SAT, UNSAT and Coloring

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In this survey, we study recent developments on the CNF satisfiability problem. The first one is about deterministic (and of course exponential-time) algorithms for k-SAT. The most recent improvement for $k = 3$ and 4 is based on the nontrivial combination of the Schöning's local search algorithm and the backtrack-type algorithm by Paturi, Pudlák, Saks, and Zane. This approach is due to Iwama and Tamaki and the current fastest algorithm, based on the same method, is due to Rolf, which runs in time $O(1.32216^n)$.

The second topic is on the inapproximability of MAX-3SAT and related problems, based on the famous PCP Theory. Recently, there was an important progress in this field, the Unique Games Conjecture (UGC), by Khot. UGC implies several optimal inapproximability results, such as Vertex Cover.

The third one is the proof complexity of unsatisfiable formulas. Whether or not extended Frege systems, the most powerful proof systems ever known, are polynomially bounded is a most important open question in this field. Pitassi and Urquhart proved that the above open question is equivalent to whether the Hajós calculus, which is a simple, nondeterministic procedure for generating non-3-colorable graphs, is polynomially bounded. Thus, the famous open question in proof complexity is beautifully linked to the open question in graph theory; in order to prove superpolynomial lower bounds for the extended Frege systems, it now suffices to find a "hard example" from the set of non-3-colorable graphs. Thanks to the long and extensive research history of graph theory and graph algorithms, this is hopefully easier than finding a hard example from the set of formulas. Recently Iwama and Tamaki made another step toward this direction by showing that it still suffices if Hajós calculus is restricted to within the class of planar graphs, not only for the final graph but also intermediate ones.

* Supported in part by Scientific Research Grant, Ministry of Japan, 16300002 and 19200001.

H. Kleine Büning and X. Zhao (Eds.): SAT 2008, LNCS 4996, p. 153, 2008.