inf\limits_{t_1,\ldots,t_{q+1} \in \Delta^{\mathcal{I}}} \max_{i=1}^{q+1}\{1 - R^{\mathcal{I}}(s,t_i)\}$	
$\geq qR.C$	$(\geq qR.C)^{\mathcal{I}}(s) = \sup\limits_{t_1,\ldots,t_q \in \Delta^{\mathcal{I}}} \min_{i=1}^{q}\{R^{\mathcal{I}}(s,t_i), C^{\mathcal{I}}(t_i)\}$	$\mathcal{Q}$
$\leq qR.C$	$(\leq qR.C)^{\mathcal{I}}(s) = \inf\limits_{t_1,\ldots,t_{q+1} \in \Delta^{\mathcal{I}}} \max_{i=1}^{q+1}\{1 - R^{\mathcal{I}}(s,t_i), C^{\mathcal{I}}(t_i)\}$	
$\{o\}$	$\{o\}^{\mathcal{I}}(s) = \begin{cases} 1 \text{ if } s = o^{\mathcal{I}} \\ 0 \text{ if } s \neq o^{\mathcal{I}} \end{cases}$	$\mathcal{O}$
$\forall R.P$	$(\forall R.P)^{\mathcal{I}}(s) = \inf_{t \in P^{\mathcal{I}}(x)}\{1 - R^{\mathcal{I}}(s,t)\}$	$\mathcal{AL}(\mathcal{S})$
$\exists R.P$	$(\exists R.P)^{\mathcal{I}}(s) = \sup_{t \in P^{\mathcal{I}}(x)}\{R^{\mathcal{I}}(s,t)\}$	$\mathcal{E}(\mathcal{S})$
$\geq qR.P$	$(\geq qR.P)^{\mathcal{I}}(s) = \sup\limits_{t_1,\ldots,t_q) \in P^{\mathcal{I}}(x)} \min_{i=1}^{q}\{R^{\mathcal{I}}(s,t_i)\}$	$\mathcal{Q}$
$\leq qR.P$	$(\leq qR.P)^{\mathcal{I}}(s) = \inf\limits_{t_1,\ldots,t_{q+1} \in P^{\mathcal{I}}(x)} \max_{i=1}^{q+1}\{1 - R^{\mathcal{I}}(s,t_i)\}$	$\mathcal{Q}$
$[C \bowtie n]$	$[C \bowtie n]^{\mathcal{I}}(s) = \{t \mid C^{\mathcal{I}}(t) \bowtie n\}$	
$[C \bowtie D]$	$[C \bowtie D]^{\mathcal{I}}(s) = \{t \mid C^{\mathcal{I}}(t) \bowtie D^{\mathcal{I}}(t)\}$	
$[C \bowtie D^{\uparrow}]$	$[C \bowtie D^{\uparrow}]^{\mathcal{I}}(s) = \{t \mid C^{\mathcal{I}}(t) \bowtie D^{\mathcal{I}}(s)\}$	
$\neg P$	$(\neg P)^{\mathcal{I}}(s) = \Delta^{\mathcal{I}} \setminus P^{\mathcal{I}}(s)$	
$P \sqcap Q$	$(P \sqcap Q)^{\mathcal{I}}(s) = P^{\mathcal{I}}(s) \cap Q^{\mathcal{I}}(s)$	$\mathcal{AL}(\mathcal{S})$
$P \sqcup Q$	$(P \sqcup Q)^{\mathcal{I}}(s) = P^{\mathcal{I}}(s) \cup Q^{\mathcal{I}}(s)$	$\mathcal{U}(\mathcal{S})$
$R \sqsubseteq S$	$\forall s,t \in \Delta^{\mathcal{I}},\ R^{\mathcal{I}}(s,t) \leq S^{\mathcal{I}}(s,t)$	$\mathcal{H}$
$(a,b) : R \bowtie n$	$R^{\mathcal{I}}(a^{\mathcal{I}}, b^{\mathcal{I}}) \bowtie n$	
$C \sqsubseteq D$	$\forall s \in \Delta^{\mathcal{I}}, C^{\mathcal{I}}(s) \leq D^{\mathcal{I}}(s)$	
$a : C \bowtie n$	$C^{\mathcal{I}}(a^{\mathcal{I}}) \bowtie n$	
$P \sqsubseteq Q$	$P^{\mathcal{I}} \subseteq Q^{\mathcal{I}}$ where P, Q are absolute cuts	
$a : P(b)$	$a^{\mathcal{I}} \in P^{\mathcal{I}}(b^{\mathcal{I}})$	

3 Reasoning Algorithm

Here presents an algorithm to decide the consistency for $\mathcal{ALC}_{fc}$ ABoxes by constructing completion graphs.

Definition 7. *A* completion graph *is* $T = \langle S, E, L, \delta \rangle$*, where* S *is a set of nodes in the graph.* E *is a set of edges (pairs of nodes) in the graph.* L *is a function:*

R1	if	$\neg C \in L(x)$, and not $C(x) =_\delta 1 - (\neg C)(x)$
	then	$L(x) \to L(x) \cup \{C\}$, and $C(x) =_\delta 1 - (\neg C)(x)$
R2	if	$C \sqcap D \in L(x)$, and not $\min(C(x), D(x)) =_\delta (C \sqcap D)(x)$
	then	$L(x) \to L(x) \cup \{C, D\}$, and $\min(C(x), D(x)) =_\delta (C \sqcap D)(x)$
R3	if	$C \sqcup D \in L(x)$, and not $(C \sqcup D)(x) =_\delta 1 - (\neg C \sqcap \neg D)(x)$
	then	$L(x) \to L(x) \cup \{C \sqcup D\}$, and $(C \sqcup D)(x) =_\delta 1 - (\neg C \sqcap \neg D)(x)$
R4	if	$\exists R.C \in L(x)$, and there is y with $R \in L(x, y)$
		but not $X \leq_\delta (\exists R.C)(x)$ for some $X \in \{R(x, y), C(x)\}$
	then	$L(y) \to L(y) \cup \{C\}$, and $X \leq_\delta (\exists R.C)(x)$
R5	if	$\exists R.C \in L(x)$, and there is no y with $X =_\delta (\exists R.C)(x)$
		or $X \lessdot_\delta (\exists R.C)(x)$, for some $X \in \{R(x, y), C(x)\}$
	then	add a new node y with $L(x, y) = \{R\}, L(y) = \{C\}$,
		and $X =_\delta (\exists R.C)(x)$ or $X \lessdot_\delta (\exists R.C)(x)$
R6	if	$\forall R.C \in L(x)$, and not $(\forall R.C)(x)_\delta = 1 - (\exists R.\neg C)(x)$
	then	$L(x) \to L(x) \cup \{\exists R.\neg C\}$, and $(\forall R.C)(x) =_\delta 1 - (\exists R.\neg C)(x)$
R7	if	$\exists R.P \in L(x)$, and there is y with $R \in L(x, y)$
		but not $R(x, y)_\delta \leq (\exists R.C)(x)$ nor $\neg P(x)_\delta \in L(y)$
	then	$R(x, y) \leq_\delta (\exists R.C)(x)$, or $L(y) \to L(y) \cup \{\neg P(x)\}$
R8	if	$\exists R.P \in L(x)$, and there is no y with $P(x) \in L(y)$,
		$R(x, y) =_\delta (\exists R.C)(x)$ or $R(x, y) \lessdot_\delta (\exists R.C)(x)$
	then	add a new node y with $L(x, y) = \{R\}, L(y) = \{P(x)\}$,
		and $R(x, y) =_\delta (\exists R.C)(x)$ or $R(x, y) \lessdot_\delta (\exists R.C)(x)$
R9	if	$\forall R.P \in L(x)$, and not $(\forall R.P)(x) =_\delta 1 - (\exists R.\neg P)(x)$
	then	$L(x) \to L(x) \cup \{\exists R.\neg P\}$, and $(\forall R.P)(x) =_\delta 1 - (\exists R.\neg P)(x)$
R10	if	$[C \bowtie n] \in L(x)$, and not $C(x) \bowtie_\delta n$
	then	$L(x) \to L(x) \cup \{C\}$, and $C(x) \bowtie_\delta n$
R11	if	$[C \bowtie D] \in L(x)$, and not $C(x) \bowtie_\delta D(x)$
	then	$L(x) \to L(x) \cup \{C, D\}$, and $C(x) \bowtie_\delta D(x)$
R12	if	$[C \bowtie D^\uparrow](y) \in L(x)$, and not $C(x) \bowtie_\delta D(y)$
	then	$L(x) \to L(x) \cup \{C\}$, $L(y) \to L(y) \cup \{D\}$, and $C(x) \bowtie_\delta D(y)$
R13	if	$(P \sqcap Q)(y) \in L(x)$, and not $\{P(y), Q(y)\} \subseteq L(x)$
	then	$L(x) \to L(x) \cup \{P(y), Q(y)\}$
R14	if	$(P \sqcup Q)(y) \in L(x)$, and $\{P(y), Q(y)\} \cap L(x) = \emptyset$
	then	$L(x) \to L(x) \cup \{X\}$ for some $X \in \{P(y), Q(y)\}$
R15	if	$C \sqsubseteq D \in \mathcal{T}$, and there is x with no $C(x) \leq_\delta D(x)$
	then	$L(x) \to L(x) \cup \{C, D\}$, and $C(x) \leq_\delta D(x)$
R16	if	$C \sqsubset D \in \mathcal{T}$, and there is x with no $C(x) <_\delta D(x)$
	then	$L(x) \to L(x) \cup \{C, D\}$, and $C(x) <_\delta D(x)$
R17	if	$C \in L(x)$ or $R \in L(x, y)$, and let $X = C(x)$ or $R(x, y)$
		there is no i such that $v_i <_\delta X <_\delta v_{i+1}$, or $X =_\delta v_i$
	then	$v_i <_\delta X <_\delta v_{i+1}$ for some v_i, v_{i+1}, or $X =_\delta v_i$ for some v_i

Fig. 1. Expansion rules for $\mathcal{ALC}_{fc}$

for every node $x \in S$*,* $L(x)$ *is a set of concepts or absolute cuts; for every edge* $(x, y) \in E$*,* $L(x, y)$ *is a set of roles.* δ *is a set of formulas of the form* $X \leq Y$*,* $X \neq Y$ *or* $X \lessdot Y$*, where* $X, Y ::= n|C(x)|R(x, y)|1 - X$ *such that* $n \in [0, 1]$*,* C *is a concept,* R *is a role,* $x, y \in S$*, and for any* X*,* $1 - (1 - X) = X$*.*

The completion graph T of an ABox $\mathcal{A}$ w.r.t. a TBox $\mathcal{T}$ initializes with: $S = \{a \in N_I | a \text{ occurs in } \mathcal{A}\}$; for any $\langle a : P(b)\rangle \in \mathcal{A}$, $P(b) \in L(a)$; for any $\langle (a, b) : R \bowtie n\rangle \in \mathcal{A}$, $R \in L(a, b)$ and $R \bowtie_\delta n$. Let $V_0 = \{v_1, v_2, \ldots, v_k\} = \{0, 1, 0.5\} \cup \{n \in [0, 1]|n$ or $1 - n$ occurs in $\mathcal{A}$ or $\mathcal{T}\}$, where $0 = v_1 < v_2 < \cdots < v_k = 1$. For any $v_i < v_j$, let $v_i <_\delta v_j$. Then the graph grows up by applying the *expansion rules* showed in Fig. 1. If a rule applied to x creates a new node y, then y is a *successor* of x. Let *descendant* be transitive closure of successor. Several abbreviations are defined below:

$$X \leq_\delta Y =_{def} X \leq Y \in \delta, \text{ or } X \leq_\delta Y, Y \leq_\delta Z, \text{ or } 1 - Y \leq_\delta 1 - X$$

$$\min(X, Y) =_\delta Z =_{def} Z \leq_\delta X, Z \leq_\delta Y, W \leq_\delta Z \text{ for some } W \in \{X, Y\};$$

$$X \neq_\delta Y =_{def} X \neq Y \in \delta; \qquad X \lessdot_\delta Y =_{def} X \lessdot Y \in \delta;$$

$$X \geq_\delta Y =_{def} Y \leq_\delta X; \qquad X =_\delta Y =_{def} X \leq_\delta Y, Y \leq_\delta X;$$

$$X <_\delta Y =_{def} X \leq_\delta Y, X \neq_\delta Y; \quad X_\delta > Y =_{def} Y \leq_\delta X, X \neq_\delta Y.$$

The $\lessdot_\delta$ relation is used to simulate the infinite supreme. For any $a \in N_I$, $\text{lev}(a) = 1$. If $\text{lev}(x) = i$, y is a successor of x by updating $\lessdot_\delta$, then $\text{lev}(y) = i+1$. For any X of the form $C(x)$, $1 - C(x)$, $R(y, x)$, or $1 - R(y, x)$, if $\text{lev}(x) = i$, then $X \in V_i$. If $X \lessdot_\delta Y$ and $Y \in V_i$, then for any $Z \in V_j$ such that $j \leq i$, $Z <_\delta Y \to Z <_\delta X$ and $Z >_\delta X \to Z \geq_\delta Y$. So $X \lessdot_\delta Y$ means X is greater than any $Z < Y$ such that $Z \in V_0 \cup V_1 \cup \cdots \cup V_i$ and $Y \in V_i$. It ensures that for any given constant ε, we can assign values to the variables in V such that $X - Y < \varepsilon$ without inducing any conflict.

Since there are variables, the blocking condition in $\mathcal{ALC}_{fc}$ is different from classical DLs. It has to consider the comparisons between degrees. For any x, let $\delta(x) = \{X \bowtie Y | X \bowtie_\delta Y$, X, Y are of the form $C(x)$, $1 - C(x)$, or v_i. A node x is *blocked* by y, iff x is an descendant of y, and $\delta(x) = [x/y]\delta(y)$, where $[x/y]\delta(y)$ means to replace any y in $\delta(y)$ by x. Then we call *y blocks x*. When x is blocked, all descendants of x is also blocked. No rules in Fig. 1 can be applied to blocked nodes. T is said to contain a *clash* if $\{X \neq_\delta Y, X =_\delta Y\} \subseteq \delta$, or $X >_\delta 1$, or $X <_\delta 0$. T is said to be *clash-free* if it contains no clash. If none of the expansion rules can be applied to T, then T is said to be *complete*.

From the blocking condition and the number of concepts in any $L(x)$ is finite, the algorithm terminates. An $\mathcal{ALC}_{fc}$ ABox $\mathcal{A}$ is consistent w.r.t. TBox $\mathcal{T}$ iff a complete and clash-free completion graph can be constructed from $\mathcal{A}$ w.r.t. $\mathcal{T}$.

4 Conclusions

It is important to compare fuzzy membership degrees in representation of expressive fuzzy knowledge. This paper defines fuzzy comparison cuts to represent comparison expressions on fuzzy membership degrees, and extends fuzzy DLs by importing them and introducing new constructors. They enable representation and reasoning for expressive fuzzy knowledge on the semantic web. The future work is to design reasoning algorithms for more expressive FCDLs, implement reasoners for FCDLs and construct fuzzy knowledge systems based on FCDLs.

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